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# Field-theoretic methods in nuclear physics: A reappraisal

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**Abstract.** We discuss here some non-perturbative techniques of field theory as applied to nuclear physics. We first describe Walecka's mean field approach and its generalizations with a many-body description. We next apply to nuclear matter a new approach to strongly interacting systems, of which Walecka's method becomes a specific approximation. The phenomenological implications of the same are also discussed.

**Keywords.** Field theory; nuclear matter; non-perturbative methods.

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## 1. Introduction

Field theoretic techniques in nuclear physics started with the advent of meson theory by Yukawa (1935) to explain the short range nature of the nuclear forces. At present this method has many facets and assumptions, and we shall discuss some of these here.

The nucleus is highly complicated. In fact even the nucleons which are made up of quarks and gluons interact non-perturbatively and are understood only at a phenomenological level. They may have an effective interaction through mesons or have a residual interaction due to the substructure of quarks and gluons. We shall here consider the effective meson interactions at a field theoretic level.

The paper is organized as follows. In §2 we first recapitulate the mean field approach of Walecka (1974) and Chin and Walecka (1974). We then discuss the general self-consistent approaches using Schwinger Dyson equations (Horowitz and Serot 1983, ter Haar and Malfliet 1987) and touch upon the general philosophy of the Bonn school (Machleidt *et al* 1987). In §3 we apply to nuclear matter a new approach to strongly interacting systems (Misra 1987a, b). The mean field approach of Walecka with classical fields becomes a particular approximation of the same. In §4 we discuss the results, and indicate how we may apply the same to finite nuclear systems.

## 2. Conventional methods and their limitations

We discuss here the mean field approach and then some aspects of many body techniques (Fetter and Walecka 1971).

### 2.1 *Mean-field approach*

We shall first discuss the mean field approach of Walecka as an extremely simple and nonperturbative field theoretic technique of looking at nuclear matter (Serot and

Walecka 1986). One takes here the interaction of nucleons with a neutral scalar field corresponding to the $\sigma$-particle which gives rise to the attractive force, and the neutral vector meson, the $\omega$, which yields the repulsion. Thus the Lagrangian density becomes (with $\hbar = c = 1$),

$$\mathscr{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - M)\psi + \tfrac{1}{2}[(\partial_\mu \phi)(\partial^\mu \phi) - \mu^2 \phi^2] \\ - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}m^2 V_\mu V^\mu] \\ + g_s \bar{\psi}\psi\phi + g_v \bar{\psi}\gamma^\mu \psi A_\mu. \tag{1}$$

When $m > \mu$ and $g_v > g_s$, the potential is repulsive at short distances, and is attractive at large distances.

For nuclear matter ground state given as $|\psi_0\rangle$, we now proceed to consider a trial solution with

$$\langle \psi_0 | \phi | \psi_0 \rangle = \phi_0; \quad \langle \psi_0 | V^\mu | \psi_0 \rangle = \delta_0^\mu V^0. \tag{2}$$

The above are related to the baryonic densities

$$\rho_B = \langle \psi_0 | \psi^+ \psi | \psi_0 \rangle; \quad \rho_s = \langle \psi_0 | \bar{\psi}\psi | \psi_0 \rangle. \tag{3}$$

We may note in particular $V^0 = (g_v/m^2)\rho_B$. The thermodynamic quantities, pressure and energy density, are given in the covariant form as (Weinberg 1972)

$$\langle \psi_0 | T_{\mu\nu} | \psi_0 \rangle = -P g_{\mu\nu} + (\rho + P) u_\mu u_\nu, \tag{4}$$

where the $u$'s are covariant velocities. This yields

$$P = \tfrac{1}{3}\langle \psi_0 | T_i^i | \psi_0 \rangle; \quad \varepsilon = \rho = \langle \psi_0 | T_{00} | \psi_0 \rangle. \tag{5}$$

We may now expand the nucleon field operator as

$$\psi(\mathbf{x}, 0) = (2\pi)^{-3/2} \int \mathrm{d}\mathbf{k}\, \tilde{\psi}_r(\mathbf{k}) U_r(\mathbf{k}) \exp(i\mathbf{k}\cdot\mathbf{x}) \tag{6}$$

with the normalizations

$$U_r(\mathbf{k})^\dagger U_s(\mathbf{k}) = \delta_{rs}, \tag{7a}$$

$$\bar{U}_r(\mathbf{k}) U_s(\mathbf{k}) = \delta_{rs} M^*/(\mathbf{k}^2 + M^{*2})^{1/2}. \tag{7b}$$

In the above, $M^*$ is the effective mass of the nucleons in nuclear matter. From (1) and (3) we obtain

$$M^* = M - g_s \phi_0 = M - \frac{g_s}{\mu^2}\rho_s. \tag{8}$$

We now assume nuclear matter to be at zero temperature. From (3) and (6) we have

$$\rho_B = \langle \psi_0 | \psi(\mathbf{x})^\dagger \psi(\mathbf{x}) | \psi_0 \rangle \\ = \frac{\gamma}{(2\pi)^3} \int_0^{k_F} \mathrm{d}\mathbf{k} = \gamma k_F^3 / 6\pi^2. \tag{9}$$

In the above, $k_F$ is the Fermi momentum, and $\gamma = 4$ for nucleons. Similarly, for the scalar density $\rho_s$ we have

$$\rho_s = \langle \psi_0 | \bar{\psi}(\mathbf{x}) \psi(\mathbf{x}) | \psi_0 \rangle$$

$$= \frac{\gamma}{(2\pi)^3} \int_0^{k_F} \frac{M^*}{(\mathbf{k}^2 + M^{*2})^{1/2}} \mathrm{d}\mathbf{k}, \tag{10}$$

with $M^*$ as in (8). The energy density, from (5) is given as

$$\varepsilon = \frac{g_v^2}{2m^2} \rho_B^2 - \frac{g_s^2}{2\mu^2} \rho_s^2 + \frac{\gamma}{(2\pi)^3} \int_0^{k_F} (\mathbf{k}^2 + M^{*2})^{1/2} \mathrm{d}\mathbf{k}. \tag{11}$$

Further, the pressure is given as

$$P = \frac{g_v^2}{2m^2} \rho_B^2 - \frac{g_s^2}{2\mu^2} \rho_s^2 + \frac{1}{3} \int_0^{k_F} \frac{\mathbf{k}^2}{(\mathbf{k}^2 + M^{*2})^{1/2}} \mathrm{d}\mathbf{k}, \tag{12}$$

which agrees with the thermodynamic expression $P = \rho_B^2 \partial(\varepsilon/\rho_B)/\partial \rho_B$.

With (8), for a given $k_F$, (10) involves a self-consistent determination of $\rho_s$, and further, $k_F$ gets determined when we consider this in conjunction with the minimization of $\varepsilon$ in (11). Thus the only input parameters are $g_s^2/\mu^2 = 12{\cdot}10\,\mathrm{fm}^2$ and $g_v^2/m^2 = 8{\cdot}86\,\mathrm{fm}^2$, which have been adjusted so as to obtain the appropriate binding energy as well as the density of nuclear matter given as

$$\varepsilon/\rho_B - M \simeq -15{\cdot}75\,\mathrm{MeV} \tag{13a}$$

and

$$k_F \simeq 1{\cdot}42\,\mathrm{fm}^{-1}. \tag{13b}$$

One may also calculate the incompressibility of nuclear matter as

$$K = k_F^2 \mathrm{d}^2(\varepsilon/\rho_B)/\mathrm{d}k_F^2 \simeq 550\,\mathrm{MeV}, \tag{14}$$

which is rather high.

In the context of later discussions, Walecka's method consists of dressing of nuclear matter with a classical field. We shall give a quantum description of the same in the next section, as well as *not* take a $\sigma$-meson which is not observed. Before that, however, in the next sub-section we shall note the relativistic Hartree Fock corrections to the above (Horowitz and Serot 1983).

## 2.2 *Self-consistent Green's functions approach*

Green's function approaches have emerged from many-body formulations of temperature-dependent field theories (Matsubara 1954; Martin and Schwinger 1959). These are governed by the baryon propagator $G$ (which may include delta with $m_\Delta = 1236\,\mathrm{MeV}$), the proper self-energy $\Sigma$, and the meson propagators $K_M$. The first two are related by the Dyson equations given as

$$G(k) = G_0(k) + G_0(k)\Sigma(k)G(k), \tag{15}$$

which has the formal solution

$$G^{-1}(k) = G_0^{-1}(k) + \Sigma(k) \\ = \gamma_\mu[k^\mu + \Sigma^\mu(k)] - [M + \Sigma^s(k)]. \quad (16)$$

In the above we have substituted (neglecting the tensor part of self-energy),

$$\Sigma(k) = \Sigma^s(k) - \gamma_\mu \Sigma^\mu(k) \\ = \Sigma^s(|\mathbf{k}|, k^0) - \gamma^0 \Sigma^0(|\mathbf{k}|, k^0) + (\boldsymbol{\gamma}, \mathbf{k})\Sigma^v(|\mathbf{k}|, k^0). \quad (17)$$

$G_0(k)$ is the free baryon propagator given as

$$G_0(k) = G_0^F(k) + G_0^D(k). \quad (18)$$

The propagation of real nucleons is given by

$$G_0^D(k) = (\gamma^\mu k_\mu + M)\frac{i\pi}{E_k^0}\delta(k^0 - E_k^0)\theta(k_F - |\mathbf{k}|). \quad (19)$$

For the description of the baryon in nuclear matter, (16) and (17) lead to the effective variables

$$M^* = M + \Sigma^s(k); \quad \mathbf{k}^* = \mathbf{k}[1 + \Sigma^v(k)], \quad (20a,b)$$

$$E_k^* = (\mathbf{k}^{*2} + M^{*2})^{1/2}; \quad k^{*\mu} = k^\mu + \Sigma^\mu(k). \quad (20c,d)$$

This enables us to give a break-up parallel to (18) for $G(k)$.

The above formulation is so far exact. We arrive at the Hartree Fock approximation when we retain for the self-energy only the tadpole term and the exchange term. We shall have, e.g. the self-energy due to scalar exchange given as

$$\Sigma(k) = ig_s^2 \int \frac{d^4q}{(2\pi)^4}\left[\frac{\exp(iq^0\varepsilon)\mathrm{Tr}[G(q)]}{\mu^2} + \frac{G(q)}{(k-q)^2 - \mu^2 + i\varepsilon}\right]. \quad (21a)$$

The first term above, the tadpole term, is related to the baryon density. The specific way in which $q^0$ integration to be performed is to be noted. When we approximate $G$ by $G^D$ and perform integrations, we obtain a self-consistent on mass-shell determination of self-energies at the single particle level. Corresponding to the Lagrangian quoted in the last subsection, we in fact have, e.g. (with $k = |\mathbf{k}|$, $q = |\mathbf{q}|$

$$\Sigma^s(k, E_k) = \frac{-\gamma}{2\pi^2}\frac{g_s^2}{\mu^2}\int_0^{k_F} q^2\,dq\,\frac{M_q^*}{E_q^*} \\ + \frac{1}{4\pi^2 K}\int_0^{k_F} q\,dq\,\frac{M_q^*}{E_q^*}\{\tfrac{1}{4}g_s^2\Theta_s(k,q) - g_v^2\Theta(k,q)\}, \quad (21b)$$

where we have the substitutions (with $m_i^2 = \mu^2, m^2$),

$$A_i(k, q) = k^2 + q^2 + m_i^2 - (E_q - E_k)^2 \quad (22a)$$

and next

$$\Theta_i(k,q) = \ln\left|\frac{A_i(k,q)+2kq}{A_i(k,q)-2kq}\right|. \tag{22b}$$

There are two other equations for $\Sigma^0(k, E_k)$ and $\Sigma^v(k, E_k)$ parallel to (21a). The single particle energies above are obtained from the transcendental equation

$$\begin{aligned} &E_k(E_k^* - \Sigma^0(k))|_{k^0=E_k} \\ &\quad = [\mathbf{k}^{*2} + M^{*2}]^{1/2} - \Sigma^0(k, E_k). \end{aligned} \tag{23}$$

One can then calculate the energy of the nuclear medium by considering (Horowitz and Serot 1983)

$$T^N_{\mu\nu} = -i\int \frac{d^4k}{(2\pi)^4}\mathrm{Tr}[\gamma_\mu G^D(k)]k_\nu \exp(ik_0\varepsilon) \tag{24}$$

and

$$T^s_{\mu\nu} = -i\int \frac{d^4k}{(2\pi)^4}[\tfrac{1}{2}(k^2-\mu^2)g_{\mu\nu} - k_\mu k_\nu]K_M(k). \tag{25}$$

One may also take the self-energy contribution of the meson as

$$\pi_s(k) = -ig_s^2\int \frac{d^4q}{(2\pi)^4}\mathrm{Tr}[G^D(q)G^D(k+q)]. \tag{26}$$

One then obtains the energy density, taking only neutral scalar and vector mesons as earlier,

$$\begin{aligned} \varepsilon = {} & \frac{\gamma}{2\pi^2}\int_0^{k_F} \mathbf{k}^2 E_k^*\, dk - \frac{g_s^2}{2\mu^2}\rho_s^2 + \frac{g_v^2}{2m^2}\rho_B^2 \\ & + \frac{\gamma}{2(2\pi)^6}\int_0^{k_F}\frac{d\mathbf{k}}{E_k^*}\int_0^{k_F}\frac{d\mathbf{q}}{E_q^*}\{g_s^2 D_s^0(k-q) \\ & \times [\tfrac{1}{2} - (E_k - E_q)^2 D_s^0(k-q)](k_\mu^* q^{*\mu} + M_k^* M_q^*) \\ & + \text{the corresponding contributions from vector mesons}\}. \end{aligned} \tag{27}$$

In the above, $D^0(k) = (k^{0^2} - \mathbf{k}^2 - \mu^2)^{-1}$, and $\rho_B$ and $\rho_s$ are given by (9) and (10). Extremization of $\varepsilon$ in (27) or any approximation of it will determine $k_F$, and hence the nuclear density. Here pressure $P$ may be calculated from the thermodynamic equation $P = \rho_B^2\partial(\varepsilon/\rho_B)/\partial\rho_B$ as stated earlier. The results do not alter very much from the simple calculations of the last subsection, and are here meant to illustrate the ideas. The renormalization effects have been absorbed in cut-off due to Fermi momentum. These were discussed in detail earlier by Chin (1977).

### 2.3 *Meson exchange theory* (*Bonn School*)

We shall now very briefly recapitulate some features of meson exchange theory of the Bonn School (Machleidt *et al* 1987). They use conventional Feynman diagrams including higher order terms, along with a phenomenological form factor given as

$$F_\alpha(k^2)=\left(\frac{\Lambda_\alpha^2-m_\alpha^2}{\Lambda_\alpha^2+k^2}\right)^{n_\alpha}, \tag{28}$$

where $\alpha$ denotes the meson considered, and $\Lambda_\alpha$ and $n_\alpha$ are suitable parameters. They include in their analysis two pion contributions. The model parameters

$$g_\pi^2/4\pi=14{\cdot}4,\quad f_{N\Delta\pi}^2/4\pi=0{\cdot}224,\quad m_\pi=138\,\text{MeV},$$
$$g_\rho^2/4\pi=0{\cdot}55,\quad f_\rho/g_\rho=6.1,\quad m_\rho=769\,\text{MeV},\quad \Gamma_\rho=154\,\text{MeV},$$
$$g_{\sigma'}^2/4\pi=10,\quad m_{\sigma'}=662{\cdot}5\,\text{MeV},\quad \Gamma_{\sigma'}=524{\cdot}5\,\text{MeV},\quad g_w^2/4\pi=5{\cdot}7,$$

are taken so as to provide a realistic description of long and intermediate range nuclear reactions: One here includes as many meson exchanges as possible.

## 3. Nuclear matter with constituent mesons

In this section we shall develop a field theoretic approach for strongly interacting systems which is non-perturbative and which has elsewhere yielded conventional results when we make a perturbative approximation (Misra 1987a, b; Mishra *et al* 1988c). We shall show that Walecka's mean field approach becomes a particulars approximation in two respects, such as $\sigma$-meson, which is not observed, need not be used, and, the description becomes quantum mechanical with no classical approximation anywhere.

To see this, let us consider the effective Lagrangian for pion nucleon interaction given as

$$\mathscr{L}=\bar\psi(i\gamma^\mu\partial_\mu-M-g\gamma_5\phi)\psi+\tfrac{1}{2}\{(\partial_\mu\phi_i)(\partial^\mu\phi_i)-\mu^2\phi_i\phi_i\}. \tag{29}$$

In the above, $\psi$ is the nucleon doublet, $\phi=\tau_i\phi_i$ and $\gamma_5=\begin{pmatrix}0 & -i\\ -i & 0\end{pmatrix}$, the other notations being obvious. We now rewrite (29) in terms of the "large" and "small" components of the nucleon. We shall use the notation $E=i(\partial/\partial t)$ and $\mathbf{p}=-i(\partial/\partial\mathbf{x})$, where $E$ will be ultimately identified as an effective Hamiltonian. Equation (29) now yields that

$$(E-M)\psi_{\rm I}-(\boldsymbol{\sigma}\cdot\mathbf{p}-ig\phi)\psi_{\rm II}=0 \tag{30}$$

and

$$(E+M)\psi_{\rm II}-(\boldsymbol{\sigma}\cdot\mathbf{p}+ig\phi)\psi_{\rm I}=0. \tag{31}$$

The above two equations then yield that

$$(E-M)\psi_{\rm I}-(\boldsymbol{\sigma}\cdot\mathbf{p}-ig\phi)(E+M)^{-1}(\boldsymbol{\sigma}\cdot\mathbf{p}+ig\phi)\psi_{\rm I}=0, \tag{32}$$

which we rewrite as

$$(E^2-M^2)\psi_{\rm I}-(E+M)(\boldsymbol{\sigma}\cdot\mathbf{p}-ig\phi)(E+M)^{-1}(\boldsymbol{\sigma}\cdot\mathbf{p}+ig\phi)\psi_{\rm I}=0. \tag{33}$$

$E$ as an effective Hamiltonian will involve space differentiation operators for nucleons.

We shall assume that the momentum scales of the meson field for nuclear matter is small compared to the nucleon mass. In that case, $E+M$ commutes with $\boldsymbol{\sigma}\cdot\mathbf{p}-ig\phi$ and we thus rewrite (33) as

$$(E^2-M^2-\mathbf{p}^2-ig((\boldsymbol{\sigma}\cdot\mathbf{p})\phi)-g^2\phi_i\phi_i)\psi_1=0. \tag{34}$$

From the above equation, we identify the effective Hamiltonian for the interacting pion nucleon system with the two-component notation as

$$\mathscr{H}_N=\psi_1(\mathbf{x})^\dagger\left[\varepsilon_x+\frac{ig}{2\varepsilon_x}((\boldsymbol{\sigma}\cdot\mathbf{p})\phi)+\frac{g^2}{2\varepsilon_x}(\phi_i\phi_i)\right]\psi_1(\mathbf{x}), \tag{35}$$

where $\varepsilon_x=(M^2-\nabla_{\mathbf{x}}^2)^{1/2}$. We may also separate the "free" and the "interaction" parts of the above Hamiltonian as

$$\mathscr{H}_N^0=\psi_1(\mathbf{x})^\dagger\varepsilon_x\psi_1(\mathbf{x}) \tag{36}$$

and

$$\mathscr{H}_{\text{int}}=ig\psi_1(\mathbf{x})^\dagger\frac{1}{2\varepsilon_x}((\boldsymbol{\sigma}\cdot\mathbf{p})\phi)\psi_1(\mathbf{x})+g^2\psi_1(\mathbf{x})^\dagger\frac{1}{2\varepsilon_x}(\phi_i\phi_i)\psi_1(\mathbf{x}). \tag{37}$$

The first term above is the usual $p$-wave coupling of the isovector pion. The second term in (37) corresponds to a scalar isoscalar coupling of nucleons with *two* $s$-wave pions, which, as we shall see, will simulate the effect of the $\sigma$-meson.

Now we shall consider nuclear matter. For the purpose of normalization we shall take $N$ nucleons occupying a spherical volume $V=4\pi R^3/3$, where $N/(4\pi R^3/3)=\rho$ remains constant as $N\to\infty$, and we neglect the surface effects. We describe this system with the density operator $\hat{\rho}_N$ such that

$$\mathrm{Tr}[\hat{\rho}_N\psi_\beta(\mathbf{y})^\dagger\psi_\alpha(\mathbf{x})]=\rho_{\alpha\beta}(\mathbf{x},\mathbf{y}), \tag{38}$$

so that

$$\mathrm{Tr}(\hat{\rho}_N\hat{N})=\int\rho_{\alpha\alpha}(\mathbf{x},\mathbf{x})\mathrm{d}\mathbf{x}=N=\rho V. \tag{39}$$

We thus have $\rho_{\alpha\alpha}(\mathbf{x},\mathbf{x})=\rho=$ constant. Throughout $\alpha$ and $\beta$ will stand for both spin and isospin indices of the two-component nucleons, where, we shall illustrate the present ideas with a non-relativistic description.

We shall now "dress" nuclear matter with mesons (Misra 1987a, b), calculate the total energy density, and minimize the same to obtain the equilibrium configuration. Conceptually, the "exact" result will correspond to considering all possible configurations. This being impossible, we shall extremize the energy density for a specific ansatz for a class of configurations. Variation over $\rho$ will then give the equation of state for nuclear matter. We note that while calculating the energy densities, in addition to the Hamiltonian densities of (36) and (37), we should also take

$$\mathscr{H}_M(\mathbf{x})=a_i(\mathbf{x})^\dagger\omega_x a_i(\mathbf{x}), \tag{40}$$

with $\omega_x=(\mu^2-\nabla_{\mathbf{x}}^2)^{1/2}$, where $a_i(\mathbf{x})^\dagger$ and $a_i(\mathbf{x})$ are the creation and annihilation operators of the pions.

We note that the two pions in (37) should simulate the $\sigma$-meson interaction. With this in mind, let us consider a two-pion creation operator given as

$$B^\dagger = \frac{1}{2}\int f(\mathbf{z}_1 - \mathbf{z}_2) a_i(\mathbf{z}_1)^\dagger a_i(\mathbf{z}_2)^\dagger \,\mathrm{d}\mathbf{z}_1\,\mathrm{d}\mathbf{z}_2, \tag{41}$$

where we have chosen a form consistent with translational invariance and the arbitrary function $f(\mathbf{r})$ will be determined by a variational procedure. With the construction of coherent states of scalar particles in mind (Misra 1987a, b), we now consider a meson "dressing" of nuclear matter through the state

$$|f\rangle = N_R \exp(B^\dagger)|\mathrm{vac}\rangle, \tag{42}$$

where $N_R$ is a normalization constant. In the construction of the above state, our objective has been to include many mesons with as few parameters or functions as possible. We take a state with an even number of pions because of parity, and because we expect to generate the effect of the $\sigma$-meson.

We now note that

$$[B, B^\dagger] = \tfrac{3}{2} b^2 V, \tag{43}$$

where we have substituted

$$b^2 = \int |f(\mathbf{x})|^2 \,\mathrm{d}\mathbf{x}. \tag{44}$$

We choose the normalization

$$\langle f|f\rangle = 1, \tag{45}$$

so that we have $N_R = \exp(-\tfrac{3}{4}b^2 V)$. We may calculate the meson number density as

$$\rho_M = \langle f|a_i(\mathbf{z})^\dagger a_i(\mathbf{z})|f\rangle = 3b^2, \tag{46}$$

which thus gives a meaning to the constant $b$ in (44).

From (40) we now obtain the kinetic energy density due to the mesons as

$$\begin{aligned} h_K &= \langle f|\mathscr{H}_M(\mathbf{z})|f\rangle \\ &= 3\int \tilde{f}(\mathbf{q})^* \tilde{f}(\mathbf{q})\omega(\mathbf{q})\mathrm{d}\mathbf{q}. \end{aligned} \tag{47}$$

We have obviously written the above in the momentum space. We next proceed to evaluate from (37) the interaction energy density

$$h_{\mathrm{int}} = \langle f|\mathscr{H}_{\mathrm{int}}(\mathbf{z})|f\rangle. \tag{48}$$

For this purpose we first note that

$$:\phi_i(\mathbf{x})\phi_i(\mathbf{x}): = \{\phi_i^{\mathrm{an}}(\mathbf{x})\phi_i^{\mathrm{an}}(\mathbf{x}) + \phi_i^{\mathrm{cr}}(\mathbf{x})\phi_i^{\mathrm{cr}}(\mathbf{x})\} + 2\phi_i^{\mathrm{cr}}(\mathbf{x})\phi_i^{\mathrm{an}}(\mathbf{x}), \tag{49}$$

where we have substituted e.g.

$$\phi_i^{\mathrm{an}}(\mathbf{x}) = \frac{1}{(2\omega_x)^{1/2}} a_i(\mathbf{x}).$$

Clearly we have on simplification

$$h_{\text{int}} = h_{\text{int}}^{(1)} + h_{\text{int}}^{(2)}$$

where

$$h_{\text{int}}^{(1)} = \frac{G^2}{2M} \cdot \rho \cdot (2\pi)^{-3/2} \cdot \frac{3}{2} \int \{ \tilde{f}(\mathbf{q})^* + \tilde{f}(\mathbf{q}) \} \frac{d\mathbf{q}}{\omega(\mathbf{q})}, \tag{50}$$

and

$$h_{\text{int}}^{(2)} = \frac{G^2}{2M} \cdot \rho \cdot 3 \int \tilde{f}(\mathbf{q})^* \tilde{f}(\mathbf{q}) \frac{d\mathbf{q}}{\omega(\mathbf{q})}. \tag{51}$$

With a conventional assumption of zero temperature for nuclear matter, we also obtain the total nucleon energy as

$$h_F = \frac{\gamma}{6\pi^2} k_F^3 \left( M + \frac{3}{10M} k_F^2 \right), \tag{52}$$

where as before $\gamma = 4$.

In the three equations (47), (50) and (51) we may vary with respect to $\tilde{f}(\mathbf{q})^*$ and thus obtain the solution

$$\tilde{f}(\mathbf{q}) = -\frac{G^2}{2M} \cdot \frac{\rho}{2} \cdot (2\pi)^{-3/2} \left[ \omega(\mathbf{q})^2 + \frac{G^2}{2M} \rho \right]^{-1}. \tag{53}$$

We then obtain the corresponding energy density from the kinetic and interaction terms as

$$\begin{aligned} h_M &= h_K + h_{\text{int}}^{(1)} + h_{\text{int}}^{(2)} \\ &= -\frac{3}{4} \cdot \left( \frac{G^2}{2M} \right)^2 \cdot \rho^2 \cdot (2\pi)^{-3} \int \frac{d\mathbf{q}}{\omega(\mathbf{q}) \left[ \omega(\mathbf{q})^2 + \frac{G^2}{2M} \rho \right]}. \end{aligned} \tag{54}$$

We now note that (54) is not acceptable since the energy density diverges. Further, (53) gives a short distance correlation for the two pions as compared to their size, and thus cannot be accepted. This happens because we have taken the pions to be point-like, and assumed that they can approach as near each other as possible, which is physically not correct. We now introduce the effect of compositeness in an ad hoc manner, and replace (54) by

$$h_M = -\frac{3}{2} \cdot \frac{(G^2/4\pi)^2}{M^2} \cdot \rho^2 \int \frac{q^2\, dq}{\omega(\mathbf{q}) \left[ \omega(\mathbf{q})^2 + \frac{G^2}{2M} \rho \right] [1 + \frac{1}{6} R_\pi^2 q^2]}, \tag{55}$$

where the last factor is introduced as a form factor to suppress the high momenta. The pion number density then becomes

$$\rho_M = 3 \cdot \frac{(G^2/4\pi)^2}{2M^2} \cdot \rho^2 \int \frac{q^2\, dq}{\left( \omega(\mathbf{q})^2 + \frac{G^2}{2M} \rho \right)^2 (1 + \frac{1}{6} R_\pi^2 q^2)^2}. \tag{56}$$

We now proceed to extremize the energy density as a function of the nucleon density $\rho$. This now consists of the kinetic energy part of the nucleon as in (52), and the energy density optimized over the pion configuration as given in (55). As advertised earlier, (55) simulates the effect of the $\sigma$-meson. We next have to include the energy of repulsion, which may arise from $\omega$-interactions as in Walecka (1974), or from the composite nature of nucleons. We parametrize the effect of such a repulsive contribution by the simple form

$$h_R = \lambda \rho^2, \tag{57}$$

where $\lambda$ is an arbitrary constant to be fixed from phenomenology. We note that (57) can arise from a Hamiltonian density given as

$$\mathscr{H}_R(\mathbf{x}) = \psi(\mathbf{x})^\dagger \psi(\mathbf{x}) \int v_R(\mathbf{x}-\mathbf{y})\psi(\mathbf{y})^\dagger \psi(\mathbf{y})\,d\mathbf{y}, \tag{58}$$

where, when the density is constant, we in fact have

$$\lambda = \int v_R(\mathbf{r})d\mathbf{r}. \tag{59}$$

We next minimize the energy density per nucleon given by

$$E = (h_M + h_F + h_R)/\rho, \tag{60}$$

as a function of $\rho$ or $k_F$ related by the equation $\rho = (\gamma/6\pi^2)k_F^3$ with the parameters $R_\pi$ and $\lambda$ to be subsequently fixed. For $\lambda = 0{\cdot}04\,\mathrm{fm}^2$ and $R_\pi = 1{\cdot}8\,\mathrm{fm}$, we then obtain $k_F = 1{\cdot}42\,\mathrm{fm}^{-1}$ corresponding to $\rho = 0{\cdot}19\,\mathrm{fm}^{-3}$ along with single particle energy being given as $E = -15\,\mathrm{MeV}$. The corresponding equation of state is of the conventional form. Further the incompressibility becomes (Blaizot *et al* 1976)

$$K = k_F^2(\partial^2 E/\partial k_F^2) \approx 91\,\mathrm{MeV}. \tag{61}$$

Thus nuclear matter appears to be rather too soft. Further we can make an estimate of $g_\omega$ if we identify the repulsion term as completely due to $\omega$-exchange. Then we shall have, in fact, $g_\omega^2/4\pi = \lambda m_\omega^2/(2\pi)$. This yields the value of $g_\omega^2/4\pi \simeq 0{\cdot}09$ for $m_\omega \simeq 770\,\mathrm{MeV}$ and with the present parameters, which again is too small. Thus nuclear matter becomes equivalent to nucleons with number density around $0{\cdot}19\,\mathrm{fm}^{-3}$ along with "dipions" with pion density being as small as 0·01 pions per nucleon.

The arbitrary feature of the present scheme is the ad hoc manner for the introduction of the pion form factor. We may contrast this with the obvious comment that if we bring two pions very close to each other, there will be an effective force of repulsion since they are composite, and the system is no longer in equilibrium. Such a repulsive effect should generate what we have approximated with a form factor. We then may have a form for $\mathscr{J}(\mathbf{q})$ as, with $a$ and $R_\pi$ as parameters

$$\mathscr{J}(\mathbf{q}) = -\frac{1}{2}\cdot\frac{G^2\rho}{2M}\cdot(2\pi)^{-3/2}\left[\omega(\mathbf{q})^2 + \frac{G^2}{2M}\rho + a\omega(\mathbf{q})\exp(R_\pi^2\mathbf{q}^2)\right]^{-1}. \tag{62}$$

This yields, on energy extremization that $E = -15{\cdot}1\,\mathrm{MeV}$ along with $K = 142\,\mathrm{MeV}$. In the above $a = 0{\cdot}13\,\mathrm{GeV}$ and $R_\pi = 1{\cdot}2\,\mathrm{fm}$ along with $\lambda = 0{\cdot}35\,\mathrm{fm}^2$. The corresponding model will be reported elsewhere (Mishra *et al* 1988a). We have here $g_\omega^2/4\pi = 0{\cdot}79$, which is more reasonable.

## 4. Discussion

Let us note the new features introduced in the last section. First, we have located in interaction of nucleons with *pairs* of pions by using the corresponding field equations. Pions earlier played no role in the mean field approaches since they have odd parity. This interaction, along with an even number of pions, do not have this handicap. It is shown here that the corresponding expectation values generate the effect of the $\sigma$-meson. The state construction here is very similar to the consideration of fermion pairs in the solvable 1 + 1 dimensional Gross–Neveu model discussed earlier (Mishra *et al* 1988b). The pions exist inside the nuclear matter, and have a number density as given by (56) which is a non-trivial function of the number density of the nucleons. This may have relevance in heavy ion collisions when enough energy is pumped in to bring the pions on mass-shell, but, simultaneously, the energy is not too much to destabilize them and convert them to quark gluon plasma.

We particularly note that we dress the nuclear matter as a whole, and the idea of pion "exchange" or two-body potential has disappeared. The same procedure also applies for finite nucleus, where the pions may simultaneously "see" many nucleons. Many-body forces have earlier been considered (see e.g. Rajaraman and Bethe 1967). The above comment implies that they need not be negligible. The pions also have a testable momentum distribution here, as given in (56). This can be seen in experiments for finite nuclei, the details of which will be given elsewhere (Mishra *et al* 1988a).

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