Pramāṇa – J. Phys., Vol. 32, No. 4, April 1989, pp. 319–330. © Printed in India.

Infinite nuclear matter model and a new mass formula for atomic nuclei

L SATPATHY
Institute of Physics, Bhubaneswar 751 005, India

Abstract. The ground-state energy of an atomic nucleus with asymmetry β is considered to be equivalent to the energy of a perfect sphere made up of the infinite nuclear matter of the same asymmetry plus a residual energy η called the local energy, η represents the energy due to shell, deformation, diffuseness and exchange Coulomb effect etc. Using this picture and the generalized Hugenholtz-Van Hove theorem of many-body theory a new mass formula has been developed. Based on this, a mass table containing the mass excesses of 3481 nuclei in the range $18 \leqslant A \leqslant 267$ has been made. This mass formula is compared with other mass models.

Keywords. Mass formula for nuclei; saturation properties; nuclear matter model; atomic nuclei; ground-state energy.

PACS Nos 21·10, 21·65

1. Introduction

A mass formula for nuclei would generally predict the mass of a nucleus or equivalently its binding energy which is its most important property. Thus, the role of mass formulae and mass relations in nuclear physics is of central importance from both the theoretical and experimental points of view. Our inability to predict the ground-state energies of nuclei by solving the complex many-body problem satisfactorily with nucleon-nucleon potential has led us into constructing mass formulae based on various models. Study of nuclear physics over the years has revealed two main features of nuclear dynamics which are the shell and liquid drop features. Thus, the two main philosophies around which the various mass formulae and mass relations have been developed are based on these two features. The classic Bethe-Weizsacker (BW) mass formula, the droplet model (Myers and Swiatecki 1966; Myers 1977) and the Yukawa-plus exponential model (Moler and Nix 1981) employ liquid drop feature as the basis. In the last two models, the shell effect has been considered as a correction resulting in their improvement, Garvey *et al* (1969; hereafter referred to as Garvey-Kelson (GK)), Zeldes and Liran (1976) and Janeeke and Eynon (1975) have exclusively used the shell feature of nuclear dynamics in writing mass relations for nuclei. Although these two classes of models are based on two mutually contradictory pictures, they have enjoyed the success in ample measures. The origin of the shells in nuclei or liquid-like behaviour lies in the microscopic many-body dynamics. Looked at a fundamental level, a nuclear system consists of a set of fermions interacting through a two-body interaction. The shell or liquid-like features are just the macroscopic manifestation of the many-body dynamics. Hence, it is quite natural to expect that if a mass formula is constructed with many-body dynamics as its basis, then these two features of nuclear dynamics could be taken into account non-perturbatively, and that mass formula could be quite successful. During the last several years, we have attempted to develop (Satpathy and Nayak 1983, 1984, 1988; Satpathy 1984, 1987) such a mass formula. Here we report on this mass formula, in particular, how it is being compared with other mass models in this field and what new features of nuclear dynamics emerge through it.

We believe the nuclei possess two categories of properties namely, the universal and the individualistic. All nuclei have something common with one another, that is, they are made up of nuclear matter. The properties relating to this aspect of the nucleus are universal. The properties relating to the shell effect, deformation and diffuseness etc belong to the category of individualistic properties. Such properties contain the fingerprint of the nucleus and determine why one nucleus is different from the other. We have proposed a model called infinite nuclear matter (INM) model (Satpathy 1987) in which the ground-state energy of a nucleus is considered to be equal to the energy of a perfect sphere made up of infinite nuclear matter plus the residual characteristic energy η called the local energy. η represents the energy due to shell, deformation and diffuseness etc. We have extended the Hugenholtz Van-Hove (HVH) theorem to asymmetric nuclear matter which is referred to as generalized HVH theorem (Satpathy and Nayak 1983). The present mass formula is based on the INM model and the above theorem. Here it has been possible to separate the two categories of the properties and obtain their respective contribution to the binding energies. Since the mass formula has been built in terms of the properties of nuclear matter, it has been found quite suitable to extract the saturation properties of nuclear matter namely, the binding energy and saturation density from the fit of the masses of finite nuclei. It is found that the saturation properties so obtained are different from the accepted empirical values, but agree with the recent estimate of Day (1983) based on his sophisticated many-body calculations using the modern two-body potentials. In §2, the INM model has been described. A comparison of the present mass formula with other mass models is presented in §3. Section 4 presents the new saturation properties of nuclear matter as determined through the present mass formula. The concluding remarks are presented in §5.

2. The infinite nuclear matter model

2.1 *The foundation*

Consider a nucleus with neutron number N, proton number Z, mass number $A = N + Z$ and asymmetry parameter $\beta = (N - Z)/(N + Z)$. We are interested in the ground-state energy of this nucleus. Then we take a sea of infinite nuclear matter with

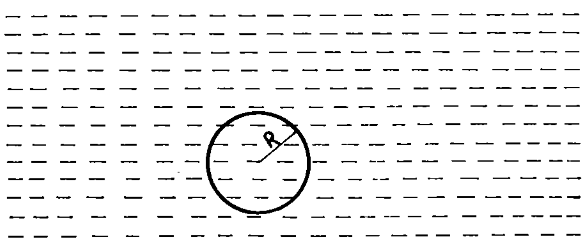

Figure 1. The dashed lines represent a sea of infinite nuclear matter of density ρ_0. The circle represents a sphere of radius R which contains N neutrons and Z protons.

the same asymmetry parameter β and consider a perfectly spherical volume of radius R inside it (see figure 1) which contains N neutrons and Z protons. R is related to A through $R = \gamma_0 A^{1/3}$ with γ_0 being a constant. The density of the nuclear matter is ρ_0. Let the energy contained in this volume be $E(A, Z)$. Then we cut out this perfect sphere from the sea of infinite nuclear matter by switching on the surface force, subsequently switch on the Coulomb force and the pairing force. While the surface force will tend to contract the sphere and increase its density, the Coulomb force will tend to expand and decrease the density. It has been shown by Brandow (1964) and as is generally believed these two effects nearly cancel each other. It is therefore reasonable to assume that our isolated sphere will have the same density ρ_0 of infinite nuclear matter sea. Let $E^s(A, Z)$ be the energy of this cut out, sphere hereafter referred to as 16 INM sphere or nuclear drop. Thus $E^s(A, Z)$ is given by

$$E^s(A,Z) = E(A,Z) + a_s A^{2/3} + a_c \frac{Z^2}{A^{1/3}} - \delta(A,Z), \tag{1}$$

where a_s and a_c are the surface and Coulomb coefficients of the INM sphere and $\delta(A, Z)$ is the pairing term given by

$$\delta(A,Z) = \begin{cases} +\Delta A^{-0\cdot 5} & \text{for even-even nuclei,} \\ 0 & \text{for odd-}A\text{ nuclei,} \\ -\Delta A^{-0\cdot 5} & \text{for odd-odd nuclei.} \end{cases} \tag{2}$$

Here Δ is a parameter. The Coulomb coefficient a_c is given by $3e^2/5\gamma_0$ where, γ_0 is the nuclear radius constant. The nuclear matter density ρ_0 is related to the Fermi momentum k_f through $\rho_0 = 2k_f^3/3\pi^2$ and to γ_0 through $\rho_0 = 2k_f^3/3\pi^2$. How we visualize that the ground-state energy $E^F(A, Z)$ of a real finite nucleus of mass number A and charge number Z is equal to the energy $E^s(A, Z)$ of the INM sphere plus the characteristic residual energy $\eta(A, Z)$ called the individualistic or local energy. The superscript F hereafter denotes the finite nucleus. η as pointed out before represents all the contributions arising from all the individualistic properties like shell, deformation and diffuseness etc. Then the energy of a finite nucleus splits into global (universal) and local (individualistic) parts:

$$\begin{aligned} E^F(A,Z) &= E^s(A,Z) + \eta(A,Z) \\ &= E(A,Z) + a_s A^{2/3} + [a_c(Z^2/A^{1/3})] - \delta(A,Z) + \eta(A,Z). \end{aligned} \tag{3}$$

Putting

$$f(A,Z) = a_s A^{2/3} + [a_c(Z^2/A^{1/3})] - \delta(A,Z). \tag{4}$$

Equation (3) is rewritten as

$$E^F(A,Z) = E(A,Z) + f(A,Z) + \eta(A,Z). \tag{5}$$

Thus (5) represents a mass formula with two unknown functions $E(A, Z)$ and $\eta(A, Z)$ and a known function $f(A, Z)$ with three unknown parameters. In the following we describe how these unknowns are determined.

2.2 The Hugenholtz-Van Hove theorem and its generalization

The Hugenholtz-Van Hove theorem (Hugenholtz and Van Hove 1958) in general deals with the single particle properties of a self-bound interacting Fermi gas at absolute zero of the temperature. It states that for a system of A particles and total energy E

$$\frac{E}{A}+\rho\frac{\mathrm{d}(E/A)}{\mathrm{d}\rho}=\left.\frac{\delta E}{\delta A}\right|_V, \tag{6}$$

where ρ and V are the number density and volume respectively. Hugenholtz and Van Hove (1958) that showed (6) is equal to the Fermi energy ε which is the same as the separation energy with negative sign. At equilibrium i.e for ground state $\mathrm{d}(E/A)/\mathrm{d}\rho$ vanishes and one obtains as a special case the average energy is equal to the Fermi energy

$$E/A=\varepsilon. \tag{7}$$

This theorem is a rare exact theorem in many-body theory. It is valid for any Fermi system and therefore is applicable to ^3He and in particular to nuclear matter. To emphasize the dynamical content of this theorem it is useful to recall here that it helped Hugenholtz and Van Hove (1958) to discover the internal inconsistency in the early nuclear matter calculation of Brueckner (1958). This theorem in the present form is applicable to a Fermi system consisting of one kind of particles or to symmetric nuclear matter. We have extended (Satpathy and Nayak 1983) this theorem to asymmetric nuclear matter which assumes the following form for the ground-state

$$E/A=\tfrac{1}{2}[(1+\beta)\varepsilon_n+(1-\beta)\varepsilon_p], \tag{8}$$

where β, ε_n and ε_p are the asymmetry parameter, neutron and proton Fermi energies respectively. For symmetric system $\beta=0$ and $\varepsilon_n=\varepsilon_p$ and for a system consisting of one kind of particle $\beta=1$, $\varepsilon_n=\varepsilon$ and $\varepsilon_p=0$ then (8) reduces to the usual theorem given by (7).

2.3 Determination of $f(A,Z)$ and derivation of a new mass relation

Equation (5) can be used as a transformation equation between energy $E^F(A,Z)$ of a finite nucleus (A,Z) and the corresponding infinite nuclear matter energy $E(A,Z)$. Thus by using (5) the INM Fermi energies ε_n and ε_p can be expressed in terms of their counterparts ε_n^F and ε_p^F for finite nuclei as

$$\begin{aligned}\varepsilon_n&=\varepsilon_n^F-f(A,Z)+f(A-1,Z)-\eta(A,Z)+\eta(A-1,Z),\\ \varepsilon_p&=\varepsilon_p^F-f(A,Z)+f(A-1,Z-1)-\eta(A,Z)+\eta(A-1,Z-1).\end{aligned} \tag{9}$$

Thus with the help of (5) and (9), (8) reduces to

$$\begin{aligned}\frac{E^F(A,Z)}{A}-\frac{\eta(A,Z)}{A}=\tfrac{1}{2}[(1+\beta)\varepsilon_n^F(A,Z)&+(1-\beta)\varepsilon_p^F(A,Z)+S(A,Z)\\ &-\frac{N}{A}[\eta(A,Z)-\eta(A-1,Z)]\\ &+\frac{Z}{A}[\eta(A,Z)-\eta(A-1,Z-1)],\end{aligned} \tag{10}$$

where

$$S(A,Z)=\left(\frac{1-A}{A}\right)f(A,Z)+\frac{N}{A}f(A-1,Z)+\frac{Z}{A}f(A-1,Z-1) \tag{11}$$

$\eta(A,Z)$ is expected to be a smooth function of A and Z and further it is expected to be much smaller than the corresponding total energy $E^F(A,Z)$. Hence terms like $\eta(A,Z)-\eta(A-1,Z)$ and $\eta(A,Z)/A$ in (10) can be neglected. More appropriately, it is assumed here that the sum of the terms involving η on the right side is equal to similar terms on the left side. This yields

$$\frac{\eta(A,Z)}{A}=\frac{N}{A}[\eta(A,Z)-\eta(A-1,Z)]+\frac{Z}{A}[\eta(A,Z)-\eta(A-1,Z-1)] \tag{12}$$

or equivalently

$$\eta(N,Z)=\frac{N}{A-1}\eta(N-1,Z)+\frac{Z}{A-1}\eta(N,Z-1). \tag{13}$$

Hence (10) reduces

$$\frac{E^F(N,Z)}{A}=\tfrac{1}{2}[(1+\beta)\varepsilon_n^F(N,Z)+(1-\beta)\varepsilon_p^F(N,Z)]+S(A,Z). \tag{14}$$

Equation (14) can be used as a mass relation if we express the Fermi energies ε_n^F and ε_p^F in terms of the binding energies of the neighbouring nuclei. Then (14) would read as

$$\frac{E^F(N,Z)}{A}-\tfrac{1}{2}\{(1+\beta)[E^F(N,Z)-E^F(N-1,Z)]$$
$$+(1-\beta)[E^F(N,Z)-E^F(N,Z-1)]\}=S(A,Z). \tag{15}$$

The above equation describes a relation among the masses of three neighbouring nuclei with neutron and proton numbers (N,Z), $(N-1,Z)$ and $(N,Z-1)$. It is remarkable to note that the mass relation (15) does not contain any local energy parameter η. It relates the three parameters a_s, and a_c and Δ which are universal in character being the properties of INM sphere, to the ground-state energies of three nuclei. Thus, through (13) and (14) the separation of local and global energies has been achieved. The least square fitting of (14) to all the data on binding energies with error bars less than 30 keV from the recent mass table of Wapstra and Audi (1985) was performed. There are 756 such cases ranging from $Z=8$ and $A=20$ to $Z=101$ and $A=255$. The value of the parameters obtained are $a_c=0{\cdot}841$ MeV, $a_s=25{\cdot}846$ MeV and $\Delta=11{\cdot}709$ MeV. The goodness of this mass relation is well tested elsewhere (Satpathy and Nayak 1983). It can be profitable used to predict the mass of an unknown nucleus provided the masses of two neighouring nuclei are known.

2.4 *Determination of* $E(A,Z)$

First we have to find out how $E(A,Z)$ depends upon A and Z. Since $E(A,Z)$ is the energy of nuclear matter, it must satisfy the generalized HVH theorem (equation (8)).

The solution of this equation is of the form

$$E = -a_v A + a_a \beta^2 A \tag{16}$$

where, a_v and a_a are two constants which can be identified as volume and symmetry coefficient. It is indeed very interesting to note that these two terms which were taken into account in 1936 by Bethe and Weiszacher in their liquid drop model form the solution of the many-body HVH theorem which was established 22 years after in 1958. Using (5), (8) and (16) we obtain

$$-a_v + a_a\beta^2 = \tfrac{1}{2}[(1+\beta)\varepsilon_n^F + (1-\beta)\varepsilon_p^F] - f(A,Z) + \left(\frac{N}{A}\right) f(A-1,Z) + \left(\frac{Z}{A}\right) f(A-1,Z-1). \tag{17}$$

a_v and a_a are the only two unknowns in the above equation since $f(A,Z)$ has been determined before. Using the neutron and proton separation energies of the same 756 nuclei we least square fit (17) and obtain $a_v = 18{\cdot}335$ MeV and $a_a = 36{\cdot}211$ MeV. Thus $E(A,Z)$ is determined.

2.5 *Determination of $\eta(A,Z)$*

$\eta(A,Z)$ represents the local energy of the nucleus which comprises the contributions due to shell, deformation, diffuseness effects etc and any other characteristic effect which as yet might not have been identified. It is in fact the fingerprint of the nucleus. We feel it is impossible to find a general expression in A and Z which will truly represent η for all nuclei. We therefore determine it locally from the experiment as follows. Since by now $E(A,Z)$ and $f(A,Z)$ are known, using the experimental ground-state energy for $E^F(A,Z)$ in (5) we can obtain the value of η which we call experimental value η_{ex}. Then, η_{ex} for all known nuclei can be determined. The systematics of η_{ex} for the entire periodic table is shown elsewhere (Satpathy 1987). This is a new quantity in nuclear physics. Equation (13) relates the η^{1s} of three neighbouring nuclei with neutron and proton number (N,Z), $(N-1,Z)$ and $(N,Z-1)$. It has been shown (Satpathy 1987) elsewhere that not only this relation is very well satisfied, but it also has excellent extrapolation properties. Hence it can be used as a recursion relation to determine η of unknown nuclei using those of the known nuclei. Thus, the mass of any nuclei can be predicted. In practical application the whole periodic table is divided into 25 regions with $\Delta A = 8$ or 10. In each region the η^{1s} of all the nuclei are expressed in terms of the η^{1s} of few nuclei using the relation (13) which could be considered as local parameters. These parameters are determined by fitting the η_{ex}^{1s} of all the known nuclei in the region. The details of this procedure can be found in Satpathy (1987).

The predictive power of this mass formula has been compared (Satpathy 1987) with the celebrated GK mass relation and the droplet model in the different regions of the periodic table. It has been shown that the present mass formula does remarkably better in predicting the masses of nuclei far from stability compared to the other two. Based on this mass formula, a mass table containing the predictions of 3481 nuclei in the range $18 \leqslant A \leqslant 267$ has been prepared (Satpathy and Nayak 1988).

3. Comparison with other contemporary mass models

The 1986–87 mass prediction project which was undertaken by the Atomic Data and Nuclear Data Table with Peter Haustein as co-ordinator, has just produced (July 1988 vol. 39, No. 2) a comprehensive update of atomic masses. Altogether ten groups from all over the world have made predictions using their models starting with a common data base supplied by the organiser. At the fifth International Conference on "Nuclei far from stability" held recently, a comparative study on the performance of the ten models was presented by Haustein (1988a). A more detailed version has appeared elsewhere (Haustein 1988b) Table 1 is taken from the latter article, where the various features of the ten models are compared. A remarkable feature of the present mass formula is that the mean deviation is only 1 keV which is far too small compared to any other mass model in this group of ten. It is well known that if a formula is expected to correctly describe a set of data, then the mean deviation obtained in the least square fit should ideally be zero. This ensures that no systematic physical effect has been missing. In view of this, the extremely small value of mean deviation is quite reassuring in regard to the soundness of the physical basis of the present mass formula. When one compares the r.m.s. deviation obtained in the present mass formula with those of Comay *et al* (1988), Tachibania *et al* (1988) and Janecke and Masson (1988) (which are generally used by experimentalists for quantitative comparison) one finds that it is quite low even though the number of parameters is much less. This is yet another good signal about the present mass formula.

It is well known that Na isotopes present a serious challenge to all the mass formulae. Thus, the prediction of the masses of Na isotopes provides a very good testing ground. As a measure of the success of the present mass formula, we compare in table 2 our predictions on the Na isotopes with those of the recent predictions in four popular mass models in the group of ten. It can be seen that errors have systematically built up in all

Table 1. Comparison of the various mass models. This table is from Haustein (1988a).

Model	Parameters used	Database nuclei used	RMS deviation (keV)	Average deviation (keV)	Nuclei[a] predicted
Pape and Antony	(?)	85	271	123	381
Dussel, Caurier and Zuker	45	1328	287	−13	1984
Moller and Nix	26	1593	849	13	4635
Moller, Myers, Swiatecki and Treiner	29	1593	777	14	4635
Comay, Kelson and Zidon	[b]	1632	424	13	6537
Satpathy and Nayak	238	1593	456	1	3481
Yachibana, Uno Yamada and Yamada	281	1657	538	22	7204
Spanier and Johannson	12	886	699	−111	4162
Janecke and Masson	928	1633	339	19	5860
Masson and Janecke	471	1582	344	14	4383

[a]Includes the database nuclei used. [b]Subset of known masses.

Table 2. Comparison with experiment the predictions of the masses of Na isotopes in various mass models. The predictions: Columns 2–6 give respectively the differences between the calculated and experimental mass excesses due to the present model Comay *et al* 1988 (CKZ); Janecke and Masson 1988 (JM); Moller and Nix 1988 (MN); and Moller *et al* 1988 (MMST). The last but one and the last rows give respectively the total deviation of the predictions from experiment and the mean deviation respectively.

Nucleus	Calc.—expt. (MeV)	CKZ—expt. (MeV)	JM—expt. (MeV)	MN—expt. (MeV)	MMST—expt. (MeV)
^{28}Na	0·23	0·09	0·27	1·13	1·06
^{29}Na	0·3	−0.39	−0·53	·21	0·07
^{30}Na	−0·61	0·6	0·15	·49	0·27
^{31}Na	−0·9	1·75	0·91	1·65	0·62
^{32}Na	2·03	5·87	4·82	2·99	1·84
^{33}Na	0·4	7·15	6·22	3·51	2·26
^{34}Na	−0·66	10·57	9·83	4·44	3·08
Total deviation (MeV)	0·79	25·66	21·67	14·42	9·2
Mean deviation (MeV)	113	−3666	3096	2060	1314

Table 3. Same as table 2 but for different nuclei.

Nucleus	Calc.—expt. (MeV)	CKZ—expt. (MeV)	JM—expt. (MeV)	MN—expt. (MeV)	MMST—expt. (MeV)
^{19}C	0·25	0·17	0·27		
^{27}Ne	0·59	1·00	1·10	2·37	2·32
^{28}Ne	−2·11	−0·30	−0·29	0·94	0·35
^{32}Al	0·04	0·52	0·04	1·56	1·31
^{33}Al	−0·58	−0·18	−0·30	−0·73	−1·08
^{34}Al	−0·01	1·28	0·34	0·82	0·12
^{34}Si	−0·73	1·08	1·03	1·56	0·83
^{37}P	0·7	0·67	0·78	0·79	0·25
Total deviation (MeV)	−1·84	4·424	3·47	7·29	4·6
Average deviation (keV)	−230	530	434	1041	657

these four cases as one moves from ^{29}Na to ^{34}Na. In the case of Kelson group, the error is as high as 10·57 MeV for ^{34}Na. However, the present mass formula is free from such discomfitures. To obtain a quantitative idea about the degree of discrepancy we have calculated the total and the mean deviations which are 0·79 MeV and 113 keV as against 25·66 MeV and 3666 keV for Kelson group respectively. In table 3 we compare eight predictions on light nuclei with the same four models which have been recently measured at Los Alamos and Ganil. Similar analysis as in table 2 was performed in table 3 for these eight cases. It clearly bears out the relative success of the present mass formula.

4. Saturation properties of nuclear matter

Infinite nuclear matter is a hypothetical object created by theorists to understand the effective interaction and nuclear dynamics. Its closest resemblances in real life may be the matter in a neutron star and the one in the centre of a heavy nucleus. Saturation properties of nuclear matter are of fundamental importance and intimately related to the equation of state. The two saturation properties of nuclear matter are the binding energy per nucleon w and the saturation density or equivalently the Fermi momentum k_f. Traditionally the value of k_f is obtained from experiment by identifying with the volume term of BW mass formula whose recent value from droplet model (Myers 1977) is 15·97 MeV. The value of k_f similarly can be obtained from the Coulomb coefficient of the same BW mass formula. The value so obtained is 1·26 fm^{-1}. The Coulomb coefficient of the droplet model gives 1·29 fm^{-1}. These values are not accepted. On the other hand, the value of k_f is determined from the data on electron scattering on heavy nuclei assuming the density of infinite nuclear matter to be the same as the central density of a large nucleus. The value so obtained is $k_f = 1{\cdot}36\ \text{fm}^{-1}$. These two values $w = 15{\cdot}97$ MeV and $k_f = 1{\cdot}36\ \text{fm}^{-1}$ are the accepted empirical values which the many-body calculations using the Brueckner theory try to reproduce over the last several decades.

The traditional procedure of determining the properties of nuclear matter as described above has the following two major drawbacks at a fundamental level: (i) The properties pertaining to nuclear matter should be extracted from the experimental binding energies through a model which is built specifically incorporating these features. For example consider the spectrum of ^{20}Ne. If one describes the low lying states $0^+, 2^+, 4^+, 6^+$ and 8^+ in a rotational model, then one can extract a moment of inertia for this nucleus from these levels. However, if one describes them in a configuration-mixing shell model, then the moment of inertia is non-existent. (ii) The value of the two quantities w and k_f should be simultaneously determined through one model and not deduced from two different sources. In view of this, we feel the present model which is based on a many-body theoretic foundation and built specifically in terms of the properties of infinite nuclear matter should be eminently suitable to extract the saturation properties from the binding energies of nuclei. We obtain the value of w which is equal to a_v to be 18·33 MeV. The value of k_f obtained from our value of the Coulomb coefficient a_c is 1·48 fm^{-1}, we refer to these values as the new saturation properties. In the following, we discuss how the results of the present day many-body calculation compare with it.

Recently Day (1983) made an exhaustive many-body calculation of nuclear matter including the higher order coupled clustered terms. In his calculation the modern two-body interactions like Paris, Bonn etc were used and he concluded that there is no more room for improving the calculation, and if the nuclear force is of the two-body type then the saturation point would lie in an oval shaped area (see figure 2) in the w-k_f plane. He has beautifully summarised the present status of the nuclear matter calculation with various forces in this figure which is taken from his article (Day 1983). The saturation point corresponding to our above values of w and k_f falls inside this oval which is shown as a solid triangle. This agreement is not accidental and is being viewed with seriousness.

It is now desirable to understand exactly what differences in physics have led into the value of w determined in the present model being different from that obtained through BW-like mass formulae. There are the following two essential additional features.

(i) In the case of the BW-like mass formulae, the only input is the total energy of the nucleus which contains the average property of nuclear matter namely the average

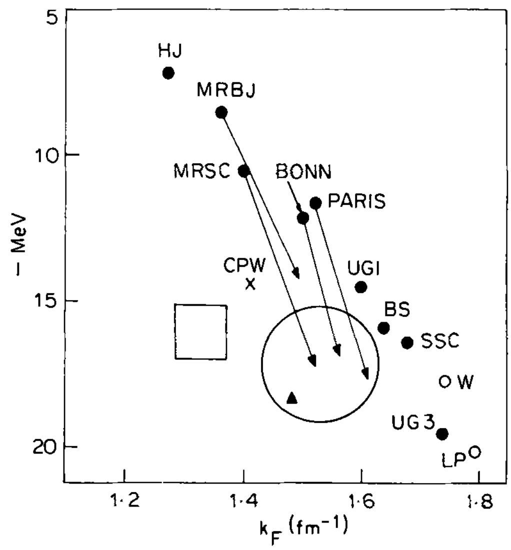

Figure 2. Nuclear matter saturation points from Day (1983). The potentials are HJ (Hamada and Johnston 1962); MRSC, the modified Reid soft core potential (Day 1981); MRBJ, the modified MRSC potential by (Bethe and Johnson (1979); CPW (Carlson *et al* 1983); BONN (Holinde and Machleidt 1975); PARIS (Lacombe *et al* 1980), UG1 and UG3 (Ueda and Green 1968); BS (Baryan and Scott 1969); SSC (de Tourriel and Sprung 1973), LP (Lagaris and Pandaripande 1981) and W (Wiringa 1983). For four potentials the arrows pointing to the oval show the shift in the saturation point when higher order cluster terms are included by Day (1983) and the region defined by the oval is the uncertainty assigned by him. The square represents the usual empirical saturation point. The solid triangle is the new empirical saturation point obtained in the present study.

energy per nucleon. The present model, in addition, contains the energy of the Fermi state which can be considered as a differential property. There is no concept of Fermi energy in the liquid drop which is a very important single particle property of an interacting Fermi system. It has been demonstrated by Hugenholtz and Van Hove (1958) that in an interacting Fermi system the true single particle state is the Fermi state which has infinite lifetime while others are metastable. The lifetime of the single particle state approaches ∞ in the limit $k \to k_f$. This property of the infinite nuclear matter is not contained in the BW-like mass formula which is an important ingredient in the present model. To put the matter more quantitatively, through equation (14), one can see that the parameters a_s, a_c and Δ are the function of E^F/A and ε^F which is not the case with BW mass formula. The same is true for a_v and a_a also.
(ii) By the use of HVH theorem, we have been able to separate the total energy into local and global parts, which has helped us in the relatively clean determination of the global parameter. Further by determining a_s, a_c and Δ in one step (equation (14)) and a_v and a_a in another step (equation (17)), the cross-correlation amongst parameters are avoided.

5. Concluding remarks

The present mass formula is unique in two ways. Firstly, it is the only mass formula which is based on a many-body theoretic foundation and uses explicitly the concept of infinite nuclear matter, and secondly, it has much better ability for predicting masses of nuclei in the unknown region far from stability compared to the often used G. K mass relation and droplet model etc. The success of the mass formula inspires confidence in the INM model, and hence the values of the universal parameters determined through it. So the saturation properties (w and k_f) determined in this model should be more reliable. These also agree with the result obtained in the modern many-body calculation. Every model of nucleus is expected to unravel certain new features about its properties. The present model has shown why the liquid drop model is inadequate to represent the properties of nuclear matter. It has found in a consistent manner the binding energy and saturation density of nuclear matter which should be considered as the true empirical saturation properties.

Finally, we would like to emphasize that the universal parameter a_s and a_c refer to the hypothetical INM sphere and not to the real nuclei. They can be used successfully in the present model to compute the ground-state energy of nuclei. Application of the present model to the study of fission barrier is presented elsewhere (Gupta and Satpathy 1987).

Acknowledgement

This paper was presented at the Nuclear/Particle Physics Symposium in honour of Prof S P Pandya. I would like to thank Dr R C Nayak for his association with many phases of this work.

References

Baryan R and Scott B L 1969 *Phys. Rev.* **177** 1435
Bethe H A and Johnson M B 1979 *Nucl. Phys.* **A230** 1
Brandow B H 1964 Ph.D. Thesis Cornel University
Brueckner K A 1958 *Phys. Rev.* **110** 597
Carlson J, Pandaripande V R and Wiringa R B 1983 *Nucl. Phys.* **A401** 591
Comay E, Kelson I and Zidon A 1988 *At. Data Nucl. Data. Table* **39** No. 2
Day B D 1981 *Phys. Rev.* **C24** 1203
Day B D 1983 *Comments Nucl. Phys. Part. Phys.* **11** 115
de Tourriel R and Sprung D 1973 *Nucl. Phys.* **A201** 193
Garvey *et al* 1969 *Rev. Mod. Phys.* **41** S1
Gupta S K and Satpathy L 1987 *Z. Phys.* **A326** 221
Hamada T and Johnston I P 1962 *Nucl. Phys.* **34** 382
Haustein P 1988a *AIP Conf. Proc.* **164** 53 (Fifth Int. Conf. on Nuclei far from stability 1987) (ed.) Jan S. Towner
Haustein P 1988b *At. Data Nucl. Data Table* **39** No. 2
Holinde K and Machleidt R 1975 *Nucl. Phys.* **A247** 495
Hugenholtz N H and Van Hove W 1958 *Physica* **24** 263
Janecke J and Masson P J 1988 *At. Data and Nucl. Data. Table* **39** No. 2
Janecke J and Eynon B P 1975 *Nucl. Phys.* **A243** 326
Lagaris L E and Pandaripande V R 1981 *Nucl. Phys.* **A359** 349
Lacombe M *et al* 1980 *Phys. Rev.* **C21** 861
Moller P, Myers W D, Swiatecki W J and Treiner J 1988 *At. Data Nucl. Data Table* **39** No. 2
Moller P and Nix J R 1981 *At. Data Nucl. Data Table* **26** 165
Moller P and Nix J R 1988 *At. Data Nucl. Data Table* **39** No. 2
Myers W D 1977 *Droplet model of atomic nuclei* (New York: Plenum)
Myers W D and Swiatecki W J 1966 *Nucl. Phys.* **81** 1

Satpathy L 1984 in *Proc. Int. Conf. on Nucl. Phys.* (ed.) B K Jain and B C Sinha (Singapore: World Scientific) p. 79
Satpathy L 1987 *J. Phys.* **G13** 761
Satpathy L and Nayak R 1983 *Phys. Rev. Lett.* **51** 1243
Satpathy L and Nayak R 1988 *At. Data Nucl. Data Table* **39** No. 2
Satpathy L and Nayak R 1984 *Proc. Seventh Int. Conf. on At. masses and fundamental constant* AMCO-7, (ed.) O. Klepper (Darmstadt: Technishe Hoschule) p. 436
Tachibania T, Uno M, Yamada M and Yamada S 1988 *At. Data Nucl. Data Table* **39** No. 2
Ueda T and Green A 1968 *Phys. Rev.* **174** 1394
Wapstra A H and Audi G 1985 *Nucl. Phys.* **A432** 1
Wiring R B 1983 unpublished
Zelds N and Liran S 1976 *Atomic masses and fundamental constants* (New York: Plenum) Vol 5 p. 264
Zelds N and Liran S 1976 *Phys. Lett.* **B62** 12