Pramāna - J Phys, Vol 32, No. 3, March 1989, pp 231-239 © Printed in India

Variation of interband interaction strength in odd-A nuclei

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MS received 12 May 1988, revised 7 November 1988

Abstract. Bandcrossing in 31 rotational bands of 25 different odd-A nuclei in the rare-earth region has been analysed by using a two-band mixing formalism with a constant band interaction within the framework of the effective decoupling picture. The interband interaction strength $|V|$ between the one-quasiparticle band and the three-quasiparticle band exhibits a variation with the neutron number which is not different from the oscillatory behaviour observed in even-even nuclei and does not show signs of any appreciable phase shifting as predicted by theory However, the overall range of variation of $|V|$ is greater than that observed in even-even systems

Keywords. Nuclear structure, two-band mixing; one-quasiparticle band, three-quasiparticle band; odd-A nuclei, interband interaction strength.

PACS Nos 21·10; 21·60

1. Introduction

The phenomenon of backbending (Sorensen 1973; Stephens 1975; deVoigt *et al* 1983) is a widely discussed topic and a number of attempts have been made to describe it phenomenologically or otherwise with varying degrees of success. It is now well understood that backbending is a manifestation of the crossing of the ground band having zero alignment and a superband having large aligned angular momentum. In odd-A bands the 'backbending' corresponds to the crossing of a one-quasiparticle rotational band having very little alignment and a three-quasiparticle aligned band having a large aligned angular momentum. The degree of 'backbending' clearly depends on the interaction between the two crossing bands. This interband interaction has recently been analysed in detail for even-even nuclei (Bengtsson *et al* 1978; Almberger *et al* 1979; Bengtsson and Frauendorf 1979a, b; Bonatsos 1985) and exhibits an oscillatory behaviour as a function of the neutron number. It would be of great interest to study the behaviour of the interband interaction in odd-A systems and how this interaction compares with that in the even-even systems. An added advantage in studying the odd-A bands is that many rotational bands are usually known in a single nucleus and it would be interesting to compare the interband interaction among bands of the same nucleus.

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2. The model

We have pointed out earlier (Jain 1983, 1984, 1987; Jain and Jain 1984) that a sizeable number of rotational bands in odd-A rare-earth nuclei exhibit an effective decoupling. This allows us to describe the odd-A bands in a particularly simple way in terms of an even-even rotational band expression, once we are able to extract the effective aligned angular momentum from the experimental data. The aligned angular momentum (i_n) of the one-quasiparticle band is obtained by assuming the total angular momentum of the levels to be composed of $I = R' + i_n$, where R' is the effective rotational angular momentum. Usually one takes R' to be zero for the bandhead of a given odd-A band and it is assumed that the total angular momentum is that of the odd-particle. However, we find that $R' = 0$, 2 or 4 depending on the bands concerned and this value is obtained directly from a comparison of the experimental data on odd-A band $\Delta I = 2$ transition energies with that of the even-even core ground band transition energies. This procedure works very well for a large number of odd-A bands (Jain 1983, 1984, 1987; Jain and Jain 1984) and leads to a well-defined value of R' in all the cases discussed here. The values of R' so obtained are particularly meaningful in the spin region where the bandcrossing occurs. The cubic polynomial expression or a three parameter angular momentum expansion (AME3) first proposed by Satpathy and Satpathy (1971), has been found to explain successfully the ground band energy of even-even nuclei (Sood and Jain 1975; Satpathy *et al* 1978). Therefore, the levels of one or both of the $\Delta I = 2$ sequences of the one-quasiparticle band (hereafter referred to as normal band) are described by the AME3 expression

$$E(R') = aR' + bR'^2 + cR'^3, \tag{1}$$

where the bandhead now does not necessarily correspond to $R' = 0$. We note that the AME3 expression is also capable of describing backbending in even-even nuclei. We however use only the lower spin levels below the backbending when fitting the odd-A band data.

We also find that the S-band (in this case a three-quasiparticle band) can be treated to a very good degree of accuracy as a pure rotational band with almost constant moment of inertia due to the loss of pairing correlations and increased alignment. It can therefore be described by a simple expression

$$E^S(R'') = E_0 + \frac{\hbar^2}{2\mathscr{I}_S} R''(R'' + 1), \tag{2}$$

where R'' is the effective rotational angular momentum of the S-band levels obtained by subtracting the S-band alignment i_S from the total angular momentum I. The S-band alignment $i_S = i_n + \Delta i$, where Δi is the increment in alignment over and above that of the normal band. The value of Δi is obtained by the standard procedure of plotting I vs transition energy and evaluating the sudden shift in the angular momentum (Bengtsson and Frauendorf 1979a, b). Once we know the R'' values, the bandhead of the S-band is automatically known to us, provided, we assume that the S-band behaves like a $K = 0$ band. We thus require only one parameter $\hbar^2/2\mathscr{I}_S$ to describe the S-band. We further assume the R' and R'' values to be nearest even integers for simplicity.

Once we are able to fix the parameters of equations (1) and (2) by a least square fitting of the normal and the S-band levels respectively, we are left with only one variable parameter that is the interaction strength $|V|$ which is varied in a two-band mixing formalism to reproduce the levels in the bandcrossing region (Goldhaber *et al* 1976). Out of the two solutions of the secular equation for two-band mixing, the yrast level energy is given by

$$T_- = \tfrac{1}{2}[E(R') + E^s(R'')] - \{\tfrac{1}{4}[E(R') - E^s(R'')]^2 + |V|^2\}^{1/2}, \tag{3}$$

where $R' = R''$, and corresponds to the rotational band being fitted. As an example, we present in table 1 the input data used to calculate the interband interaction strength in $\frac{1}{2}$-[541] band in ^{173}W nucleus. We have divided the band in odd-A nucleus into three regions: The normal band region (n-band), the transition region and the S-band region. We have not taken into account the effect of yrast energy levels in the transition region in calculating the parameters of n-band and S-band. In the last column of the table 1, we have listed the parameters calculated by using eqs (1) and (2) for the two respective regions. We then adjust the value of the interaction strength parameter $|V|$ in such a way that all the levels including the levels in the transition region are well reproduced and a minimum value of root mean square deviation is obtained.

A similar two-band mixing calculation for even-even nuclei has also used an angular momentum-dependent interaction in addition to a constant interaction (Bonatsos 1985). We however find no compelling reason to do this. In fact we could not improve our fits by using an angular momentum-dependent interaction. Bonatsos (1985) for example uses an angular momentum-dependent interaction to achieve better fits but obtains an oscillatory behaviour of $|V|$ as predicted by theory by using a constant band interaction.

Table 1. The level energy data and the calculated parameters used to determine the interband interaction strength $|V|$ in $1/2^-[521]$ band of $^{173}_{74}W_{99}$

Regions	Energy Spin	Calculated (keV)	Parameters
Normal-band region	1/2	0·0	$a = 16{\cdot}997$
	5/2	89·7	$b = 14{\cdot}620$
	9/2	280·7	$c = -0{\cdot}3394$
	13/2	555·4	
	17/2	896·7	
	21/2	1295·5	
Transition region	25/2	1734·6	
	29/2	2213·4	
S-band region	33/2	2719·3	$E_0 = 1151{\cdot}15\,\text{keV}$
	37/2	3271·5	$\hbar^2/2\mathscr{I} = 10{\cdot}071\,\text{keV}^{-1}$
	41/2	3887·9	

3. Results and discussion

Bandcrossing in 31 rotational bands of 25 different odd-A nuclei (both odd-Z and odd-N) in the rare-earth region has been analysed by using the two-band mixing formalism. Out of these 31 bands, 6 bands have bandcrossing known in the favoured as well as the unfavoured sequences, thus giving us a total of 37 values for the interaction strength $|V|$. Qualitatively the behaviour of odd-A bands may be classified into S-shaped backbending and up-bending depending on the shape of the I vs $\Delta E(I \rightarrow I-2)$ plots. It is interesting to note that most of the odd-N odd-A nuclei are found to exhibit an upbending or, forward bending behaviour while majority of odd-Z odd-A nuclei show an S-shaped backbending.

In figure 1, we display the results for four typical cases of odd-Z nuclei. Notice the excellent fits to the experimental level energies. Also shown in the figure are the values of $|V|$ and the alignments i_n and i_s for the normal and the S-bands respectively. Figure 2 contains similar results for both the $\Delta I = 2$ (favoured and unfavoured) sequences of the $7/2^-[523]$ and $7/2^+[404]$ bands in ^{159}Tm. It may be noted that the values of $|V|$ from the favoured and the unfavoured sequences in these two cases are almost the same whereas the values for the two different bands differ widely. However, in most of the cases the values of $|V|$ for different bands in the same nucleus turn out to be

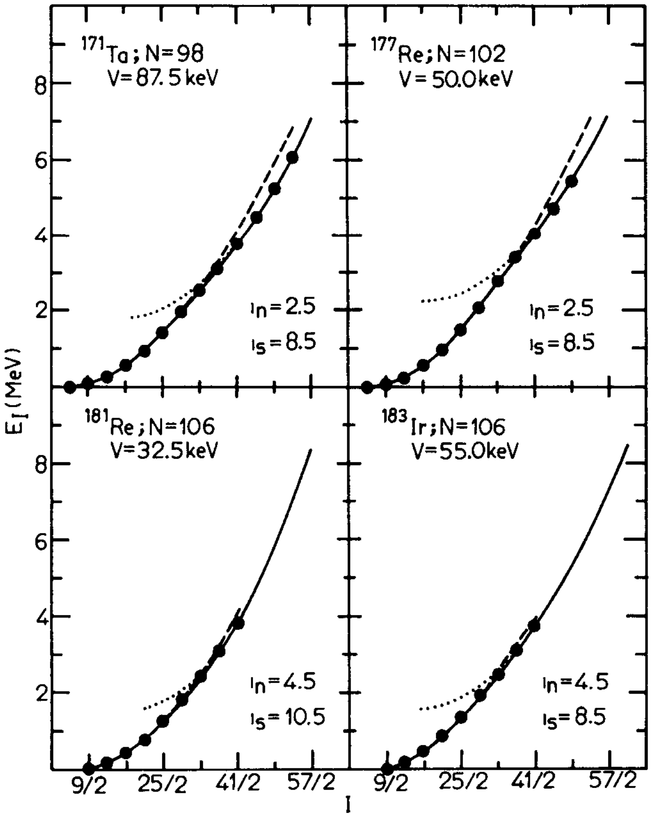

Figure 1. Energy vs angular momentum plot for four odd-A nuclei for $1/2^-[541]$ band (shown by solid lines) and their extensions by broken (normal) and dotted lines (S-band) Experimental points are shown by the filled circles

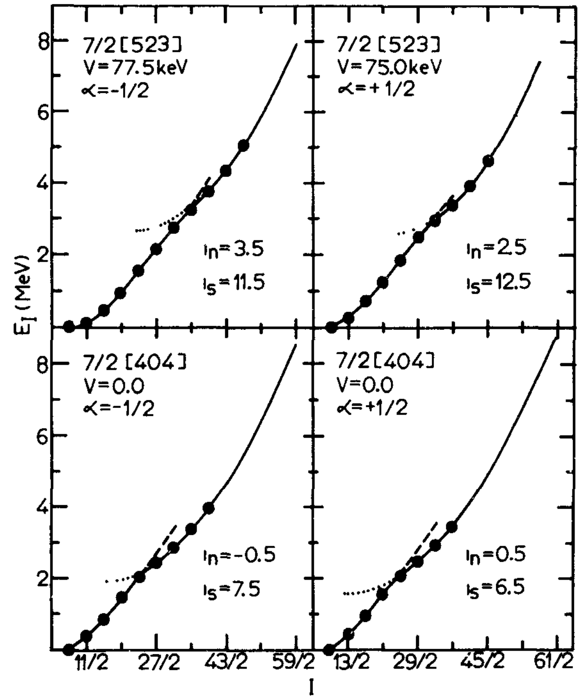

Figure 2. Similar to figure 1 but for two different bands in ^{159}Tm.

of the same order. Out of the six bands where data are available for both the favoured and the unfavoured sequences, five cases give the same interaction strength for both the sequences. The exceptions are ^{155}Ho and ^{163}Er. The $7/2^-$[523] band of ^{155}Ho gives an interaction strength $|V| = 25{\cdot}0$ keV for the unfavoured sequence and $|V| = 47{\cdot}5$ keV for the favoured sequence. The other nucleus is ^{163}Er which for $5/2^-$[523] band yields $|V| = 0{\cdot}0$ keV and $45{\cdot}0$ keV for the unfavoured and the favoured sequence respectively.

The variation of $|V|$ with neutron number has been studied theoretically by Bengtsson and Frauendorf (1979a, b) for even-even nuclei and also for $i_{13/2}$ bands by Bengtsson *et al* (1978) and Almberger *et al* (1979). An important characteristic of $|V|$ as pointed out by Bengtsson *et al* (1978) and Bengtsson and Frauendorf (1979a, b) is its oscillatory behaviour as a function of the location of the Fermi energy or, in other words, the neutron number N. The interaction strength obtained from the experimental data in general exhibits good agreement with the theoretical results in even-even nuclei. Recent analysis of Bonatsos (1985) carried out for the even-even nuclei confirms these results. However the oscillations in $|V|$ corresponding to an odd particle in the $i_{13/2}$ orbital have been predicted to be out of phase with those of the even-even system (Almberger *et al* 1979). It has also been pointed out that the values of $|V|$ are generally larger than the values in the even system. It would be very interesting to compare our results shown in tables 2 and 3 based on experimental data with these theoretical predictions.

In figures 3 and 4, we plot the interaction strength $|V|$ between the normal and the S-band as a function of the neutron number N for odd-N and odd-Z nuclei respectively. Also shown in the same figures is the isotopewise variation of $|V|$ with

Table 2. Details of odd-N bands studied in the present work. F denotes the favoured sequence and U the unfavoured sequence. $I_{\max}$ is the highest spin included in the calculations. i_n and i_s represent the alignment of n-band and s-band respectively $|V|$ represents the calculated interaction strength. The last column gives references for the experimental data.

Nucleus	Band	Sequence	$I_{\max}$	i_n	i_s	$\|V\|$(keV)	References
$^{157}\mathrm{Er}_{89}$	$i_{13/2}$	F	53/2	6·5	12·5	25·0	Holzmann *et al* (1983)
$^{161}\mathrm{Er}_{93}$	$3/2^-[521]$	F	49/2	2·5	4·5	60·0	Garrett *et al* (1982)
$^{163}\mathrm{Er}_{95}$	$i_{13/2}$	F	53/2	4·5	6·5	0·0	Bacelar *et al* (1985b)
$^{163}\mathrm{Er}_{95}$	$5/2^-[523]$	F	47/2	2·5	6·5	45·0	Bacelar *et al* (1985b)
$^{161}\mathrm{Yb}_{91}$	$i_{13/2}$	F	49/2	6·5	10·5	0·0	Riedinger *et al* (1980)
$^{161}\mathrm{Yb}_{91}$	$3/2^-[521]$	F	49/2	2·5	10·5	12·5	Riedinger *et al* (1980)
$^{163}\mathrm{Yb}_{93}$	$5/2^-[523]$	F	41/2	2·5	8·5	60·0	Kownacki *et al* (1983)
$^{165}\mathrm{Yb}_{95}$	$5/2^-[523]$	F	61/2	2·5	8·5	72·5	Roy *et al* (1982), Schuck *et al* (1984)
$^{167}\mathrm{Yb}_{97}$	$i_{13/2}$	F	61/2	4·5	8·5	72·5	Roy *et al* (1982) Bacelar *et al* (1985a)
$^{167}\mathrm{Yb}_{97}$	$5/2^-[523]$	F	57/2	2·5	6·5	75·0	Roy *et al* (1982), Bacelar *et al* (1985a)
$^{169}\mathrm{Yb}_{99}$	$1/2^-[521]$	F	53/2	0·5	6·5	5·0	Bacelar *et al* (1985a)
$^{169}\mathrm{Yb}_{99}$	$i_{13/2}$	F	61/2	2·5	6·5	40·0	Bacelar *et al* (1985a)
$^{167}\mathrm{Hf}_{95}$	$i_{13/2}$	F	45/2	4·5	8·5	10·0	Janssens *et al* (1981)
$^{169}\mathrm{Hf}_{97}$	$i_{13/2}$	F	49/2	4·5	10·5	87·5	Table of isotopes (1978)
$^{167}\mathrm{W}_{93}$	$i_{13/2}$	F	49/2	6·5	12·5	55·0	Gerl *et al* (1985)
$^{169}\mathrm{W}_{95}$	$i_{13/2}$	F	57/2	4·5	12·5	15·0	Recht *et al* (1982)
$^{173}\mathrm{W}_{99}$	$1/2^-[521]$	F	41/2	0·5	4·5	0·0	Walker *et al* (1978)
$^{175}\mathrm{W}_{101}$	$1/2^-[521]$	F	37/2	0·5	6·5	25·0	Walker *et al* (1978)
$^{181}\mathrm{Os}_{105}$	$1/2^-[521]$	F	33/2	0·5	8·5	85·0	Table of isotopes (1978)

Table 3. Similar to table 2 but for odd-Z bands.

Nucleus	Band	Sequence	$I_{\max}$	i_n	i_s	$\|V\|$(keV)	References
$^{155}\mathrm{Ho}_{88}$	$h_{11/2}$	U	45/2	0·5	8·5	25·0	Hagemann *et al* (1984)
		F	47/2	3·5	11·5	47·5	
$^{157}\mathrm{Ho}_{90}$	$h_{11/2}$	U	49/2	2·5	8·5	0·0	Hagemann *et al* (1984)
		F	47/2	1·5	7·5	0·0	Simpson *et al* (1985)
$^{159}\mathrm{Tm}_{90}$	$h_{11/2}$	U	61/2	2·5	12·5	75·0	Holzmann *et al* (1983)
		F	59/2	3·5	11·5	77·5	Larabee *et al* (1984)
$^{159}\mathrm{Tm}_{90}$	$7/2^+[404]$	U	41/2	0·5	6·5	0·0	Holzmann *et al* (1983)
		F	43/2	−0·5	7·5	0·0	Larabee *et al* (1984)
$^{161}\mathrm{Tm}_{92}$	$h_{11/2}$	U	45/2	2·5	12·5	35·0	Foin *et al* (1984)
		F	43/2	1·5	11·5	25·0	
$^{165}\mathrm{Lu}_{94}$	$h_{11/2}$	U	45/2	2·5	10·5	22·5	Jonsson *et al* (1984)
		F	43/2	1·5	9·5	30·0	
$^{171}\mathrm{Ta}_{98}$	$5/2^+[402]$	F	53/2	0·5	6·5	30·0	Yang *et al* (1983)
$^{171}\mathrm{Ta}_{98}$	$1/2^-[541]$	F	53/2	2·5	8·5	87·5	Yang *et al* (1983)
$^{177}\mathrm{Re}_{102}$	$1/2^-[541]$	F	49/2	2·5	8·5	50·0	Yang *et al* (1983)
$^{177}\mathrm{Re}_{102}$	$5/2^+[402]$	F	37/2	0·5	6·5	70·0	Yang *et al* (1983)
$^{181}\mathrm{Re}_{106}$	$1/2^-[541]$	F	41/2	4·5	10·5	32·5	Table of isotopes (1978)
$^{183}\mathrm{Ir}_{106}$	$1/2^-[541]$	F	41/2	4·5	8·5	55·0	Table of isotopes (1978)

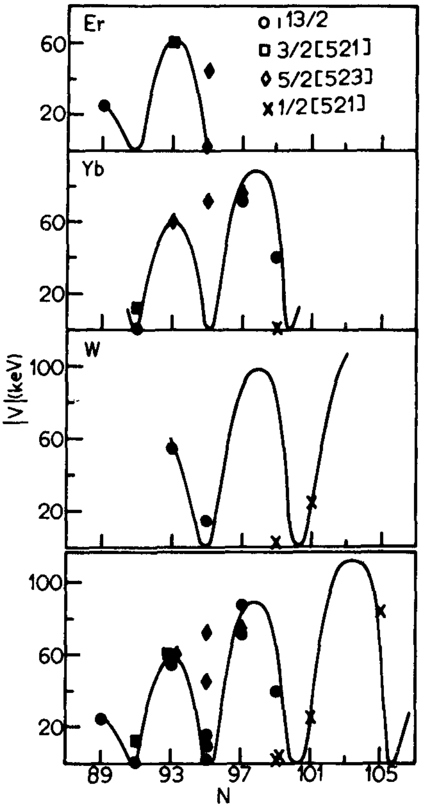

Figure 3. The interband interaction $|V|$ vs the neutron number N for odd-N bands showing the oscillatory behaviour Open and closed symbols represent the unfavoured and favoured sequences respectively. (×)s show the favoured sequence of the band. While the top panels show isotopewise variation for the cases where at least three values were known, the bottom panel combines all the data points together. The lines are drawn to roughly show the behaviour of $|V|$ as expected in the even-even systems. Our values do not show any appreciable deviation from the lines except for the two data points at $N = 95$

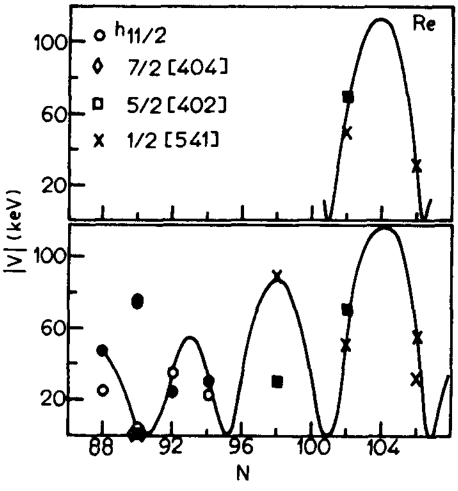

Figure 4. Similar to figure 3 but for odd-Z bands. Only two values at $N = 90$ and $N = 98$ show deviation from the behaviour for even-even system.

N provided at least three values are available. We find that $|V|$ indeed exhibits an oscillatory behaviour. However, most of the data points fall near the line showing oscillatory behaviour of interaction strength in even-even nuclei. Except for two data points for $N=95$ in figure 3 and two data points for $N=90$ and $N=98$ in figure 4, we do not observe any appreciable deviation from these lines. According to Almberger *et al* (1979), the first minimum of the fluctuation in $|V|$ for odd-A $i_{13/2}$ bands should coincide with the second maximum obtained for the even-even nuclei, while the second minimum of odd-A system should coincide with the third minimum of the even-even case. It implies that the odd-A interaction strength should exhibit a minimum at $N=92$–93 and no minimum at $N=95$. Our data on the other hand do not give a minimum at $N=92$–93 and show a minimum at $N=95$. This is just opposite to the theoretical predictions. We thus observe 'no-phase shifting' in odd-A data as compared to the even-even data. Even if we plot the strength for $i_{13/2}$ bands alone, as in the calculations of Almberger *et al*, no phase shift is observed.

That the behaviour of odd-A systems turns out to be identical to the even-even system, is perhaps due to the use of the effective decoupling mechanism wherein the odd-A band, with appropriate shifting, behaves like its even-even core band. It implies that the effective decoupling is indeed 'effective' in these nuclei. However, the interaction strength is seen to vary between 0 keV to 90 keV approximately, a range slightly wider than that for the even-even cases (0 to 70 keV), in agreement with the theoretical expectations

We note that four data points show significant deviations from the general trend. To be specific, the two data points deviating in figure 3 correspond to the $5/2^-[523]$ neutron band. We note that no such deviation is found for this band for $N=93$ and 97. It is therefore not clear why this deviation is seen at $N=95$. Both the sequences of the $h_{11/2}$ band of ^{159}Tm yield a value of $|V|=75$ keV which is very different from the value of $|V|=0{\cdot}0$ keV for the $7/2^+[404]$ band in the same nucleus (figure 4). Another deviation in figure 4 is seen to occur at $N=98$. In view of these deviations it is important that another model like the variable moment of inertia (VMI) model be used to describe the n-band (Gregory and Taylor 1972; Volkov 1972) and the interaction strength $|V|$ be evaluated and compared with the present results. Any model dependence in the values of $|V|$ will become clear from such studies. We are presently undertaking this problem for detailed investigation.

4. Conclusions

In conclusion, we find that a simple two-band mixing model based on the effective decoupling picture can describe satisfactorily the first bandcrossing in odd-A bands where sufficient data are known and the effective decoupling is applicable. Very good fits to the experimental level energies were obtained. Such an analysis provides useful information about the interaction matrix element $|V|$. We find that the values of $|V|$ for different bands in a given nucleus are almost the same. Also the value of $|V|$ for both the sequences of a band is similar. Further we observe that the behaviour of $|V|$ is not different from its behaviour in even-even systems. We do not notice any appreciable shift in the phase of oscillation of the odd-A values of $|V|$ as compared to the even-even behaviour. However, four of the cases included in this study gave values of $|V|$ which differ from the general trend.

Acknowledgements

This work was supported by the Department of Atomic Energy, Government of India, and the National Science Foundation under contract PHY-8605032 with the Florida State University.

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