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A quark model based on QCD scale anomaly

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Abstract. We consider a quark model based on QCD scale anomaly in which the quarks move in the field of an effective glueball field. Hadrons correspond to solitonic bags of higher energy density in a nonperturbative sea of condensed gluons. We calculate the static properties of nucleon in this model and find that the nucleon mass is far too large (2·4–4 GeV) and the proton charge radius (0·37–0·54 fm) is low. The proton gyromagnetic ratio and g_A/g_V are in reasonable agreement with the experimental numbers.

Keywords. Quarks; scale anomaly; glueball; soliton; nucleon; static properties; quantum chromodynamics.

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1. Introduction

At present, it is not possible to derive the low energy properties of hadrons from the first principles of QCD. This is because the effective coupling constant of QCD is large at low energies and perturbative techniques cannot be applied in this regime. Hence, one has to rely on phenomenological models in the low energy region. The various characteristic features of QCD provide the basis for the construction of such models. For instance, in the MIT bag model, confinement is assumed and the quarks are restricted to a bag which is identified with a hadron (Chodos *et al* 1974). Starting from some general arguments based on the 'diaelectric' nature of QCD vacuum, one can construct a class of solitonic bag models in which the bag formation is due to the interaction of quarks with an effective gluon condensate scalar field (see for instance Lee 1981). Here the quark-scalar field interaction is of the familiar Yukawa type but the potential corresponding to the self-interaction of the scalar field is arbitrary subject to some general restrictions. The MIT and SLAC bag models are obtained in some limiting situations in this class of models. In the Skyrme model, baryons are the topological solitons of a chiral invariant, nonlinear mesonic field theory (for a review see Zahed and Brown 1986). There are also other approaches like hybrid models in which both the quarks and mesonic degrees of freedom are present with chiral invariant couplings (Banerjee 1987). Each of the models has its own advantages and limitations and none of them can claim to have a distinct advantage over the others.

One of the distinguishing features of QCD is its anomalous behaviour under scale transformations (Nielsen 1977; Collins *et al* 1977). Indeed the vacuum expectation value of the scale anomaly sets the confinement scale which is one of the two fundamental scales of QCD, the other being the chiral symmetry-breaking scale. The

QCD scale anomaly can be incorporated in an effective Lagrangian framework through a 'glueball' field (Schechter 1980; Solomone *et al* 1981). It has been shown that a bag can be automatically formed in the Skyrme model modified to possess the correct QCD scaling behaviour (Gomm *et al* 1986; Jain *et al* 1987). In this paper we chose to work with a quark model based on the scale anomaly. The model is economical in the sense that we have only the quark and glueball degrees of freedom with the simplest possible interactions. Using suitable approximations, we find that there are nontopological solitonic solutions in this model, which we identify with hadrons. In other words, hadrons are bubbles of higher energy density in a nonperturbative sea of condensed gluons. Our model is similar in spirit to the solitonic bag models mentioned earlier but the potential corresponding to the scalar self-interactions is completely fixed by the scale anomaly. For reasonable ranges of values for the glueball mass and coupling constant associated with the quark-glueball interactions which are the only free parameters in this model, we find that the nucleon mass comes out too large (2·4–4 GeV) and the proton charge radius (0·37–0·54 fm) is smaller than the experimental value ($\approx 0{\cdot}8$ fm). However the dimensionless parameters fare better. g_A/g_V is in the range (1·1–1·26) compared with the experimental value of 1·23 and the proton gyromagnetic ratio μ_p is (1·97–3·05), the measured value being 2·79. This is to be compared with the results of the modified Skyrme model where the electric and magnetic charge radii are predicted to be within 10–15% of the experimentally measured quantities after fitting the N and Δ masses (at the cost of pushing down the value of F_π) and g_A/g_V is very low ($\approx 0{\cdot}60$).

The paper is planned as follows: in §2 we set up the effective Lagrangian and the equations of motion. We give details of the approximation scheme we employ to obtain the soliton energy in §3 and discuss the soliton solutions in §4. In §5, we calculate the static properties of the nucleon in this model and present the numerical results. We make a few concluding remarks in the end.

2. A quark-glueball interaction model

In QCD with massless quarks the divergence of the scale current or dilation current J^D_μ is zero from naive calculations. A more careful calculation yields the scale anomaly equation (Nielson 1977; Collins *et al* 1977):

$$\partial_\mu J^{\mu D} = \theta^\mu_\mu = \frac{\beta(g)}{g} \mathrm{Tr}\,(F_{\mu\nu}F^{\mu\nu}) \equiv H. \tag{1}$$

Here θ^μ_μ is the trace of the energy momentum tensor. From QCD sum rules (Shifman *et al* 1979)

$$\langle \partial_\mu J^{\mu D} \rangle \simeq (0{\cdot}34\,\mathrm{GeV})^4. \tag{2}$$

First, let us consider a simple effective Lagrangian for QCD without matter fields. We assume that the degrees of freedom reduce to that of the single order parameter field H defined in (1). Then it can be shown (Schechter 1980; Solomone *et al* 1981) that the unique Lagrangian (up to two derivatives) which satisfies the scale anomaly equation is:

$$\mathscr{L}_H = \tfrac{1}{32} b^2 H^{-3/2} (\partial_\mu H)^2 - \tfrac{1}{4} H \ln (H/\Lambda^4), \tag{3}$$

where b is a dimensionless parameter and Λ is the needed scale for QCD. The potential term in (3) has a minimum at:

$$\langle H \rangle = \Lambda^4/e \simeq (0{\cdot}34\,\mathrm{GeV})^4 \tag{4}$$

This yields a negative vacuum energy density, $-\Lambda^4/4e$.

It is convenient to introduce a glueball field χ defined by

$$H = \chi^4 \tag{5}$$

The vacuum value of χ is

$$\langle \chi \rangle \equiv \chi_{\mathrm{vac}} = \Lambda e^{-1/4} \simeq 0{\cdot}34\,\mathrm{GeV} \tag{6}$$

and the mass of the glueball quantum is seen to be

$$m_\chi = (2\langle \chi \rangle)/b \simeq (0{\cdot}68\,\mathrm{GeV})/b. \tag{7}$$

The experimental situation regarding the glueball states is quite confusing and we leave b as a parameter. Note that $b = 0{\cdot}5$ if the O^{++} glueball state is around $1{\cdot}4\,\mathrm{GeV}$.

Now we introduce the quark fields which are coupled to the glueball field. As long as we are considering light quarks, they do not contribute to scale anomaly. Subtracting the vacuum value of the potential, our Lagrangian is

$$\mathscr{L} = i\sum_i \bar{\psi}_i \gamma^\mu \partial_\mu \psi_i - f\sum_i \bar{\psi}_i \psi_i \chi + \tfrac{1}{2} b^2 (\partial_\mu \chi)^2 - \chi^4 \ln(\chi/\Lambda) - \Lambda^4/4e. \tag{8}$$

Here the summation is over flavour degrees of freedom.

In the ground state, all the quarks are in the same energy eigenstate ε, with the same wavefunction (apart from the spin function s_i),

$$\psi_i = \frac{1}{\sqrt{4\pi}} \begin{pmatrix} G(r) s_i \\ iF(r)\boldsymbol{\sigma}\cdot\hat{r} s_i \end{pmatrix} \exp(-i\varepsilon t) \tag{9}$$

Also χ is time-independent and is a function of r only in the ground state. It is convenient to introduce the dimensionless variables defined by:

$$\begin{aligned} x &= \langle \chi \rangle r, & \bar{G} &= \langle \chi \rangle^{-3/2} G, \\ \bar{F} &= \langle \chi \rangle^{-3/2} F, & \bar{\chi} &= \chi/\langle \chi \rangle \end{aligned} \tag{10}$$

and $\bar{\varepsilon} = \varepsilon/\langle \chi \rangle$. Then the equations of motion are:

$$\begin{aligned} \frac{\mathrm{d}\bar{G}}{\mathrm{d}x} &= (-f\bar{\chi} - \bar{\varepsilon})\bar{F} \\ \frac{\mathrm{d}\bar{F}}{\mathrm{d}x} + \frac{2}{x}\bar{F} &= (-f\bar{\chi} + \bar{\varepsilon})\bar{G} \end{aligned} \tag{11}$$

and

$$b^2\left(\frac{\mathrm{d}^2\bar{\chi}}{\mathrm{d}x^2} + \frac{2}{x}\frac{\mathrm{d}\bar{\chi}}{\mathrm{d}x}\right) = \frac{Nf}{4\pi}(\bar{G}^2 - \bar{F}^2) + 4\bar{\chi}^3 \ln\bar{\chi}$$

for the ground state with N quarks. The normalization condition is:

$$\int_0^\infty (\bar{G}^2 + \bar{F}^2)x^2\,dx = 1. \tag{12}$$

The ground state energy of N-quark state is given by:

$$\begin{aligned} E &= N\varepsilon + \int d^3x\left[\frac{b^2}{2}(\nabla\chi)^2 + \chi^4\ln(\chi/\Lambda) + \Lambda^4/4e\right] \\ &= \langle\chi\rangle\left[N\bar{\varepsilon} + 4\pi\int_0^\infty x^2\,dx\left(\frac{b^2}{2}\left(\frac{d\bar{\chi}}{dx}\right)^2 + \bar{\chi}^4(\ln\bar{\chi} - \tfrac{1}{4}) + \tfrac{1}{4}\right)\right]. \end{aligned} \tag{13}$$

Note that the minimization of energy with respect to the variation of the functional form of χ is equivalent to solving the equation of motion. If localized solutions to (11) exist, hadrons can be considered as bubbles of higher energy density (with respect to the vacuum) or 'solitonic bags'; they are composed of quarks held together by the glueball field. Note that the solitons are nontopological and their quantum numbers are the same as those of the N-quark systems they are made of.

3. Approximation scheme

As equations (11) are very difficult to solve even numerically, we have to make some reasonable approximations. For the variational calculation of the minimum of the total energy E in (13) we have to make a suitable ansatz for χ depending on some parameters. The quark eigen energy ε and the contribution to the energy from the glueball field can be calculated for fixed values of these parameters. The minimum of the total energy with respect to the variation of these parameters gives the soliton energy. Anticipating the fact that the minima exist and that the quark wavefunctions fall rapidly to zero outside some finite region (the bag), it is obvious that χ can differ appreciably from its vacuum value only inside this region, in the ground state. We assume that χ varies linearly inside the bag and is constant outside it where its value is equal to its vacuum value, $\Lambda\bar{e}^{1/4}$. Apart from simplicity, it should be mentioned that the solutions for χ found in the modified Skyrme model mentioned earlier are nearly linear. Also, in a nonperturbative treatment of QCD it has been claimed that the gluon and quark condensates lead to a linear confining potential (Namyslowski 1987). The linear form for χ involves two parameters namely the size of the region and the slope.

Depending on its slope $\bar{\chi}$ has the form shown in figures 1a and 1b which we will refer to as case I and case II respectively. The nature of approximation for calculating $\bar{\varepsilon}$ is different for cases I and II.

CASE I

In this case, we have:

$$\begin{aligned} &\bar{\chi}(x) = 0, && x < \xi - \eta, \\ &\bar{\chi}(x) = \frac{1}{\eta}[x - (\xi - \eta)], && \xi - \eta < x < \xi, \\ &\bar{\chi}(x) = 1, && x > \xi. \end{aligned} \tag{14}$$

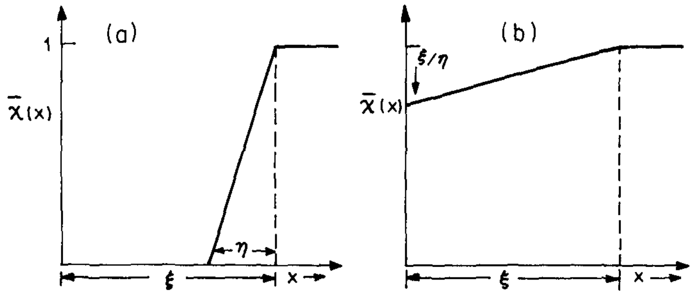

Figure 1. Assumed form of $\bar{\chi}(x)$. **a.** Case I: $\xi > \eta$. **b.** Case II: $\xi < \eta$

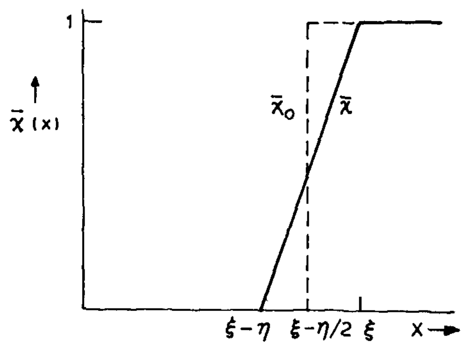

Figure 2. Solid and dotted lines correspond to $\bar{\chi}(x)$ and $\bar{\chi}_0(x)$ respectively. When $x > \xi$, $\bar{\chi}(x) = \bar{\chi}_0(x) = 1$.

Let $\bar{\varepsilon}$ and ψ be the eigenvalue and the normalized wavefunction corresponding to this potential. Consider the step function potential

$$\begin{aligned}\bar{\chi}_0(x) &= 0, \quad x < \xi', \\ &= 1, \quad x > \xi', \end{aligned} \tag{15}$$

where $\xi' = \xi - \eta/2$. $\bar{\chi}(x)$ and $\bar{\chi}_0(x)$ differ from each other in the region $\xi - \eta < x < \xi$ as shown in figure 2.

Intuitively one expects that the eigenvalues corresponding to $\bar{\chi}(x)$ and $\bar{\chi}_0(x)$ are nearly equal (if $\eta \ll \xi$ this is obviously true). Now it is easy to calculate the eigenvalue $\bar{\varepsilon}_0$ and the normalized wavefunction ψ_0 for the step function potential $\bar{\chi}_0(x)$. Then in the first order in perturbation theory ($\bar{\chi}(x) - \bar{\chi}_0(x)$ is the perturbed potential),

$$\begin{aligned}\bar{\varepsilon} &\approx \bar{\varepsilon}_0 + f\int \bar{\psi}_0\psi_0(\bar{\chi}(x) - \bar{\chi}_0(x))4\pi x^2\,dx \\ &= \bar{\varepsilon}_0 + f\int (\bar{G}_0^2 - \bar{F}_0^2)(\bar{\chi}(x) - \bar{\chi}_0(x))x^2\,dx \end{aligned} \tag{16}$$

and

$$\psi \approx \psi_0 = \frac{1}{\sqrt{4\pi}}\begin{pmatrix}\bar{G}_0 s \\ i\bar{F}\boldsymbol{\sigma}\cdot\hat{\boldsymbol{r}}s\end{pmatrix}.$$

This approximation makes sense only if $\bar{\varepsilon}$ and $\bar{\varepsilon}_0$ are close to each other. Later we will find that this is indeed so.

It is convenient to define

$$\tilde{\varepsilon} = \bar{\varepsilon}_0/f \quad \text{and} \quad \zeta = f\xi'. \tag{17}$$

For the step function potential $\bar{\chi}_0(x)$, $\tilde{\varepsilon}$ is the solution of the transcendental equation,

$$\frac{1}{\tilde{\varepsilon}\zeta} - \cot(\tilde{\varepsilon}\zeta) = \left(\frac{1-\tilde{\varepsilon}}{1+\tilde{\varepsilon}}\right)^{\frac{1}{2}} + \frac{1}{(1+\tilde{\varepsilon})\zeta}. \tag{18}$$

We find that the following fit for $\tilde{\varepsilon}$ is quite adequate:

$$\tilde{\varepsilon}(\zeta) = (2{\cdot}042/\zeta) - (0{\cdot}96/\zeta^2). \tag{19}$$

$\bar{G}_0(x)$ and $\bar{F}_0(x)$ are given by the expressions:

$$\begin{aligned} \bar{G}_0 &= Af^{1/2}[\sin(f\tilde{\varepsilon}x)/x], \\ \bar{F}_0 &= -\frac{Af^{1/2}}{x}\left(\cos(f\tilde{\varepsilon}x) - \frac{\sin(f\tilde{\varepsilon}x)}{f\tilde{\varepsilon}x}\right) \end{aligned} \qquad x < \xi' \tag{20a}$$

and

$$\begin{aligned} \bar{G}_0(x) &= Cf^{1/2}\exp[-f(1-\tilde{\varepsilon}^2)^{\frac{1}{2}}x] \\ \bar{F}_0(x) &= \frac{Cf^{1/2}}{x(1+\tilde{\varepsilon})}\left[\frac{1}{fx} + (1-\tilde{\varepsilon}^2)^{1/2}\right]\exp[-f(1-\tilde{\varepsilon}^2)^{\frac{1}{2}}x], \end{aligned} \qquad x > \xi' \tag{20b}$$

where

$$A = \left[\zeta + \sin^2\tilde{\varepsilon}\zeta\left\{\frac{1}{(1+\tilde{\varepsilon})(1-\tilde{\varepsilon}^2)^{1/2}} + \frac{1}{(1+\tilde{\varepsilon}^2)\zeta} - \frac{1}{\tilde{\varepsilon}^2\zeta}\right\}\right]^{-1/2} \tag{21}$$

and $C = A\sin\tilde{\varepsilon}\zeta\exp[(1-\tilde{\varepsilon}^2)^{\frac{1}{2}}\zeta]$. By knowing $\bar{\varepsilon}_0$, $\bar{G}_0(x)$, $\bar{F}_0(x)$ and $\bar{\chi}(x) - \bar{\chi}_0(x)$, we calculate $\bar{\varepsilon}$, which we write as $\bar{\varepsilon}(f, \xi, \eta)$, using (16). After a straightforward but tedious calculation, we find

$$\begin{aligned} \bar{\varepsilon}(f,\xi,\eta) \simeq f\tilde{\varepsilon}(\zeta) + \frac{A^2}{\eta}\Bigg[& -\frac{f\eta}{4\tilde{\varepsilon}}\sin(2\tilde{\varepsilon}\zeta) - \frac{1}{4\tilde{\varepsilon}^2}\{\cos(2\tilde{\varepsilon}\zeta) - \cos(2f\tilde{\varepsilon}(\xi-\eta))\} \\ & + \frac{f\eta}{2}\frac{\sin^2\tilde{\varepsilon}\zeta}{\tilde{\varepsilon}^2\zeta} + \frac{1}{2\tilde{\varepsilon}^2}\int_{f(\xi-\eta)}^{\zeta}\frac{(\cos 2\tilde{\varepsilon}x - 1)}{x}\,dx \\ & + \sin^2\tilde{\varepsilon}\zeta\Bigg\{\frac{-f\eta\tilde{\varepsilon}}{2(1+\tilde{\varepsilon})(1-\tilde{\varepsilon})^{1/2}} + \frac{\tilde{\varepsilon}}{2(1+\tilde{\varepsilon})(1-\tilde{\varepsilon}^2)^{+1/2}} + \frac{f\eta}{2(1+\tilde{\varepsilon})^2} \\ & - \frac{\tilde{\varepsilon}\exp[-f(1-\tilde{\varepsilon}^2)^{1/2}\eta]}{2(1+\tilde{\varepsilon})(1-\tilde{\varepsilon})} - \frac{\exp[2(1-\tilde{\varepsilon}^2)^{1/2}\zeta]}{(1+\tilde{\varepsilon})^2} \\ & \times \int_{\zeta}^{f\xi}\frac{\exp[-2(1-\tilde{\varepsilon}^2)^{1/2}x]}{x}\,dx\Bigg\}\Bigg]. \end{aligned} \tag{22}$$

The contribution of the glueball kinetic and potential energies (in units of $\langle\chi\rangle$) is given by the expression:

$$E_{\bar{\chi}} = \int_0^{\infty}\left[\frac{b^2}{2}\left(\frac{d\bar{\chi}}{dx}\right)^2 + \bar{\chi}^4(\ln\bar{\chi} - \tfrac{1}{4}) + \tfrac{1}{4}\right]4\pi x^2\,dx. \tag{23}$$

Corresponding to case I, with $\bar{\chi}(x)$ described in (14) we have:

$$E_{\bar{\chi}}^{(1)}(\xi,\eta) = 4\pi\left[\frac{b^2}{6\eta^2}\{\xi^3 - (\xi-\eta)^3\} + \frac{1}{12}\xi^3 - \frac{11}{196}\eta^3 - \frac{5}{36}\eta^2(\xi-\eta) - \frac{9}{100}\eta(\xi-\eta)^2\right]. \quad (24)$$

Then the total energy which we write as $E^{(I)}(f,\xi,\eta)$ for an N-quark bound state is given by:

$$E^{(I)}(f,\xi,\eta) = \langle\chi\rangle[N\bar{\varepsilon}(f,\zeta,\eta) + E_{\bar{\chi}}^{(I)}(\xi,\eta)]. \quad (25)$$

This has to be minimized with respect to variation of ξ and η to find the soliton energy.

For the range of values of ξ and η of interest to us, we find that $\bar{\varepsilon}$ and $\bar{\varepsilon}_0$ differ by $\approx 3\%$ for $f \approx 2\cdot5$. The difference increases with f and is ≈ 6–17% for $f = 6$. The soliton energies found using $\bar{\varepsilon}$ and $\bar{\varepsilon}_0$ also differ by nearly the same amounts. Considering the fact that our model is very incomplete and that most of the phenomenological models predict hadronic parameters which differ from the actual values by $\sim 20\%$ the approximations we have made are adequate.

CASE II

In this case corresponding to figure 1b we have

$$\bar{\chi}(x) = \frac{1}{\eta}[x - (\xi - \eta)], \quad x < \xi,$$
$$\bar{\chi}(x) = 1 \qquad x > \xi, \quad (26)$$

with $\xi < \eta$. The expression for the quark Hamiltonian H (in units of $\langle\chi\rangle$) is:

$$H = \boldsymbol{\alpha}\cdot\mathbf{p} + \beta f \bar{\chi}(x) \quad (27)$$

which we write as

$$H = \boldsymbol{\alpha}\cdot\mathbf{p} + \beta f + \beta V, \quad (28)$$

where $V = f(\bar{\chi}(x) - 1)$.

This corresponds to the Hamiltonian for a spin-$\frac{1}{2}$ particle with mass f moving in a potential V. It is difficult to find the eigenvalues of H even approximately as it involves both even and odd operators. When the potential is small compared to the mass f, one can apply Foldy-Wouthuysen transformation (see for example Bjorken and Drell 1964) on the Hamiltonian such that it contains only even operators up to some order in $1/f$. Then it is easier to find the eigenvalues of the Hamiltonian. In our case, this procedure is perfectly justified when $\xi \ll \eta$ as $V \ll f$ for all values of x. However even in an extreme case when $\xi = \eta$, the average value of V is $\approx 0\cdot5f$ in the region $0 < x < \xi$. Hence the procedure seems to be justified for finding the approximate value of eigenenergy. We find that when we work to higher orders in $1/f$, the eigenenergy seems to converge for positive energy solutions. The transformed Hamiltonian which is correct to $o(1/f^3)$ is:

$$H_{fw} = f + \frac{1}{2f}O^2 - \frac{1}{8f^3}O^4 + V - \frac{1}{8f^2}(O^2 + 2OVO + VO^2) + \frac{1}{8f^3}\{O, V\}^2 + O(1/f^4), \tag{29}$$

where $O = \alpha \cdot \mathbf{p}$.

We find the eigenvalue of this Hamiltonian by variational method. We assume an exponential trial wavefunction (normalized),

$$\psi = \left(\frac{\lambda^3}{\xi^3 \pi}\right)^{1/2} \exp(-\lambda x/\xi), \tag{30}$$

where λ is the variational parameter. Then,

$$\begin{aligned}\langle \psi | H_{fw} | \psi \rangle = f + \frac{1}{2}\frac{f\xi}{\eta}\Bigg[& \frac{3\eta}{4f^4\xi^5}\lambda^4 + \left(\frac{\eta}{\xi^3 f^2} + \frac{1}{f^2\xi^2} + \frac{1}{f^2\xi\eta}\right)\lambda^2 \\ & - \left(\frac{1}{f^2\xi^2} + \frac{2}{f^2\xi\eta}\right) + \frac{3}{\lambda} - 2 - \frac{7}{4f^2\xi\eta} \\ & + \exp(-2\lambda)\left\{ -2\lambda - 4 - \frac{3}{\lambda} + \frac{1}{f^2\xi^2}(\lambda^3 + \lambda^2 + \lambda) \right. \\ & \left. - \frac{1}{4f^2\xi\eta}(2\lambda^2 + 3\lambda + 7)\right\}\Bigg].\end{aligned} \tag{31}$$

The quark eigenenergy $\bar{\varepsilon}(f, \xi, \eta)$ is the minimum of this when λ is varied:

$$\bar{\varepsilon}(f, \xi, \eta) = \langle \psi | H_{fw} | \psi \rangle_{\min}. \tag{32}$$

With $\bar{\chi}(x)$ given by (26), the glueball part of the energy (in units of $\langle \chi \rangle$) is given by the expression:

$$\begin{aligned}E_{\bar{\chi}}^{(\mathrm{II})}(\xi, \eta) = 4\Bigg[& \frac{b^2\xi^3}{6\eta^2} + \frac{\xi^3}{12} + \frac{1}{\eta^4}\Bigg\{ -\frac{11\eta^7}{196} - \frac{5\eta^6}{36}(\xi - \eta) - \frac{9\eta^5}{100} \\ & - (\xi - \eta)^2 - 0{\cdot}0007233(\xi - \eta)^7 + \frac{(\xi - \eta)^7}{105}\ln\left(\frac{\eta - \xi}{\eta}\right)\Bigg].\end{aligned} \tag{33}$$

Then the total energy for case II is given by:

$$E^{(\mathrm{II})}(f, \xi, \eta) = \langle \chi \rangle [N\bar{\varepsilon}(f, \xi, \eta) + E_{\bar{\chi}}^{(\mathrm{II})}(\xi, \eta)]. \tag{34}$$

4. Soliton solutions

We compute the energy for various values of ξ and η, and locate its minimum for various values of f and b. This minimum corresponds to the soliton solution. We have not found any soliton solution corresponding to case II discussed above.

Table 1. Soliton solutions for a two-quark bound state. All energies are in units of $\langle\chi\rangle$

f	b	ξ	η	$E_{\bar{\chi}}$ Glueball energy	$\bar{\varepsilon}$ Quark eigenenergy	$E = E_{\bar{\chi}} + 2\bar{\varepsilon}$ Total energy
2·5	0·25	—	—	—	—	—
2·5	0·33	—	—	—	—	—
2·5	0·5	—	—	—	—	—
2·5	0·75	—	—	—	—	—
3·0	0·25	1·1	0·55	1·30	2·08	5·46
3·0	0·33	1·1	0·66	1·38	2·21	5·80
3·0	0·5	1·1	0·88	1·46	2·48	6·42
3·0	0·75	—	—	—	—	—
4·0	0·25	1·1	0·44	1·60	2·09	5·78
4·0	0·33	1·1	0·55	1·69	2·24	6·17
4·0	0·5	1·2	0·84	2·07	2·46	6·99
4·0	0·75	1·1	0·99	2·10	3·01	8·12
5·0	0·25	1·1	0·44	1·60	2·18	5·96
5·0	0·33	1·1	0·55	1·69	2·36	6·41
5·0	0·5	1·2	0·84	2·07	2·63	7·33
5·0	0·75	1·1	0·88	2·58	3·05	8·68
6·0	0·25	1·1	0·44	1·60	2·25	6·10
6·0	0·33	1·2	0·6	2·01	2·29	6·59
6·0	0·5	1·1	0·66	2·21	2·69	7·59
6·0	0·75	1·2	0·96	2·93	3·07	9·07

Corresponding to case I we have soliton solutions (both for $N = 2$ and $N = 3$) for f greater than $\approx 2{\cdot}5$. Tables 1 and 2 given the details of the solutions for $f = 2{\cdot}5$, 3, 4, 5 and 6 and $b = 0{\cdot}25$, 0·33, 0·5 and 0·75.

5. Static properties of the nucleon

5.1 *Nucleon masses*

We have to consider the centre of mass motion of the quarks before we calculate the mass of the hadron. The hadron energy E and mass M are related through the relation:

$$E = (\mathbf{P}^2 + M^2)^{1/2}. \tag{35}$$

Here $\mathbf{P}$ is the total momentum associated with the motion of the quarks:

$$\mathbf{P} = \sum_{i=1}^{N} \mathbf{p}_i, \tag{36}$$

where $\mathbf{p}_i$ are the momenta of the individual quarks. We replace $\mathbf{P}^2$ by its expectation value in (35). As the motion of the quarks are uncorrelated,

$$\langle \mathbf{P}^2 \rangle = N \langle \mathbf{p}^2 \rangle. \tag{37}$$

Table 2. Soliton solutions for a three-quark bound state All energies are in units of $\langle \chi \rangle$

f	b	ξ	η	$E_{\tilde{\chi}}$ Glueball energy	$\tilde{\varepsilon}$ Quark eigenenergy	$E = E_{\tilde{\chi}} + 3\tilde{\varepsilon}$ Total energy
2·5	0·25	1·2	0·60	1·59	1·85	7·14
2·5	0·33	1·2	0·72	1·65	1·95	7·50
2·5	0·5	1·2	0·96	1·71	2·17	8·22
2·5	0·75	—	—	—	—	—
3·0	0·25	1·2	0·48	1·93	1·83	7·42
3·0	0·33	1·2	0·60	2·01	1·95	7·86
3·0	0·5	1·2	0·84	2·07	2·22	8·73
3·0	0·75	—	—	—	—	—
4·0	0·25	1·2	0·48	1·93	1·95	7·78
4·0	0·33	1·3	0·65	2·38	1·97	8·29
4·0	0·5	1·2	0·72	2·56	2·27	9·37
4·0	0·75	1·3	1·04	3·31	2·53	10·90
5·0	0·25	1·3	0·52	2·31	1·90	8·01
5·0	0·33	1·3	0·65	2·38	2·07	8·59
5·0	0·5	1·3	0·78	2·95	2·26	9·73
5·0	0·75	1·3	1·04	3·31	2·73	11·50
6·0	0·25	1·3	0·52	2·31	1·95	8·16
6·0	0·33	1·2	0·48	2·53	2·09	8·80
6·0	0·5	1·3	0·78	2·93	2·37	10·06
6·0	0·75	1·3	0·91	4·09	2·62	11·95

Using the components of the wavefunction given in (20), we obtain

$$\mathbf{p}^2 = f^2\left[\tilde{\varepsilon}^2 - \frac{A^2 \sin^2 \tilde{\varepsilon}\zeta}{(1+\tilde{\varepsilon})^2}\left\{\frac{1+\tilde{\varepsilon}}{(1-\tilde{\varepsilon}^2)^{1/2}} + \frac{1}{\zeta}\right\}\right]. \tag{38}$$

M can be calculated with the aid of equations (35), (37) and (38). Later we find that the correction due to the centre of mass motion is quite small.

5.2 *Proton charge radius*

The mean square radius of the charge distribution of a hadron is given by the expression

$$\langle r^2 \rangle = \sum_i Q_i \int \psi_i^\dagger r^2 \psi_i \, d^3x, \tag{39}$$

where Q_i refers to the charge of the ith quark. As all the quarks have the same spatial wavefunction,

$$\langle r^2 \rangle = \int \psi^\dagger \psi r^2 \, d^3x,$$

for a proton. Using the wavefunction already obtained,

$$\langle x^2 \rangle = \frac{A^2}{f^2}\left[\frac{\zeta^3}{3} + \frac{\zeta}{\tilde{\varepsilon}}\cos^2\tilde{\varepsilon}\zeta - \frac{\sin(2\tilde{\varepsilon}\zeta)}{2\tilde{\varepsilon}^3} + \frac{\sin^2\tilde{\varepsilon}\zeta}{(1+\tilde{\varepsilon})^2}\left\{(1+\tilde{\varepsilon})\left(\frac{\zeta^2}{(1-\tilde{\varepsilon}^2)^{1/2}} + \frac{\zeta}{(1-\tilde{\varepsilon}^2)} + \frac{1}{2(1-\tilde{\varepsilon}^2)^{3/2}}\right) + \zeta + \frac{1}{(1-\tilde{\varepsilon}^2)^{1/2}}\right\}\right] \tag{40}$$

in terms of the scaled variables.

5.3 *Magnetic moments*

When the quark wavefunctions differ only in their spin-part, it can be shown (see for instance Lee 1981) that the proton magnetic moment μ_p is given by:

$$\mu_p = \frac{e}{2}\int \psi_\uparrow^\dagger (\mathbf{r}\times\boldsymbol{\alpha})_Z \psi_\uparrow \, d^3x, \tag{41}$$

where $\uparrow$ indicates the spin-up wavefunctions. This is:

$$\mu_p = \frac{e}{2}\frac{4}{3}\frac{A^2}{f}\left[\frac{\zeta}{2\tilde{\varepsilon}}\left(1 + \frac{\cos 2\tilde{\varepsilon}\zeta}{2}\right) - \frac{3}{8}\frac{\sin 2\tilde{\varepsilon}\zeta}{\tilde{\varepsilon}^2} + \frac{\sin^2\tilde{\varepsilon}\zeta}{(1+\tilde{\varepsilon})}\left\{\frac{3}{4}(1-\tilde{\varepsilon}^2)^{-1/2} + \frac{\zeta}{2}\right\}\right] \tag{42}$$

in units of $\langle \chi \rangle^{-1}$. The neutron magnetic moment is related to μ_p through the SU(6) relation:

$$\mu_n = -\tfrac{2}{3}\mu_p. \tag{43}$$

5.4 g_A/g_V

The ratio of the axial vector and vector coupling constants is defined through the relation:

$$\frac{g_A}{g_V} = \frac{\langle p\uparrow | A_3^{1+i2} | n\uparrow \rangle}{\langle p\uparrow | V_0^{1+i2} | n\uparrow \rangle}, \tag{44}$$

where A_μ^i and V_μ^i are the axial vector and vector current operators. In the quark model this reduces to

$$g_A/g_V = \tfrac{5}{3}\int \psi_\uparrow^\dagger \Sigma_3 \psi_\uparrow \, d^3x. \tag{45}$$

In our model

$$g_A/g_V = \tfrac{5}{3}[1 - \tfrac{4}{3}\alpha], \tag{46a}$$

where

$$\alpha = A^2\left[\frac{1}{2}\left(\zeta + \frac{\sin 2\tilde{\varepsilon}\zeta}{2\tilde{\varepsilon}}\right) - \frac{1}{\tilde{\varepsilon}^2\zeta}\sin^2\tilde{\varepsilon}\zeta + \frac{\sin^2\tilde{\varepsilon}\zeta}{(1+\tilde{\varepsilon})^2}\left(\frac{(1-\tilde{\varepsilon}^2)^{\frac{1}{2}}}{2} + \frac{1}{\zeta}\right)\right]. \quad (46b)$$

5.5 *Numerical results*

The numerical values of the static parameters we obtain in our model are summarized in table 3. The nucleon mass comes out too large and is in the range of 2·4–4 GeV. (it should be remembered that the nucleon mass is not predicted unambiguously in any model). The proton charge radius 0·37–0·54 fm is low. μ_p (in units of $e/2m_p$ where m_p is the calculated value of the proton mass) and g_A/g_V are in reasonable agreement with the experimental values. The numbers get progressively worse for higher values of f. It should be noted that our approximation scheme is on a better footing for smaller values of f.

Table 3. Static parameters of the nucleon. Experimental values are given in parantheses.

f	b	M_N Nucleon mass (0·938 GeV)	r_p Proton charge radius (0·8 fm)	μ_p Proton mag. moment (2 79)	g_A/g_V (1·23)
2.5	0·25	2·38	0·54	2·75	1 20
2·5	0·33	2·50	0·53	2·85	1·21
2·5	0·5	2·73	0·53	3·05	1·26
2·5	0·75	—	—	—	—
3·0	0·25	2·50	0·51	2·56	1·15
3·0	0·33	2·65	0·49	2·65	1·16
3·0	0·5	2·92	0·46	2·83	1·19
3·0	0·75	—	—	—	—
4·0	0·25	2·64	0·46	2·23	1·13
4·0	0·33	2·81	0·47	2·39	1·12
4·0	0·5	3·18	0·42	2 55	1·14
4·0	0·75	3·69	0·40	2·89	1·15
5·0	0·25	2·72	0·47	2·10	1 11
5·0	0·33	2·92	0·45	2 18	1·11
5·0	0·5	3·30	0·42	2·39	1·12
5·0	0·75	3·91	0·37	2 65	1·13
6·0	0·25	2·76	0·46	1·97	1·10
6·0	0·33	2·99	0·43	2·03	1·11
6·0	0·5	3·42	0·41	2·24	1·11
6·0	0·75	4·06	0·39	2·56	1·11

6. Discussion

We have considered a simple quark model where the only scale is the one associated with the gluon condensate field. In constructing the soliton solutions, we have made approximations at two stages: (*i*) we have used a simple linear variation for the glueball field inside the bag and (ii) we have calculated the quark eigenenergy in an approximate manner for this linear potential. We have checked that the results are not substantially altered whether we use the eigenenergy $\bar{\varepsilon}_0$ corresponding to the step function potential $\bar{\chi}_0(x)$ in figure 2 or the eigenenergy $\bar{\varepsilon}$ to the first order perturbation corresponding $\bar{\chi}(x)$ in figure 2. Hence the approximation for quark eigenenergy is reasonable. Now the actual solution for $\bar{\chi}(x)$ in our model would be definitely smoother than the form we have used. A smoother form for $\bar{\chi}(x)$ would be expected to reduce the scalar field energy. However the scalar field contribution to the soliton energy is around 25–30% as we can see from tables 1 and 2. Moreover, with a smoother form for $\bar{\chi}$, the quark eigenenergy is likely to have a higher value. Hence it appears that the nucleon mass may not be substantially altered if we improve our approximation. The limitation is that of the model itself rather than the approximation scheme. Indeed this is not surprising. We have incorporated the confinement scale in the model and have not taken into account the scale associated with the spontaneous breaking of chiral symmetry and the associated degrees of freedom. A natural extension of the model would include the chiral invariant couplings of quarks to π and σ.

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References

Banerjee M K 1987 in *Chiral solitons* (ed.) K F Liu (Singapore: World Scientific)
Bjorken J D and Drell S D 1964 *Relativistic quantum mechanics* (New York: McGraw-Hill)
Chodos A, Jaffe R J, Johnson K, Thorn C B and Weisskoff V F 1974 *Phys. Rev.* **D9** 3471
Collins J C, Duncan A and Joglekar S D 1977 *Phys. Rev.* **D16** 438
Gomm H, Jain P, Johnson R and Schechter J 1986 *Phys. Rev.* **D33** 3476
Jain P, Johnson R and Schechter J 1987 *Phys. Rev.* **D35** 2230
Lee T D 1981 *Particle physics and introduction to field theory* (Harwood: Academic Publishers)
Namyslowski J M 1987 Warsaw University Preprint IFT/12/87.
Nielsen N K 1977 *Nucl Phys.* **B210** 212
Schechter J 1980 *Phys. Rev* **D21** 3393
Solomone A, Schechter J and Tundron T 1981 *Phys. Rev.* **D23** 1143
Shifmann M A, Vainshtein A I and Zakharov V I 1979 *Nucl. Phys.* **B147** 385, 448
Zahed I and Brown G E 1986 *Phys. Rep* **142** 1