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Nucleon–nucleon interaction with tensor forces in the quark compound bag model

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Abstract. A model for N–N interaction proposed earlier by two of us (VSB and VKG), has been extended to incorporate the tensor component of the nuclear force. Based on the quark compound bag model (QCB), the nucleon–nucleon potential has a short range repulsive core which is non-local and has a characteristic energy dependence and is expressed in terms of the parameters relating to the six-quark compound bag. To account for the low energy properties, this repulsive core interaction is supplemented by a phenomenological non-local potential containing both central (S-wave) and tensor components and operates only outside the QCB. Using this model, we analyse and compare the results with the experimental data for the electromagnetic form factors of the deuteron, the D-state observables, such as the quadrupole moment, the D-state probability, and the D/S ratio along with the n-p scattering phase shifts up to about 400 MeV.

Keywords. Nucleon–nucleon interaction; tensor forces; quark compound bag model; bound state; deuteron; electromagnetic form factors.

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1. Introduction

One of the formidable, if not impossible, tasks in quantum chromodynamics is to derive nucleon–nucleon interaction in a rigorous way starting from the complex nature of the q–q force. Indeed, besides the obvious increase in complexity due to the increasing number of quarks and the associated symmetries involved, from a dynamical standpoint, quarks are known to interact weakly at short distances, whereas at large distances they interact strongly. Consequently, the standard approximation techniques, with a limited range of validity, cannot be meaningfully applied in the entire domain.

At the level of non-relativistic constituent quark model, attempts (Harvey 1981; Oka and Yazaki 1981; Faessler *et al* 1985; Maltman and Isgur 1984; Harvey *et al* 1984) to study six quark systems with a view to extract information regarding the N–N system, has had some success in recent times. An important thrust of such investigations is that, while at short internucleon separation ($r \lesssim 1$ fm) the residual colour potential should play a dominant role; for larger distances, the quark pair creation is supposed to be important and as a consequence the residual colour potential must be supplemented by meson exchange forces. In other words, the picture which emerges from these studies is that one can possibly have a dynamical segregation of six quark into two three-quark nucleonic clusters represented by an effective intercluster potential with a strong repulsive core at short distances ($r \lesssim 1$ fm), plus an intermediate range attraction generated through meson (π, σ) exchanges. Various models, e.g., the hybrid quark hadron model (Kisslinger 1982), the resonating group method in six quark systems (Williams *et al* 1982), the *P*-matrix approach* and the quark composite bag model (Simonov 1981), proposed and widely used in recent times for studying N–N systems, are essentially attempts to impress upon this picture in one form or the other. Thus, for instance, based on the cluster decomposition of the total six-quark wavefunction into two parts, viz., (i) the inner wavefunction representing the six quark bag, and (ii) the outer two nucleon cluster component, Simonov (1981) was able to derive a short range repulsive core which is found to be energy-dependent and nonlocal in character. With an additional assumption that the interaction between quarks and nucleons is confined to the surface of quark compound bag (QCB), one can have a repulsive hard core produced by a strong coupling of the nucleons to the QCB. The assumption of a six-quark compound bag characterized by a discrete energy (or a set of discrete energy levels) and a bag radius provides a possible means for comparing these parameters with the information drawn from other independent studies, as for instance the study of multiquark systems in the MIT bag model (Jaffe 1977). The basic idea is to derive information about the inner quark region and to see how much can be learnt, in a self-consistent way, about the q–q contribution in the nuclear force from our knowledge of the asymptotic behaviour of nucleon-nucleon scattering with minimal assumptions regarding the dynamics of quarks.

Having adopted this approach in the spirit outlined above, we had earlier proposed a model to study the N–N interaction (Bhasin and Gupta 1985). It has a short-range repulsive core characterized in terms of the parameters relating to the six quark compound bag and a long-range S-wave separable potential, operative only outside the bag radius, to account for the low energy properties. This model was found to be fairly realistic in explaining the behaviour of $^3S_1$ and $^1S_0$ scattering phase shifts all the way up to ~ 400 MeV. However, it predicts a dip structure in the electromagnetic form factor of deuteron at $q^2 \sim 25\,\mathrm{fm}^{-2}$, but no such structure is observed experimentally. The object of the present investigation is to extend the model incorporating the tensor component of the N–N interaction with a view to study the D-state observables, such as the quadrupole moment of the deuteron, D/S asymptotic ratio etc., besides analysing the behaviour of electromagnetic form factors at large momentum transfers where the central part of the N–N interaction is found to be inadequate.

In § 2, we formulate the model, briefly reviewing the essentials of the composite bag model for the N–N system and incorporate the tensor component of the nuclear force which is operative only outside the QCB. We also introduce the definitions of various D-state observables such as the D-state probability (p_D), the D/S asymptotic ratio (η), the electric quadrupole moment (Q_D) and the magnetic moment (μ_D) of the deuteron. Finally, in § 3, we analyse and compare our results with experimental data and discuss the main points of the present investigation in the light of other investigations carried out on this topic.

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* This was initiated by Jaffe and Low (1979) and has since been applied in several processes (see, for instance, Mulders 1983).

2. The six-quark composite bag model for the N–N system

2.1 *Formal approach*

We first review, for the sake of completeness, some of the basic steps introduced earlier (Bhasin and Gupta 1985), to formulate the nucleon–nucleon force in the framework of QCB model.

We have here the basic ansatz that the total wavefunction can be represented as a sum of two parts, viz.,

$$\psi = \psi_{\text{int}} + \psi_{\text{ext}}, \tag{1}$$

which satisfies the Schrödinger equation

$$H\psi = E\psi, \tag{2}$$

where H is the total Hamiltonian given by

$$H = H_0 + \sum_{i<j=1,2,\ldots,6} V_{ij}. \tag{3}$$

and V_{ij}'s in (3) represent the pair-wise q–q interactions.

In equation (1), ψ_{int} is the internal part of the six-quark state, a composite bag state with the boundary condition that it vanishes on the surface of the bag and can, therefore, be expanded in a complete set of discrete energy eigenstates in the internal region, i.e.,

$$\psi_{\text{int}} = \sum_{\nu} a_\nu \psi_\nu, \tag{4}$$

where ψ_ν are the energy eigenstates satisfying the equation

$$H\psi_\nu = E_\nu \psi_\nu \tag{5}$$

in the confined region.

The ψ_{ext} in (1) is the wavefunction representing the external (outer) region in which nucleons, each consisting of a cluster of three quarks, are supposed to remain undeformed. Assuming a cluster decomposition, ψ_{ext} can be expressed as

$$\psi_{\text{ext}} = A[\psi_{NN}(\mathbf{r})\psi_{N_1}(3q)\psi_{N_2}(3q)], \tag{6}$$

where A is the antisymmetrizer (operating on the quark coordinates) and $\psi_{NN}(\mathbf{r})$ is the wavefunction of the relative motion between the centres of the two clusters, $\psi_{N_1}(3q)$ and $\psi_{N_2}(3q)$. It is indeed implied here that the wavefunction describing the relative coordinates between the two colour-singlet configurations can be defined even when the two nucleon bags overlap each other at short distances.

With the above basic ingredients one can derive, in a cluster model, a Schrödinger equation for $\psi_{NN}(r)$:

$$(H_0 + V_{NN} + V_{NqN} - E)\psi_{NN}(r) = 0. \tag{7}$$

Here V_{NqN} is the short-range component of the nucleon–nucleon interaction, simulated through q–q potentials, and is given by

$$V_{NqN} = -\sum_{\nu} V^{(\nu)}_{Nq}(\mathbf{r}, E) V^{(\nu)}_{qN}(\mathbf{r}', E)/(E_\nu - E), \tag{8}$$

where $V^{(\nu)}_{Nq}(\mathbf{r}, E)$ is the nucleon-quark transition vertex

$$V^{(\nu)}_{Nq}(\mathbf{r}, E) = \int \psi^*_\nu (H - E) A[\psi_{N_1}(3q)\psi_{N_2}(3q)]\delta(\mathbf{r} - \mathbf{r}') \prod_{i=1}^{6} \mathrm{d}r'_i, \tag{9}$$

which is supposed to be zero outside the $6 - q$ composite bag. One of the simplifying assumptions, as suggested by Simonov (1981), is to consider, assuming a sharp transition from quark to hadrons on the surface of the bag, a form for $V^{(\nu)}_{Nq}$ to be

$$V^{(\nu)}_{Nq} = c_\nu \delta(r - b), \tag{10}$$

which effectively gives rise to a hard core repulsion in the N–N interaction. The term V_{NN} in (7) represents the long-range component of the N–N potential simulated through meson exchanges.

2.2 *Bound state problem—the deuteron*

For the present investigation we adopt here a non-local seperable potential operative only at the internucleon distances r, r' greater than the bag 'radius'. Thus

$$V_{NN}(r, r') = -\frac{\lambda}{m}\tilde{g}(r)\tilde{g}(r')\theta(r - b)\theta(r' - b), \tag{11}$$

where

$$\tilde{g}(r) = \tilde{c}(r) + \frac{1}{\sqrt{8}} S_{12}(\hat{r})\tilde{T}(r), \tag{12}$$

and S_{12} is the usual tensor operator

$$S_{12}(\hat{r}) = 3(\boldsymbol{\sigma}_1 \cdot \hat{r})(\boldsymbol{\sigma}_2 \cdot \hat{r}) - (\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2), \tag{13}$$

$\boldsymbol{\sigma}_{i,j}$'s ($i,j = 1,2$) being the Pauli spin operators.

The $\tilde{c}(r)$ and $\tilde{T}(r)$ are the radial functions, assumed to be of Yukawa type, and given by

$$\tilde{c}(r) = \frac{\pi}{2}\exp(-\beta r)/r \tag{14}$$

$$\tilde{T}(r) = \frac{\pi}{2} t(\gamma r)^2 \exp(-\gamma r)/r. \tag{15}$$

Similarly, the short range component of the N–N potential, V_{NqN} operative only at distances r, r' less than b and acting in s-state only, can be given as

$$V_{NqN}(r, r') = [\tilde{f}_\nu(r)\tilde{f}_\nu(r')/(E_\nu - E)]\theta(b - r)\theta(b - r') \tag{16}$$

with

$$\tilde{f}_\nu(r) = \frac{1}{\sqrt{4\pi}} c_\nu \delta(r-b)/r. \tag{17}$$

It is important to note that the potential V_{NqN}, as given in (16), is not only energy-dependent but is also repulsive as long as the energy E is less than the eigenenergy E_ν of the state ψ_ν representing the 6-q bag state. [For the bound state deuteron case $E = -\alpha^2/m$ whereas $E_\nu > 0$.]

The above expressions, equation (11) to (17), can be written down in momentum space where the analytic structure of the solution of the Schrödinger equation becomes rather transparent. Thus

$$V_{NN}(\mathbf{p}, \mathbf{p}') = -\frac{\lambda}{m} g(\mathbf{p}) g(\mathbf{p}') \tag{18}$$

and

$$V_{NqN}(p, p') = f_\nu(p) f_\nu(p')/(E_\nu - E), \tag{19}$$

where

$$g(\mathbf{p}) = c(p) + \frac{1}{\sqrt{8}} S_{12}(\hat{p}) T(p) \tag{20}$$

and

$$f_\nu(p) = \frac{c_\nu}{\pi\sqrt{2}} \sin(bp)/p. \tag{21}$$

In (20)

$$c(p) = \frac{\exp(-b\beta)}{(p^2+\beta^2)} [\cos(bp) + \beta \sin(bp)/p] \tag{22}$$

and

$$T(p) = \frac{t\gamma^2 \exp(-b\gamma)}{(p^2+\gamma^2)} [t_0(p,b,\gamma] j_0(bp) - t_1(p,b,\gamma) j_1(bp) - t_2(p,b,\gamma) j_2(bp)], \tag{23}$$

where j_0, j_1, j_2 are the standard spherical Bessel functions of order zero, one and two respectively and t_0, t_1 and t_2 are the coefficients written down explicitly in Appendix A.

The Schrödinger equation (7) written in momentum space and with the input potentials (18) and (19), admits a simple analytical solution:

$$\psi(\mathbf{p}) = \frac{N}{(p^2+\alpha^2)} \left[g(\mathbf{p}) + \left\{ \frac{\lambda^{-1} - I_{11}(\alpha^2)}{I_{12}} \right\} f_\nu(p) \right], \tag{24}$$

where N is a normalization constant and, in addition, we have the relation

$$[\lambda^{-1} - I_{11}(\alpha^2)]/I_{12}(\alpha^2) = -\frac{m}{(E_v - E)}\frac{I_{21}(\alpha^2)}{[1 + mI_{22}(\alpha^2)/(E_v - E)]}, \tag{25}$$

where I_{11}, I_{12}, I_{22} are the integrals

$$\begin{aligned} I_{11}(\alpha^2) &= \int d^3q\, g^2(q)/(q^2 + \alpha^2) \\ I_{12}(\alpha^2) = I_{21}(\alpha^2) &= \int d^3q\, g(q) f_v(q)/(q^2 + \alpha^2) \\ I_{22}(\alpha^2) &= \int d^3q\, f_v^2(q)/(q^2 + \alpha^2), \end{aligned} \tag{26}$$

which can be worked out analytically, the details of which are given in Appendix B.

By taking the Fourier transform of the wavefunction (equation (24)) we recast the solution in the configuration space and express the wavefunction in the standard notation

$$\psi(\mathbf{r}) = \frac{1}{\sqrt{4\pi}}\left[\frac{u(r)}{r} + \frac{1}{\sqrt{8}} S_{12}(\hat{r}) w(r)/r\right], \tag{27}$$

where, in the present notation, the radial components, $u(r)$ and $w(r)$, can each be decomposed as a sum of two parts, one representing the short range ($r < b$) and the other the long range ($r > b$) behaviour of the radial functions. Thus

$$u(r) = u_s(r)\theta(b - r) + u_l\theta(r - b)$$

and

$$w(r) = w_s(r)\theta(b - r) + w_l\theta(r - b) \tag{28}$$

where the detailed structure of u_s, u_l, w_s, w_l are given in Appendix C.

An immediate advantage of writing the wavefunction in configuration space is that one can straightaway write down the asymptotic ($r \to \infty$) part of the wavefunction. Thus

$$u_l(r) \underset{r\to\infty}{\longrightarrow} A_S \exp(-\alpha r)$$

while

$$w_l(r) \underset{r\to\infty}{\longrightarrow} A_D\left(1 + \frac{3}{\alpha r} + \frac{3}{\alpha^2 r^2}\right)\exp(-\alpha r), \tag{29}$$

where the asymptotic normalization constants A_S and A_D, which are essentially dependent on the parameters of the potential are given in Appendix D. With A_S and A_D as introduced above, one can define the asymptotic ratio [Ericson and Rosa-Clot

1985], η, as

$$\eta = A_D/A_S. \tag{30}$$

This parameter, which can be determined experimentally to high precision, may provide important information on the external properties of the deuteron.

2.3 *The electromagnetic form factors of the deuteron*

As the next step, we evaluate the electromagnetic form factors of the deuteron by using the radial components of the wavefunction obtained above (cf. (28)). For this purpose we employ the standard definitions of the charge $[F_E(q^2)]$, quadrupole moment $[F_Q(q^2)]$, and magnetic moment $(F_M(q^2))$ form factors given by (Elias *et al* 1969, Atti 1980)

$$F_E(q^2) = G_E^S(q^2)C_E(q^2),$$

$$F_Q(q^2) = G_E^S(q^2)C_Q(q^2),$$

$$F_M(q^2) = (M_D/M)[G_M^S(q^2)C_S(q^2) + \tfrac{1}{2}G_E^S(q^2)C_L(q^2)], \tag{31}$$

where

$$C_E(q^2) = \int_0^\infty (u^2 + w^2)j_0(qr/2)\mathrm{d}r,$$

$$C_Q(q^2) = \frac{3}{\eta_D\sqrt{2}}\int_0^\infty (uw - w^2/2\sqrt{2})j_2(qr/2)\mathrm{d}r, \tag{32}$$

$$C_S(q^2) = \int_0^\infty (u^2 - w^2/2)j_0(qr/2)\mathrm{d}r + \frac{1}{\sqrt{2}}\int_0^\infty (uw + w^2/\sqrt{2})j_2(qr/2)\mathrm{d}r$$

and

$$C_L(q^2) = \tfrac{3}{2}\int_0^\infty w^2[j_0(qr/2) + j_2(qr/2)]\mathrm{d}r.$$

In (31) G_E and G_M are respectively the isoscalar electric and magnetic form factors of the nucleons. For these we adopt the dipole approximation (Cheng and Kisslinger 1987), viz.,

$$G_E^S(q^2) = (1 + q^2/M_e^2)^{-2}; \quad G_M^S(q^2) = 0{\cdot}88(1 + q^2/M_e^2)^{-2} \tag{33}$$

with $M_e^2 = 18{\cdot}23\,\mathrm{fm}^{-2}$ and $\eta_D = q^2/4M_D^2$.

The form factors defined above are normalized as follows:

$$C_E(q^2) \xrightarrow[q^2\to 0]{} 1; \quad C_Q(q^2) \xrightarrow[q^2\to 0]{} M_D^2 Q_D,$$

$$C_S(q^2) \xrightarrow[q^2\to 0]{} 1 - \tfrac{3}{2}P_D; \quad C_L(q^2) \xrightarrow[q^2\to 0]{} \tfrac{3}{2}P_D,$$

$$F_M(q^2) \xrightarrow[q^2\to 0]{} (M_D/M)[(\mu_p + \mu_n) - \tfrac{3}{2}P_D(\mu_p + \mu_n - 1/2)]. \tag{34}$$

In terms of the form factors given above (cf. (31) and (32)), we define the electric structure function

$$A(q^2) = F_E^2(q^2) + \tfrac{8}{9}\eta_D^2 F_Q^2(q^2) + \tfrac{2}{3}\eta_D F_M^2(q^2) \tag{35}$$

and the magnetic structure function

$$B(q^2) = \tfrac{4}{3}\eta_D(1 + \eta_D)F_M^2(q^2). \tag{36}$$

It is in terms of these quantities that the experimental data for elastic e–d scattering can be extracted out and can thus be compared with the theoretical predictions.

2.4 *Nucleon–nucleon scattering*

In the scattering sector, as the inclusion of tensor force in N–N interaction influences only the spin-triplet ($^3S_1$) scattering, we have, therefore, restricted ourselves to studying the behaviour of $^3S_1$ phase shifts by incorporating the tensor part of the interaction. The on-shell S-wave scattering amplitude is thus written down as

$$\begin{aligned} a(k) = 2\pi^2\lambda h^{-1}[c^2(k) + T^2(k) - \{(E_v - E)/m + I_{22}(-k^2 - i\varepsilon)\}^{-1} \\ \times [c(k)f_v(k)I_{12}(-k^2 - i\varepsilon) + c(k)f_v(k)I_{21}(-k^2 - i\varepsilon) \\ + f_v^2(k)(\lambda^{-1} - I_{11}(-k^2 - i\varepsilon))]], \end{aligned} \tag{37}$$

where

$$h = [1 - \lambda I_{11}(-k^2 - i\varepsilon) + mI_{12}^2(-k^2 - i\varepsilon)/\{E_v - E + mI_{22}(-k^2 - i\varepsilon)\}]$$

and I_{11}, I_{12}, I_{22} are the integrals defined in (26). The amplitude $a(k)$ is related to the phase shifts through the expression

$$k\cot\delta = \mathrm{Re}[1/a(k)]. \tag{38}$$

3. Results and discussion

The behaviour of the vertex functions defined through equations (14)–(23) in configuration and momentum space, is shown in figures 1 and 2 respectively by using the parameters (set I) of the potential given in table 1. Figure 3 depicts the structure of the radial components $u(r)$ and $w(r)$ corresponding to the S- and D-state wavefunctions of the deuteron. The dashed curves refer to parameters of set II and indicate the sensitiveness of the D-state wavefunction to these parameters. On comparing these radial components with the ones obtained with the Paris potential (Lacombe *et al* 1980), one finds that while the D-state part in the later case is somewhat more pronounced around $1\,\mathrm{fm} < r < 3\,\mathrm{fm}$, the S-state component in the present model contributes slightly more for smaller values of r (up to 1 fm). The corresponding behaviour of the wavefunction in the momentum space is shown in figure 4. The overall structure of these functions is found to be in qualitative agreement with the findings of other models such as the Paris potential (Lacombe *et al* 1980), Reid softcore (Reid

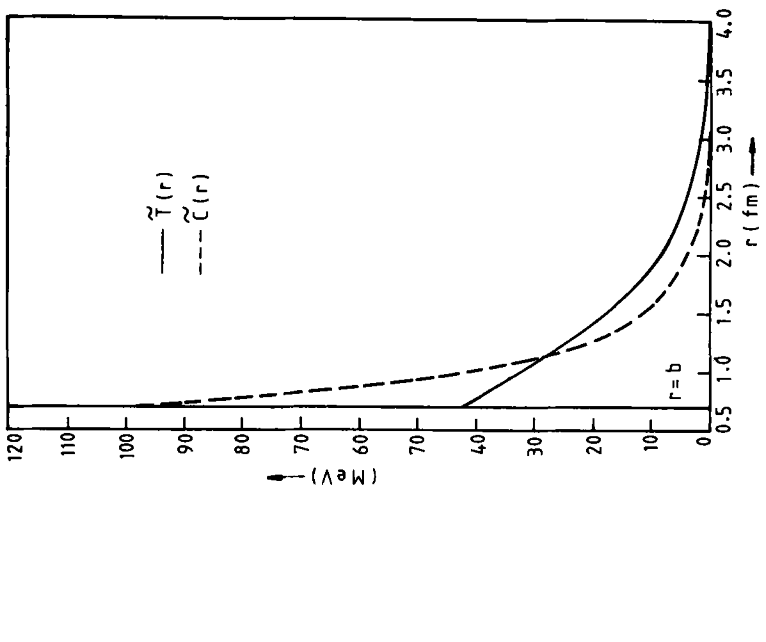

Figure 1. Vertex functions $\tilde{c}(r)$, $\tilde{T}(r)$ in configuration space vs r(fm). The ordinate at $r = b$ represents the constant contribution of $\tilde{f}_v(r)$.

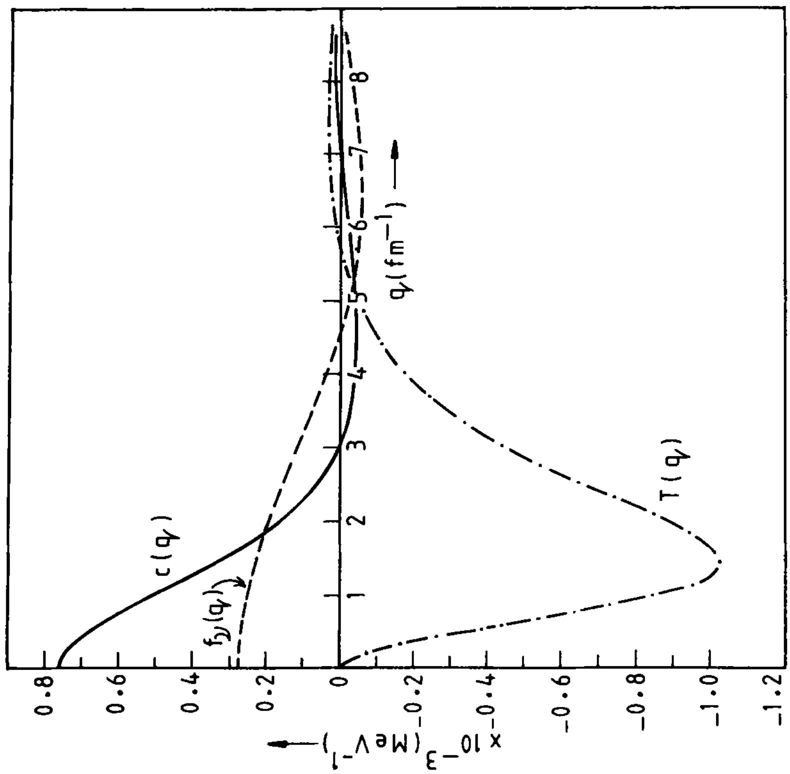

Figure 2. Functions $c(q)$, $T(q)$ and $f_v(q)$ in momentum space vs $q(\mathrm{fm}^{-1})$.

Table 1. Potential parameters. Note that E_v here represents the relative energy between the two nucleonic clusters, exluding the energy of the nucleon masses.

	E_v (MeV)	c_v(MeV)$^{1/2}$	b(fm)	γ(fm^{-1})	β(fm^{-1})	λ(fm^{-3})
I	375	6·76	0·691	2·061	1·783	2·569
II	297	6·08	0·734	2·177	1·760	2·457

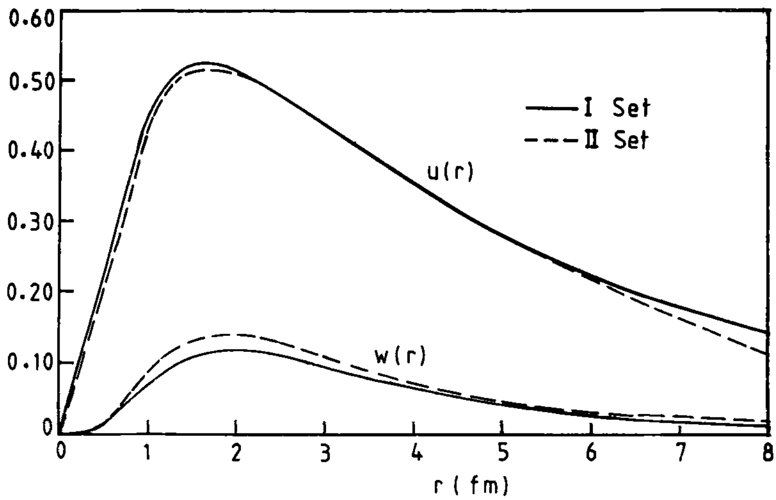

Figure 3. Deuteron wavefunction in configuration space vs r(fm).

1968) or the one proposed recently (Beyer and Weber 1987) to account for the quark degrees of freedom in the N–N system.

The plot of various form factors defined through (31) and (32) along with the electric structure function $A(q^2)$ is shown and compared with the experimental data (Arnold *et al* 1975; Elias *et al* 1969; Buchnan and Yearian 1965) in figure 5. Clearly, the charge form factor which dominates at low $q^2(<15\,\mathrm{fm}^{-2})$ shows a dip structure at $q^2 \sim 30\,\mathrm{fm}^{-2}$. At large momentum transfers, the quadrupole and the magnetic form factors take the lead. The result predicted for the electric structure function $A(q^2)$, is about 15–25% less in comparison with the experimental data, particularly at large q^2. This may be due to two reasons: first, the relativistic effect, which has not been incorporated in the present analysis, and second, the representations (in the dipole approximation) used for the isoscalar electric and magnetic form factors of the nucleon may not be too accurate. In figure 6, the magnetic structure function $B(q^2)$ is plotted against q^2 and compared with the experimental data (Auffret *et al* 1985). The overall behaviour of the structure functions predicted in figures 5 and 6 can be compared reasonably well with the results obtained, as for instance, in the hybrid quark-hadron model (Cheng and Kisslinger 1987).

To further check the sensitivity to the parameters, we have in figure 7, the plot of $^3S_1$ phase-shifts versus (lab.) energy up to about 400 MeV for both parameter sets. The

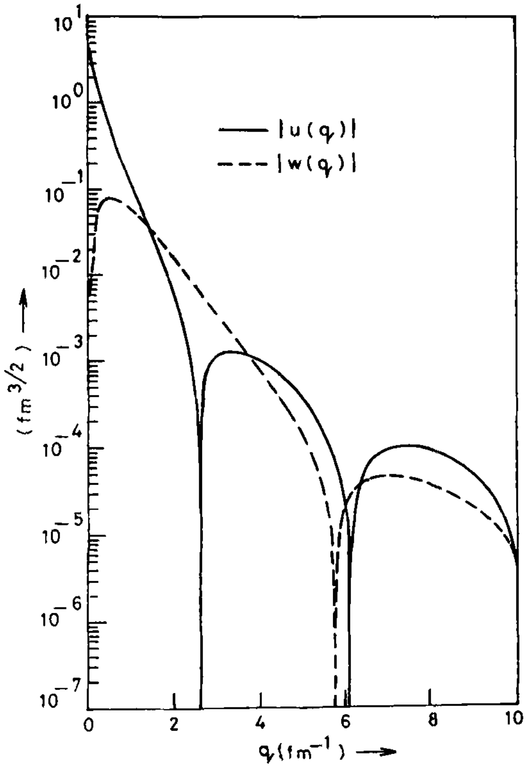

Figure 4. Deuteron wavefunction in momentum space vs $q(\mathrm{fm}^{-1})$.

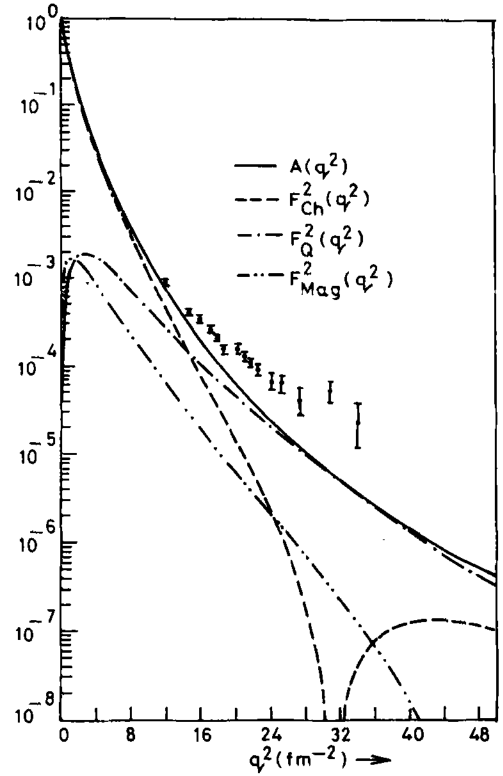

Figure 5. Structure function $A(q^2)$, deuteron charge form factor $|F_{ch}(q^2)|$, magnetic form-factor $|F_{mag}(q^2)|$ and quadrupole form factor $|F_Q(q^2)|$ vs $q^2(\mathrm{fm}^{-2})$.

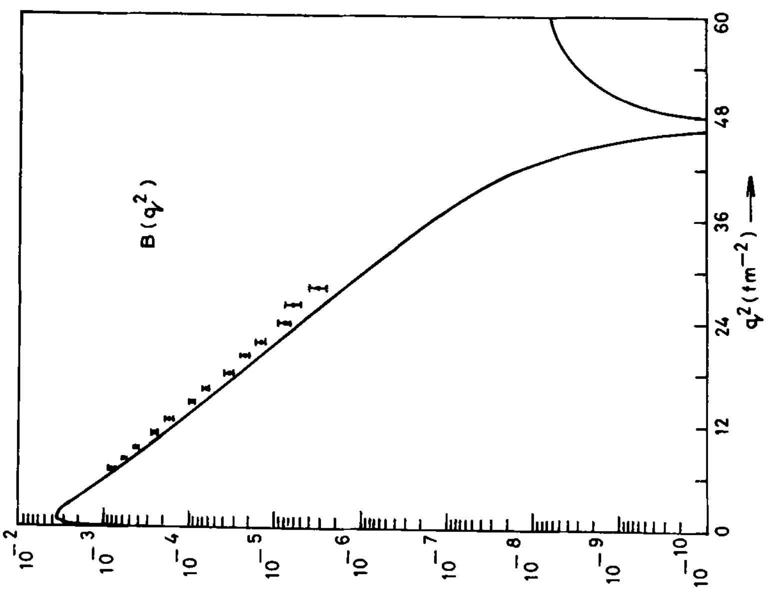

Figure 6. Structure function $B(q^2)$ vs $q^2(\mathrm{fm}^{-2})$.

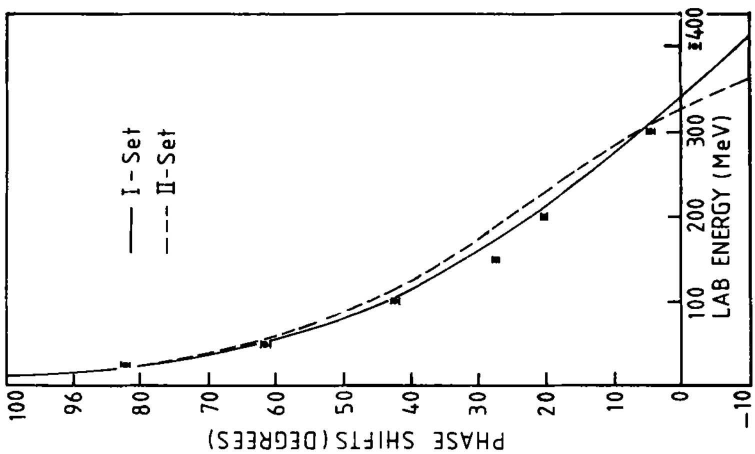

Figure 7. $^3S_1$ n–p scattering phase-shifts (deg) vs Lab. energy (MeV).

phase-shift is found to change sign around 330 MeV. We note that whereas the set II parameters slightly enhance the D-state wavefunction (see figure 3) and consequently reproduce the quadrupole moment in better agreement with experiment than the parameters of set I, the scattering phase-shifts, on the other hand, show a marked deviation, especially at higher energies.

In table 2 we present the values of the various observables, such as the deuteron binding energy, electric quadrupole moment, magnetic dipole moment, D-state probability, the triplet scattering length and the effective range etc., and compare them with the experimental data as well as with the values obtained by the other standard models. Almost all the parameters are found to be in reasonable agreement with the data.

As regards the parameters of the repulsive core potential which originates from the six-quark compound bag state, one can compare them with the values obtained in the P-matrix approach. For instance, the parameter E_v, which corresponds to the energy of the compound bag state (excluding the energy of the two-nucleon masses), is found to be around 375 MeV as compared to 230 MeV obtained in the P-matrix analysis (Simonov 1981). Similarly the coupling strength of the quark-nucleon transition vertex in the present case is 6·7 $\text{MeV}^{1/2}$ which is to be compared to 10·4 $\text{MeV}^{1/2}$. The marked difference is, however, observed in the value of b, the bag range parameter. In the P-matrix approach it is about 1·46 fm, whereas the present model requires it to be 0·69 fm. This may presumably be sensitive to the model used for the long range part of the N–N interaction.

In addition, we have computed the probability for the six-quark content in the deuteron by employing the definition

$$P_{6q} = 1 - \int_b^\infty [u_l^2(r) + w_l^2(r)]\,\mathrm{d}r.$$

We find that this comes out to be 2·1%, which is in rather good agreement with the findings of other workers (Beyer 1987; Simonov 1984).

In conclusion, we have here a fairly realistic and analytically simple model incorporating the six-quark repulsive core as well as the tensor component of the N–N force. It would be interesting to check the sensitivity of the three-body observables, especially the behaviour of H^3 and He^3 electromagnetic form factors at large q^2, using this as the input N–N interaction. Efforts in this direction are already in progress.

Table 2. Static parameters of the deuteron and low energy scattering parameters.

	Present analysis	RSC	PARIS	Expt.
E_D(MeV)	−2·2218	−2·25	−2·2249	−2·2249
P_D(%)	3·44	6·47	5·77	3–6
Q_D(fm^{-2})	0·2698	0·279	0·279	0·272
μ_D(nm)	0·8601	0·8428	0·853	0·8574
$\eta(D/S)$	0·0271	0·0262	0·0260	0·0271
a(fm)	5·400	5·390	5·427	5·38
r(fm)	1·748	1·720	1·765	1·748

Appendix A

In momentum space the tensor function $T(p)$ is given by

$$T(p) = t\gamma^2 \exp(-b\gamma)(\gamma^2 + p^2)^{-1}[t_0 j_0(bp) - t_1 j_1(bp) - t_2 j_2(bp)]$$

where t_0, t_1 and t_2 are functions of b, γ and p:

$$t_0 = 8\gamma^2(b\gamma + 1)(\gamma^2 + p^2)^{-2} + 4(b^2\gamma^2 - 2b\gamma - 2)(\gamma^2 + p^2)^{-1} - 4b^2$$

$$t_1 = p[8b\gamma^2(\gamma^2 + p^2)^{-2} + 4b(b\gamma - 2)(\gamma^2 + p^2)^{-1} + b^3]$$

and

$$t_2 = \gamma b^3.$$

In the limit $b \to 0$, $T(p)$ takes the form

$$T(p) = -8t\gamma^2 p^2(\gamma^2 + p^2)^{-3}.$$

Appendix B

The integrals $I_{11}(\alpha^2)$, $I_{12}(\alpha^2)$ and $I_{22}(\alpha^2)$ are defined as:

$$\begin{aligned} I_{11}(\alpha^2) &= \int_0^\infty d^3p g^2(p)(p^2 + \alpha^2)^{-1} \\ &= \int_0^\infty d^3p c^2(p)(p^2 + \alpha^2)^{-1} + \int_0^\infty d^3p T^2(p)(p^2 + \alpha^2)^{-1} \\ &= I_{11}^C(\alpha^2) + I_{11}^T(\alpha^2) \end{aligned}$$

(the cross term being zero).

$$\begin{aligned} I_{12}(\alpha^2) &= \int_0^\infty d^3p g(p) f_v(p)(p^2 + \alpha^2)^{-1} \\ &= \int_0^\infty d^3p \left[c(p) + \frac{1}{\sqrt{8}} S_{12}(\hat{p}) T(p) \right] f_v(p)(p^2 + \alpha^2)^{-1} \\ &= \int_0^\infty d^3p c(p) f_v(p)(p^2 + \alpha^2)^{-1} \end{aligned}$$

(again the second term vanishes). Lastly,

$$I_{22}(\alpha^2) = \int_0^\infty d^3p f_v^2(p)/(p^2 + \alpha^2).$$

Integrating over p we obtain

$$I^C_{11}(\alpha^2) = \pi^2 \exp(-2b\beta)[1/\{\beta\alpha(\beta+\alpha)\} - \exp(-2b\alpha)/\{\alpha(\beta+\alpha)^2\}],$$

$$I_{12}(\alpha^2) = \pi c_v \exp(-b\beta)(1-\exp(-2b\alpha))/\{\alpha\sqrt{2}(\beta+\alpha)\},$$

$$I_{22}(\alpha^2) = c_v^2[1-\exp(-2b\alpha)]/2\alpha,$$

$$\begin{aligned} I^T_{11}(\alpha^2) &= 4\pi\int_0^\infty \mathrm{d}pp^2 T^2(p)/(p^2+\alpha^2) \\ &= 4\pi t^2\gamma^4 \exp(-2b\gamma)\int_0^\infty \mathrm{d}pp^2[t_0 j_0(bp) - t_1 j_1(bp) \\ &\quad - t_2 j_2(bp)]^2/(p^2+\alpha^2)(p^2+\gamma^2)^2 \\ &= 4\pi t^2\gamma^4 \exp(-2b\gamma)\int_0^\infty \mathrm{d}pp^2[t_0^2 j_0^2(bp) + t_1^2 j_1^2(bp) \\ &\quad + t_2^2 j_2^2(bp) - 2t_0 t_1 j_0(bp) j_1(bp) \\ &\quad - 2t_0 t_2 j_0(bp) j_2(bp) + 2t_1 t_2 j_1(bp) j_2(bp)]/ \\ &\quad (p^2+\alpha^2)(p^2+\gamma^2)^2. \end{aligned}$$

The expressions for t_0, t_1 and t_2 are given in Appendix A. Let us define the following integrals:

$$\begin{aligned} X_1 &= \int_0^\infty \mathrm{d}pp^2 j_0^2(bp)/[(p^2+\alpha^2)(p^2+\gamma^2)] \\ &= \frac{\pi}{2b}[I_{1/2}(b\alpha)K_{1/2}(b\alpha) - I_{1/2}(b\gamma)K_{1/2}(b\gamma)]/(\gamma^2-\alpha^2), \end{aligned}$$

$$\begin{aligned} X_2 &= \int_0^\infty \mathrm{d}pp^4 j_1^2(bp)/[(p^2+\alpha^2)(p^2+\gamma^2)] \\ &= \frac{\pi}{2b}\left[\left(\frac{\gamma^2}{\gamma^2-\alpha^2}\right)I_{3/2}(b\gamma)K_{3/2}(b\gamma) - \left(\frac{\alpha^2}{\gamma^2-\alpha^2}\right)I_{3/2}(b\alpha)K_{3/2}(b\alpha)\right], \end{aligned}$$

$$\begin{aligned} X_3 &= \int_0^\infty \mathrm{d}pp^2 j_2^2(bp)/[(p^2+\alpha^2)(p^2+\gamma^2)] \\ &= \frac{\pi}{2b}[I_{5/2}(b\alpha)K_{5/2}(b\alpha) - I_{5/2}(b\gamma)K_{5/2}(b\gamma)]/(\gamma^2-\alpha^2), \end{aligned}$$

$$X_4 = \int_0^\infty \mathrm{d}pp^3 j_0(bp) j_1(bp)/[(p^2+\alpha^2)(p^2+\gamma^2)]$$

$$= \left[\frac{\pi}{2b^2} \{ I_{1/2}(b\alpha) K_{1/2}(b\alpha) - I_{1/2}(b\gamma) K_{1/2}(b\gamma) \} - \frac{\pi}{4b^2} \{ \exp(-2b\alpha) - \exp(-2b\gamma) \} \right] \Big/ (\gamma^2 - \alpha^2),$$

$$X_5 = \int_0^\infty \mathrm{d}p p^2 j_0(bp) j_2(bp) / [p^2 + \alpha^2)(p^2 + \gamma^2)]$$

$$= \left[-\frac{\pi}{4b^2\alpha}(1 + 3/b^2\alpha^2)[1 - \exp(-2b\alpha)] - \frac{3}{b\alpha}[1 + \exp(-2b\alpha)] + \frac{\pi}{4b^2\gamma}(1 + 3/b^2\gamma^2)[1 - \exp(-2b\gamma)] - \frac{3}{b\gamma}[1 + \exp(-2b\gamma)] \right] \Big/ (\gamma^2 - \alpha^2),$$

$$X_6 = \int_0^\infty \mathrm{d}p p^3 j_1(bp) j_2(bp) / (p^2 + \alpha^2)(p^2 + \gamma^2)$$

$$= \left[\frac{3\pi}{2b^2} I_{3/2}(b\alpha) K_{3/2}(b\alpha) - I_{3/2}(b\gamma) K_{3/2}(b\gamma) - \left(\frac{\pi}{4b^3\alpha} - \frac{\pi}{4b^3\gamma} \right) + \frac{\pi}{4b^2}(1 + 1/b\alpha)\exp(-2b\alpha) - (1 + 1/b\gamma)\exp(-2b\gamma) \right] \Big/ (\gamma^2 - \alpha^2).$$

$I^T_{11}(\alpha^2)$ can be expressed in terms of the derivatives (w.r.t. γ) of the six basic integrals $X_1, X_2, X_3, X_4, X_5, X_6$. Since it involves derivatives up to the fifth order, the algebra is extremely cumbersome and hence not reproduced here.

Appendix C

The wavefunctions corresponding to the S- and the D-state are usually written as $u(r)$ and $w(r)$, where $u(r)$ and $w(r)$ have the following forms:

$$u(r) = u_s \theta(b - r) + u_l \theta(r - b)$$

and

$$w(r) = w_s \theta(b - r) + w_l \theta(r - b),$$

where u_s and w_s correspond to the region $r < b$ and u_l and w_l to the region $r > b$. The two parts of the S-state contribution to the wavefunction are

$$u_s = N(c_1 + c_2)[\exp(r\alpha) - \exp(-r\alpha)]$$

and

$$u_l = N[d_1 \exp(-r\alpha) - d_2 \exp(-r\beta)].$$

In the above expressions c_1, c_2, d_1 and d_2 are constants depending on α, β, λ, c_v etc.:

$$c_1 + c_2 = \frac{\pi^2}{\alpha(\alpha+\beta)} \frac{\exp(-b(\alpha+\beta))}{(2\pi)^{3/2}} + \frac{c_v}{\alpha} \frac{1}{4\sqrt{\pi}} \left[\frac{\lambda^{-1} - I_{11}(\alpha^2)}{I_{12}(\alpha^2)} \right] \exp(-b\alpha),$$

$$d_1 = \frac{\pi^2 \exp(-b\beta)}{\alpha(\beta^2-\alpha^2)(2\pi)^{3/2}} \{(\beta+\alpha)\exp(b\alpha) - (\beta-\alpha)\exp(-b\alpha)\}$$

$$+ \frac{1}{4\sqrt{\pi}} \frac{c_v}{\alpha} \left[\frac{\lambda^{-1} - I_{11}(\alpha^2)}{I_{12}(\alpha^2)} \right] (\exp(b\alpha) - \exp(-b\alpha)),$$

$$d_2 = \frac{2\pi^2}{(2\pi)^{3/2}} \frac{1}{(\beta^2-\alpha^2)}.$$

From the D-state part we obtain

$$w_s = \frac{4\pi N}{(2\pi)^{3/2}} t\gamma^2 \exp(-b\gamma) \sqrt{\frac{r}{b}} [F_{G1} I_{AS} + F_{G2} I'_{AS} - F_{G3} I''_{AS}$$

$$+ F_{G4} I_{BS} - F_{G5} I'_{BS} + F_{G6} I''_{BS} + F_{G7} I_{CS}]$$

and

$$w_l = \frac{4\pi N}{(2\pi)^{3/2}} t\gamma^2 \exp(-b\gamma) \sqrt{\frac{r}{b}} [F_{G1} I_{Al} + F_{G2} I'_{Al} - F_{G3} I''_{Al}$$

$$+ F_{G4} I_{Bl} - F_{G5} I'_{Bl} + F_{G6} I''_{Bl} + F_{G7} I_{Cl}]$$

$F_{G1}, F_{G2}, \ldots, F_{G7}$ are constants depending on b, γ, α:

$$F_{G1} = 4\left\{ b^2 - \frac{(b^2\gamma^2 - 2b\gamma - 2)}{(\gamma^2-\alpha^2)} - \frac{2\gamma^2(b\gamma+1)}{(\gamma^2-\alpha^2)^2} \right\},$$

$$F_{G2} = \frac{2(b^2\gamma^2 - 2b\gamma - 2)}{\gamma} + \frac{(b\gamma+1)(5\gamma^2-\alpha^2)}{\gamma(\gamma^2-\alpha^2)},$$

$$F_{G3} = (b\gamma + 1)$$

$$F_{G4} = b^3 + \frac{4b(b\gamma-2)}{(\gamma^2-\alpha^2)} + \frac{8b\gamma^2}{(\gamma^2-\alpha^2)^2},$$

$$F_{G5} = \frac{2b(b\gamma-2)}{\gamma} + \frac{b(5\gamma^2-\alpha^2)}{\gamma(\gamma^2-\alpha^2)},$$

$$F_{G6} = b$$

and

$$F_{G7} = \gamma b^3.$$

I_{AS}, I_{BS} and I_{CS} are the contribution from $r < b$ part of I_A, I_B and I_C respectively.

I_{Al}, I_{Bl}, and I_{Cl}, are the corresponding parts from the $r > b$ region. I'_A, I'_B and I''_A, I''_B, are respectively, the first order and second order derivatives w.r.t. γ of I_A and I_B.

The expressions for I_{AS}, I_{BS}, I_{CS} and I_{Al}, I_{Bl}, I_{Cl} are of the form:

$$I_{AS} = -\frac{1}{(\gamma^2-\alpha^2)}[\{I_{1/2}(r\alpha) - \frac{3}{r\alpha}I_{3/2}(r\alpha)\}K_{1/2}(b\alpha)$$

$$-\{I_{1/2}(r\gamma) - \frac{3}{r\gamma}I_{3/2}(r\gamma)\}K_{1/2}(b\gamma)],$$

$$I_{BS} = -\frac{1}{(\gamma^2-\alpha^2)}\left[\alpha\left\{I_{1/2}(r\alpha) - \frac{3}{r\alpha}I_{3/2}(r\alpha)\right\}K_{3/2}(b\alpha)\right.$$

$$\left.-\gamma\left\{I_{1/2}(r\gamma) - \frac{3}{r\gamma}I_{3/2}(r\gamma)\right\}K_{3/2}(b\gamma)\right],$$

$$I_{CS} = \frac{1}{(\gamma^2-\alpha^2)}\left[\left\{I_{1/2}(r\alpha) - \frac{3}{r\alpha}I_{3/2}(r\alpha)\right\}K_{5/2}(b\alpha)\right.$$

$$\left.-\{I_{1/2}(r\gamma) - \frac{3}{r\gamma}I_{3/2}(r\gamma)\}K_{5/2}(b\gamma)\right]$$

and

$$I_{Al} = -\frac{1}{(\gamma^2-\alpha^2)}\left[\left(1+\frac{3}{r\alpha}+\frac{3}{r^2\alpha^2}\right)I_{1/2}(b\alpha)K_{1/2}(r\alpha)\right.$$

$$\left.-\left(1+\frac{3}{r\gamma}+\frac{3}{r^2\gamma^2}\right)I_{1/2}(b\gamma)K_{1/2}(r\gamma)\right],$$

$$I_{Bl} = \frac{1}{(\gamma^2-\alpha^2)}\left[\alpha\left(1+\frac{3}{r\alpha}+\frac{3}{r^2\alpha^2}\right)I_{3/2}(b\alpha)K_{1/2}(r\alpha)\right.$$

$$\left.-\gamma\left(1+\frac{3}{r\gamma}+\frac{3}{r^2\gamma^2}\right)I_{3/2}(b\gamma)K_{1/2}(r\gamma)\right],$$

$$I_{Cl} = \frac{1}{(\gamma^2-\alpha^2)}\left[\left(1+\frac{3}{r\alpha}+\frac{3}{r^2\alpha^2}\right)I_{5/2}(b\alpha)K_{1/2}(r\alpha)\right.$$

$$\left.-\left(1+\frac{3}{r\gamma}+\frac{3}{r^2\gamma^2}\right)I_{5/2}(b\gamma)K_{1/2}(r\gamma)\right].$$

We have checked that the long range (u_l, w_l) and short range (u_s, w_s) parts of the wavefunction match exactly at $r = b$.

The functions $I_n(x)$ $(n = 1/2, 3/2, 5/2)$, appearing in the above expressions, are the modified spherical Bessel functions of second kind, whereas K_n's are expressible as linear combinations of second and third kind:

$$I_{1/2}(x) = (2x/\pi)^{\frac{1}{2}}\sinh(x)/x,$$

$$I_{3/2}(x) = (2x/\pi)^{\frac{1}{2}}(\cosh(x)/x - \sinh(x)/x^2),$$

$$I_{5/2}(x) = (2x/\pi)^{\frac{1}{2}}[(3/x^3 + 1/x)\sinh(x) - 3/x^2\cosh(x)]$$

and

$$K_{1/2}(x) = (\pi/2x)^{\frac{1}{2}}\exp(-x),$$
$$K_{3/2}(x) = (\pi/2x)^{\frac{1}{2}}\exp(-x)(1 + 1/x),$$
$$K_{5/2}(x) = (\pi/2x)^{\frac{1}{2}}\exp(-x)(1 + 3/x + 3/x^2).$$

Appendix D

In the asymptotic limit $u(r)$ $w(r)$ are given by

$$u(r) \xrightarrow[r\to\infty]{} A_S \exp(-r\alpha)$$

and

$$w(r) \xrightarrow[r\to\infty]{} A_D(1 + 3/r\alpha + 3/r^2\alpha^2)\exp(-r\alpha).$$

The asymptotic D-state to S-state ratio η is given by

$$\eta = A_D/A_S,$$

where

$$A_S = \frac{4\pi N}{(2\pi)^{3/2}}\frac{1}{\alpha}\left[\frac{\pi}{2}\frac{\exp(-b\beta)}{(\beta^2 - \alpha^2)}\{\beta\sinh(b\alpha) + \alpha\cosh(b\alpha)\}\right.$$
$$\left. + \frac{c_v}{2\sqrt{2}}\left(\frac{\lambda^{-1} - I_{11}(\alpha^2)}{I_{12}(\alpha^2)}\right)\sinh(b\alpha)\right]$$

and

$$A_D = \frac{4\pi N}{(2\pi)^{3/2}}\frac{t\gamma^2\exp(-b\gamma)}{(\gamma^2 - \alpha^2)}\frac{\pi}{2\alpha}\left[\sinh(b\alpha)\{b(b\gamma - 5)\right.$$
$$+ \gamma(4b\gamma - 15)/(\gamma^2 - \alpha^2) + 8\gamma^3/(\gamma^2 - \alpha^2)^2\}$$
$$+ \alpha\cosh(b\alpha)\{b^2 + (7b\gamma - 8)/(\gamma^2 - \alpha^2) + 8\gamma^2/(\gamma^2 - \alpha^2)^2\}$$
$$\left. - \frac{3b^2\gamma^3}{(\gamma^2 - \alpha^2)}\left\{\frac{\cosh(b\alpha)}{b\alpha} - \frac{\sinh(b\alpha)}{(b\alpha)^2}\right\}\right].$$

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