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Fine-hyperfine structures of $c\bar{c}$ and $b\bar{b}$ systems in a non-relativistic Hulthen plus linear potential

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Abstract. The heavy mesons of the charmonium and upsilon family are described in an alternative static potential model chosen in a combination of Hulthen and linear potential. We find that the quark-confining potential in the form of an equal admixture of vector and scalar parts successfully explains the fine-hyperfine structures of $c\bar{c}$ and $b\bar{b}$ systems in a flavour-independent manner. The leptonic decay widths of the vector mesons of ψ and Υ families are calculated taking into account the Poggio-Schnitzer correction. We obtain some of the bound states of the yet-to-be observed $t\bar{t}$ system for the t-quark mass ranging from 50 to 200 GeV.

Keywords. Quarkonium; static potential; fine-hyperfine; flavour-independent; decay width; Fermi-Breit Hamiltonian.

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1. Introduction

Non-relativistic potential model approach for the bound quark-antiquark systems like $(c\bar{c})$ and $(b\bar{b})$, representing the heavy meson families of ψ and Υ, respectively, has proved quite successful. In such models the $(Q\bar{Q})$-system is considered to interact non-relativistically through a static potential $V(r)$ together with a spin-dependent perturbation $\delta V_{\text{spin}}(r)$. Strictly speaking, the form of the static potential $V(r)$ as well as its Lorentz character are a priori unknown. The same is also true for the $\delta V_{\text{spin}}(r)$. However, theoretical and phenomenological considerations have led to various suggestions (Gunion and Willey 1975; Appelquist and Politzer 1975; Kang and Schnitzer 1975; Eichten and Gottfried 1977; Quigg and Rosner 1977; Richardson 1979; Buchmuller *et al* 1980; Martin 1980; Barik and Jena 1980) of the static potential $V(r)$. In all these prescriptions $\delta V_{\text{spin}}(r)$ is generally considered to be generated from $V(r)$ through the Breit-Fermi mechanism of non-relativistic reduction of Bethe-Salpeter equation to order v^2/c^2, where $V(r)$ is usually believed to have a mixed Lorentz character with a vector part $V_v(r)$ and a scalar part $V_s(r)$ on the basis of strong theoretical (Eichten and Feinberg 1981; Gromes 1984) and phenomenological (Martin 1986) arguments. The specific forms of $V_v(r)$ and $V_s(r)$ are, however, chosen differently in different models. Nevertheless, it is usually believed that the vector part $V_v(r)$ must contain a short-range Coulomb-like piece due to one-gluon exchange, whereas the

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scalar part $V_s(r)$ must be due only to the long range confining interaction. The confining part of the interaction potential is generally chosen in linear form (Harrington *et al* 1975; Eichten *et al* 1975; Barbieri *et al* 1976) which derived further support from lattice calculations (Wilson 1974).

In the regime of the $Q\bar{Q}$-interaction explored through the presently available data on heavy mesons (except for the p-levels of $(c\bar{c})$ and $(b\bar{b})$ spectra); most of the models proposed so far are more or less equally satisfactory. But it becomes somewhat difficult to explain satisfactorily the p-levels of $(c\bar{c})$ and $(b\bar{b})$-spectra simultaneously in almost all such models. Recently observed candidates for the 1^1P_1-levels of $(c\bar{c})$ and $(b\bar{b})$-systems (Baglin *et al* 1986; Bowcock *et al* 1987) have added further confusion in tracking down an appropriate form of $(Q\bar{Q})$-potential through phenomenological studies (Gupta and Kogerler 1988). However, Gupta and Kogerler's (1988) analysis shows a preference for the Cornell-type potential (Eichten *et al* 1978, 1980) obtained by a simple superposition of the asymptotic forms as,

$$V(r) = -(4/3)(\alpha_s/r) + ar + V_0. \tag{1}$$

Although this potential describes the $c\bar{c}$-spectrum fairly well, it fails when applied to the $(b\bar{b})$-spectrum in explaining the experimental mass difference between $1S$ and $2S$ states, unless α_s is given a value of $\sim 0{\cdot}5$ instead of $0{\cdot}2$ [which one needs to understand the total decay rates of ψ and ψ' using QCD (De Rujula *et al* 1975, 1977). It also fails in reproducing the observed $^1P_1$ level of $(b\bar{b})$-spectra. Stanley and Robson (1980) and Bhaduri *et al* (1980) have pointed out that the short-range Coulomb potential of (1) gives the spin-spin hyperfine interaction term proportional to $\delta(\mathbf{r})$ which in fact invalidates the non-relativistic approximation in addition to making the perturbation estimates of the mass splittings inaccurate for lighter hadrons. Of course, the appearance of $\delta(\mathbf{r})$ may be argued to be partly due to the unnatural non-relativistic reduction in Breit-Fermi mechanism which may become a smooth function with a finite range, if $\delta V_{\text{spin}}(r)$ is somehow obtained correctly (Ono 1983). Nevertheless, it certainly warrants a critical look at the choice of the potential in the form (1). In fact continuation of the $(1/r)$-behaviour from one-gluon exchange at short distance to all r surely does not follow from any first principle physical argument. On the other hand, at short distance, various relativistic effects such as quark pair creation must arise which might distort the original Coulomb-like behaviour. In view of this, it may be worthwhile to replace the Coulombic part in (1) by some other suitable form which may contain the Coulomb behaviour in a limiting sense only.

With this motivation in mind we would like to suggest an effective potential in the form of Hulthen plus linear instead of the Coulomb plus linear one. The task to which the present work is devoted is to use this Hulthen plus linear potential to explore the possibility of a non-relativistic description of the $Q\bar{Q}$-bound states of the charmonium and upsilon families. One of the objectives here is to understand the precise nature of the spin dependence of the potential giving rise to the fine hyperfine structures of $(c\bar{c})$ and $(b\bar{b})$-systems. In particular, we are interested in testing this potential model at the P-wave splittings of upsilon, where normally most of the potential models fail. It may always be possible to have a good fit to one particular aspect like the P-wave splitting alone. But our purpose here is to obtain, as far as possible, a good overall description of the ψ and Υ-family which would serve as a better indicator for the suitability for the model.

Some theoretical and experimental investigations give preliminary indications regarding the existence of a still heavier quark flavour (i.e., t-quark), which may possibly give rise to the still to be observed ($t\bar{t}$)-spectrum. The t-quark mass is constrained to have various lower limits such as 25 GeV from TRISTAN (Takasaki 1987), 44 GeV from UAI (Wingerter 1987) and about 50 GeV from theoretical considerations (Ellis *et al* 1987) based on the ARGUS result (Albrecht *et al* 1987) of $B\bar{B}$-mixing. The analysis of Bernabeau and Pich (1987) gave the upper bound of the t-quark mass as $m_t < 200$ GeV (180 GeV) for Higgs particle mass $m_H < 1$ TeV (100 GeV). In view of these observations, it may be worthwhile to predict the still to be observed ($t\bar{t}$)-spectrum in the present model with the t-quark mass taken in a range $50\,\text{GeV} \leqslant m_t \leqslant 200\,\text{GeV}$. This would provide a unified description of ($c\bar{c}$), ($b\bar{b}$) as well as the ($t\bar{t}$)-systems in the framework of the present non-relativistic potential model in a flavour-independent manner.

2. Potential model

In this section we shall outline the Hulthen plus linear potential with our prescription for its Lorentz structure to generate the necessary spin dependence for explaining the fine-hyperfine splittings of charmonium and upsilon systems.

2.1 *The static potential*

The static ($Q\bar{Q}$)-potential has been chosen here in the form,

$$V(r) = V_H(r) + (Kr + D), \tag{2}$$

where the Hulthen potential

$$V_H(r) = -V_0 \exp(-r/a)/[1 - \exp(-r/a)]. \tag{3}$$

At very small distances or in the limit $V_0 \to 0$ and $a \to \infty$ (such that $V_0 a$ is finite), the Hulthen potential reduces to the Coulomb potential as,

$$V_H(r) \to -V_0 a/r = -(4/3)(\alpha_s/r). \tag{4}$$

Therefore $V_H(r)$ may be treated as a smeared-out Coulomb potential with the smearing length parameter a. The confining part of the potential is retained in its usual linear form Kr. With a suitable set of parameters (a, V_0, D), the shape of the potential $V(r)$ is displayed in figure 1.

With such a choice of the static potential $V(r)$, the radial Schrödinger equation is to be solved numerically in order to obtain the spin-averaged masses $\bar{M}_{nL}(Q\bar{Q})$ and their normalized wave functions $R_{nL}(r)$ for various ($Q\bar{Q}$)-systems in different radial and orbital configurations.

2.2 *Spin-structure of the potential*

The quantitative explanation of the fine-hyperfine levels depends on the spin-dependent potential $\delta V_{\text{spin}}(r)$ which is presumed to be generated by $V(r)$ through the

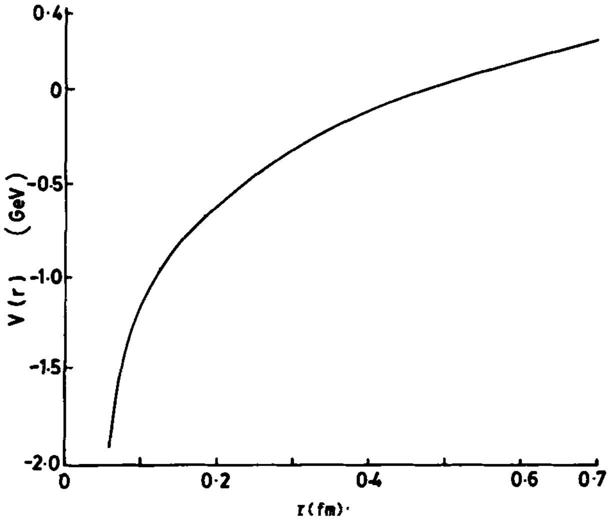

Figure 1. The potential $V(r)$ given by equation (2) is plotted against r.

usual Fermi-Breit mechanism (Schnitzer 1975 a, b; Pumplin *et al* 1975; Gromes 1977 a, b; Cellmaster and Henyey 1978). For doing this we prescribe the necessary Lorentz character of the static potential $V(r)$ as follows. The Hulthen potential $V_H(r)$ is considered as purely vector-like. The confining part $V_c(r) = (Kr + D)$, however, is assumed to be an admixture of vector and scalar parts with a vector fraction g_v. Then the static potential $V(r) = V_v(r) + V_s(r)$ can be considered to have the vector and the scalar parts, respectively, as follows:

$$V_v(r) = [V_H(r) + g_v V_c(r)],$$
$$V_s(r) = (1 - g_v)V_c(r). \tag{5}$$

This prescription would lead to the spin-dependent potential $\delta V_{\rm spin}(r)$ in its standard form

$$\partial V_{\rm spin}(r) = A_1(r)\mathbf{L}\cdot\mathbf{S} + A_2(r)\mathbf{S}_1\cdot\mathbf{S}_2 + A_3(r)S_{12}, \tag{6}$$

where the tensor operator

$$S_{12} = 3(\mathbf{S}_1\cdot\hat{r})(\mathbf{S}_2\cdot\hat{r}) - (\mathbf{S}_1\cdot\mathbf{S}_2). \tag{7}$$

Here $\mathbf{L}$ is the relative orbital angular momentum, $\mathbf{S}$ is the total spin and $(\mathbf{S}_1, \mathbf{S}_2)$ are the individual quark spins. The radial functions $A_1(r)$, $A_2(r)$ and $A_3(r)$ (with $m_{q_1} = m_q = m_{q_2}$) are as follows:

$$A_1(r) = (3/2m_q^2)[V_v'(r)/r - V_s'(r)/3r],$$
$$A_2(r) = (2/3m_q^2)\nabla^2 V_v(r),$$
$$A_3(r) = (1/3m_q^2)[V_v'(r)/r - V_v''(r)]. \tag{7a}$$

These functions expressed explicitly become,

$$\begin{aligned} A_1(r) &= [3B_1(r) + (4g_v - 1)K/r]/2m_q^2, \\ A_2(r) &= 2[2B_1(r) - B_2(r) + 2Kg_v/r]/3m_q^2, \\ A_3(r) &= [B_1(r) + B_2(r) + g_v K/r]/3m_q^2, \end{aligned} \tag{8}$$

where

$$\begin{aligned} B_1(r) &= (V_0/ar)\exp(-r/a)/[1 - \exp(-r/a)]^2, \\ B_2(r) &= (V_0/a^2)\exp(-r/a)[1 + \exp(-r/a)]/[1 - \exp(-r/a)]^3. \end{aligned} \tag{9}$$

Then the total effective potential including spin-dependent corrections to the lowest order is given by

$$V_{\text{eff}}(r) = [V(r) + \delta V_{\text{spin}}(r)]. \tag{10}$$

The spin-independent relativistic corrections are presumably very small for heavy quarkoniums, for which their exclusion may not undermine the ultimate result.

2.3 *Level splittings and mass formulas*

The spin-dependent correction $\delta V_{\text{spin}}(r)$ treated as a perturbation can yield the fine-hyperfine levels with mass formulas written conveniently in the matrix form as follows:

$$\begin{pmatrix} M(^3S_1) \\ M(^1S_0) \end{pmatrix} = \begin{pmatrix} 1 & 1/4 \\ 1 & -3/4 \end{pmatrix} \times \begin{pmatrix} \bar{M}_{ns} \\ \langle A_2(r) \rangle_{ns} \end{pmatrix}, \tag{11}$$

$$\begin{pmatrix} M(^3P_2) \\ M(^3P_1) \\ M(^3P_0) \\ M(^1P_1) \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1/4 & -1/10 \\ 1 & -1 & 1/4 & 1/2 \\ 1 & -2 & 1/4 & -1 \\ 1 & 0 & -3/4 & 0 \end{pmatrix} \times \begin{pmatrix} \bar{M}_{nP} \\ \langle A_1(r) \rangle_{nP} \\ \langle A_2(r) \rangle_{nP} \\ \langle A_3(r) \rangle_{nP} \end{pmatrix}, \tag{12}$$

$$\begin{pmatrix} M(^3D_3) \\ M(^3D_2) \\ M(^3D_1) \\ M(^1D_2) \end{pmatrix} = \begin{pmatrix} 1 & 2 & 1/4 & -1/7 \\ 1 & -1 & 1/4 & 1/2 \\ 1 & -3 & 1/4 & -1/2 \\ 1 & 0 & -3/4 & 0 \end{pmatrix} \times \begin{pmatrix} \bar{M}_{nD} \\ \langle A_1(r) \rangle_{nD} \\ \langle A_2(r) \rangle_{nD} \\ \langle A_3(r) \rangle_{nD} \end{pmatrix}. \tag{13}$$

Here $\bar{M}_{nL}$ are the spin-averaged masses to be obtained from the exact numerical solutions to the radial Schrödinger's equation with the static potential $V(r)$ in equation (2). $\langle A_i(r) \rangle_{nL}$, $i = 1, 2, 3$ are the relevant expectation values of the radial functions given in (8), which in their turn depend on $\langle B_1(r) \rangle_{nL}$, $\langle B_2(r) \rangle_{nL}$, $\langle 1/r \rangle_{nL}$ and the parameter g_v.

3. Phenomenological results and discussion

First we attempt a reasonable description of the $(c\bar{c})$ and $(b\bar{b})$-spectrum at the gross level with a suitable choice of the potential parameters and the quark masses taken to reproduce satisfactorily the spin-averaged masses $\bar{M}_{nL}$, velocity parameter $\beta_q^2 = \langle v^2/c^2 \rangle$, and the leptonic decay widths. An adequate choice of the parameters and quark masses arrived at in this process is as follows:

$$(V_0, D) = (0{\cdot}225\,\text{GeV},\ -0{\cdot}4195\,\text{GeV}),$$
$$(a, K) = (2{\cdot}22\,\text{GeV}^{-1},\ 0{\cdot}206\,\text{GeV}^2),$$
$$(m_c, m_b) = (1{\cdot}4998\,\text{GeV},\ 4{\cdot}9111\,\text{GeV}). \tag{14}$$

The results for the spin-averaged masses $\bar{M}_{nL}$, average size $\langle r \rangle$ and the quark velocity parameters β_q^2 for the $(c\bar{c})$ and $(b\bar{b})$ systems are presented in tables 1 and 2 respectively.The spin-averaged masses $\bar{M}_{nL}$ obtained here seem to be in good accord with the data. Looking at the averaged size $\langle r \rangle$ we note that

$$\langle r \rangle_\Upsilon : \langle r \rangle_\psi \simeq 1:2,$$
$$\langle r \rangle_{\Upsilon'} : \langle r \rangle_{\psi'} \simeq 1:2. \tag{15}$$

This implies that the members of the upsilon family are approximately half the size of

Table 1. Spin-averaged mass $\bar{M}_{nL}$ and the values of $\langle B_1(r) \rangle$, $\langle B_2(r) \rangle$, $\langle 1/r \rangle$, $\langle r \rangle$ and β_q^2 for the $c\bar{c}$ system.

Bound state nL	$\bar{M}_{nL}$ (GeV)	$\langle B_1(r) \rangle$ (GeV^3)	$\langle B_2(r) \rangle$ (GeV^3)	$\langle 1/r \rangle$ (GeV)	$\langle r \rangle$ (GeV^{-1})	β_q^2
1S	3·0678	4·1204	7·8564	0·7317	1·8681	0·2065
2S	3·7005	1·4527	2·7666	0·4407	3·7001	0·2560
3S	4·1702	0·6769	1·2884	0·3413	5·1933	0·3074
4S	4·5725	0·3844	0·7314	0·2893	6·4666	0·3526
5S	4·9349	0·2485	0·4725	0·2575	7·5638	0·3854
1P	3·5102	0·0248	0·0523	0·3815	—	—
2P	4·0021	0·0032	0·0066	0·2999	—	—
1D	3·8200	0·0028	0·0063	0·2785	—	—

Table 2. Spin averaged mass $\bar{M}_{nL}$, and the values of $\langle B_1(r) \rangle$, $\langle B_2(r) \rangle$, $\langle 1/r \rangle$, $\langle r \rangle$ and β_q^2 for the $b\bar{b}$ system.

Bound state nL	$\bar{M}_{nL}$ (GeV)	$\langle B_1(r) \rangle$ (GeV^3)	$\langle B_2(r) \rangle$ (GeV^3)	$\langle 1/r \rangle$ (GeV)	$\langle r \rangle$ (GeV^{-1})	β_q^2
1S	9·4170	40·287	76·337	1·5191	0·9448	0·0897
2S	10·0066	7·2913	13·806	0·7259	2·2490	0·0774
3S	10·3672	2·8210	5·340	0·5403	3·2682	0·0874
4S	10·6601	1·3847	2·6206	0·4425	4·2031	0·0996
5S	10·9175	0·8304	1·5712	0·3854	5·0156	0·1113
1P	9·9022	0·0991	0·2019	0·6433	—	—
2P	10·2729	0·0092	0·0187	0·4785	—	—
1D	10·1589	0·0077	0·0162	0·4393	—	—

the corresponding members of the ψ-family. Again the justification of the non-relativistic approach adopted here seems quite evident from the tabulated values of the velocity parameters β_q^2. Tables 1 and 2 further provide the relevant expectation values $\langle B_1(r)\rangle_{nL}$, $\langle B_2(r)\rangle_{nL}$ and $\langle 1/r\rangle_{nL}$ for various bound states of $(c\bar{c})$ and $(b\bar{b})$-systems. We also calculate the leptonic decay widths of vector mesons of $(c\bar{c})$ and $(b\bar{b})$-systems. For doing this we use the Poggio-Schnitzer corrected form of the Van Royen-Weisskopf formula (Schnitzer 1978; Poggio and Schnitzer 1979)

$$\Gamma(V_{nS}\to e^+e^-) = \frac{4\alpha^2 e_q^2}{M_v^2}|R_{nS}(r\simeq 1/m_q)|^2, \tag{16}$$

where e_q is the quark charge and M_v is the mass of the vector meson of the $(Q\bar{Q})$-systems. However, this formula should not be trusted too much in absolute terms since the corrections are large and not quite certain. In such circumstances leptonic decay width ratios, such as,

$$\frac{\Gamma(V_{nS}\to e^+e^-)}{\Gamma(V_{1S}\to e^+e^-)} = \left[\frac{M_v(1S)}{M_v(nS)}\right]^2 |R_{nS}(0)|^2/|R_{1S}(0)|^2, \tag{17}$$

can serve as meaningful and reliable quantities. The radial wavefunctions $R_{nS}(0)$ and $R_{nS}(r\simeq 1/m_q)$ together with the results calculated from (16) and (17) are presented in table 3 for $(c\bar{c})$ as well as $(b\bar{b})$-systems. The agreement with the experimental data, particularly in the case of the decay width ratios, is quite satisfactory.

In order to obtain the fine hyperfine splittings, we now need to have a suitable choice of the vector fraction parameter g_v. We observe that if $g_v = 1$, ^3P_J splittings for $(c\bar{c})$ system come out too large as compared to the experimental data (Particle Data Group 1986). Again $g_v = 0$ leads to very small ^3P_J splittings for charmonium spectrum with the level ordering completely upset in comparison to what has been observed in experiment. Therefore, it is reasonable to presume that the Lorentz character of $V_c(r)$ in the form of an admixture of scalar and vector parts with $g_v < 1$. We find that $g_v = 0{\cdot}5$ gives quite reasonable values for ^3P_J- splittings of the $c\bar{c}$-system which closely agrees with the experiment. This choice of $g_v = 0{\cdot}5$ along with the parameters given in (14) provide good overall description of the fine-hyperfine levels in both $(c\bar{c})$ and $(b\bar{b})$ spectra. The results so obtained are presented in table 4, which show a good overall agreement with experimental data. The slight disagreement of some of the values can be taken to be well within the range of relativistic spin-independent corrections not considered in the present calculations.

Sometimes the fine-hyperfine splittings of quarkonium levels are analysed in terms of the relevant expectation values $\langle A_i(r)\rangle_{nL}$, $i = 1, 2, 3$. These are in fact the model-dependent quantities appearing in $\delta V_{\rm spin}(r)$ through the usual Fermi-Breit prescription, which ultimately decides the level splittings. These quantities can also be extracted from the experimental mass data of the fine-hyperfine levels to be compared with the corresponding values obtained from the model calculations. Table 5 provides such a comparison. It is observed that $\langle A_3(r)\rangle_{nL}$ in general is somewhat underestimated whereas $\langle A_2(r)\rangle_{1P}$ for $c\bar{c}$ is grossly overestimated. This is the reason for the value of $M_{c\bar{c}}(1^1P_1) = 3{\cdot}4934$ GeV as against the reported experimental value of $3{\cdot}5254$ GeV. The experimental result indicates that in $1P$-level, the spin-singlet mass is coincident with the centre of gravity of the spin-triplets for $(c\bar{c})$-system only, whereas the same is not true in the case of $(b\bar{b})$-system. In fact, our result $M_{b\bar{b}}(1^1P_1) = 9{\cdot}899$ GeV compares very

Table 3. Leptonic decay widths and their ratios for the vector mesons of $c\bar{c}$ and $b\bar{b}$ systems along with their experimental values.

Vector meson	Mass (GeV)	$R_{nS}(0)$	$R_{nS}(r \simeq (1/m_q))$	$\Gamma(V \to e^+e^-)$ keV		$\Gamma(V_{nS} \to e^+e^-)/\Gamma(V_{1S} \to e^+e^-)$	
				Predicted	Experimental	Predicted	Experimental
ψ	3·107	1·0037	0·7437	5·42	4·7 ± 0·3	1	1
ψ'	3·717	0·8433	0·5772	2·28	2·1 ± 0·2	0·414	0·447
ψ''	4·180	0·7874	0·5070	1·39	0·75 ± 0·15	0·25	0·16
ψ'''	4·579	0·7603	0·4640	0·97	0·47 ± 0·10	0·17	0·10
ψ''''	4·940	0·7471	0·4340	0·73	—	0·13	—
Υ	9·448	3·4200	2·6452	1·86	1·22 ± 0·05	1	1
Υ'	10·013	2·2737	1·7205	0·70	0·54 ± 0·03	0·375	0·443
Υ''	10·370	2·0085	1·4994	0·49	0·40 ± 0·03	0·265	0·328
Υ'''	10·661	1·8571	1·3709	0·39	0·24 ± 0·05	0·200	0·197
Υ''''	10·918	1·7721	1·2956	0·33	0·31 ± 0·07	0·180	0·254
Υ'''''	11·153	1·7131	1·2413	0·29	0·13 ± 0·03	0·150	0·106

Table 4. Fine-hyperfine structure of the $c\bar{c}$ and $b\bar{b}$ systems.

	$c\bar{c}$ system		$b\bar{b}$ system	
$q\bar{q}$ state	Predicted mass (GeV)	Experimental mass (GeV)	Predicted mass (GeV)	Experimental mass (GeV)
1^1S_0	2·949	2·981 $\pm$0·002	9·322	—
1^3S_1	3·107	3·097 $\pm$0·001	9·448	9·46 $\pm$0·0002
2^1S_0	3·649	3·594 $\pm$0·005	9·987	—
2^3S_1	3·717	3·686 $\pm$0·0001	10·013	10·023 $\pm$0·0003
3^1S_0	4·140	—	10·358	—
3^3S_1	4·180	4·028 $\pm$0·003	10·370	10·355 $\pm$0·0005
4^1S_0	4·550	—	10·655	—
4^3S_1	4·579	4·415 $\pm$0·006	10·661	10·577 $\pm$0·004
5^1S_0	4·935	—	10·914	—
5^3S_1	4·940	—	10·918	10·865 $\pm$0·008
1^3P_0	3·4302	3·4149 $\pm$ 0·0011	9·880	9·8598 $\pm$ 0·0013
1^3P_1	3·4893	3·5107 $\pm$ 0·0005	9·896	9·8919 $\pm$ 0·0007
1^3P_2	3·5484	3·5563 $\pm$ 0·0004	9·911	9·9133 $\pm$ 0·0006
1^1P_1	3·4934	3·5254 $\pm$ 0·0008	9·899	9·8948 $\pm$ 0·0015
2^3P_0	3·969	—	10·267	10·235
2^3P_1	3·994	—	10·271	10·255 $\pm$0·002
2^3P_2	4·022	—	10·276	10·271 $\pm$0·002
2^1P_1	3·988	—	10·270	—
1^3D_3	3·854	—	10·164	—
1^3D_2	3·812	—	10·158	—
1^3D_1	3·777	3·770 $\pm$0·0024	10·152	—
1^1D_2	3·807	—	10·157	—

Table 5. Some expectation values $\langle A_1(r)\rangle_{nL}$, $\langle A_2(r)\rangle_{nL}$ and $\langle A_3(r)\rangle_{nL}$ in comparison with the corresponding values extracted from experimental data.

$(Q\bar{Q})_{nL}$	$\langle A_1(r)\rangle$ Theory (Expt.) (MeV)	$\langle A_2(r)\rangle$ Theory (Expt.) (MeV)	$\langle A_3(r)\rangle$ Theory (Expt.) (MeV)
$(c\bar{c})_{1S}$	—	158·6(116 $\pm$ 5)	—
$(c\bar{c})_{1P}$	34 (35)	22·5(−0·015)	17·24(40)
$(c\bar{c})_{2S}$	—	68(92)	—
$(b\bar{b})_{1P}$	8·9(14)	3·56(5·4 $\pm$ 1·7)	5·0 (12·8)
$(b\bar{b})_{2P}$	2·6(10)	—	1·07 (9·8 $\pm$ 1·4)

well with the experimental value of 9·8948 GeV. Nevertheless, if one takes seriously the discrepancies in $\langle A_i(r)\rangle_{nL}$ as seen from table 5, then there may be some scope for improvement by fine-tuning of the model parameters. However, since these model-dependent quantities $\langle A_i(r)\rangle_{nL}$, by themselves, are not observed directly in experiments and particularly the masses of $^1P_1$ levels of $(c\bar{c})$ and $(b\bar{b})$-systems used in the extraction of $\langle A_2(r)\rangle_{1P}$ are understood to be still tentative, we do not attach much significance to the discrepancies now. It must be pointed out here that although the non-QCD inspired phenomenological potentials (such as power-law or logarithmic) broadly explain the $c\bar{c}$ and $b\bar{b}$-spectra, they fail to explain the masses of the ^3P_J levels of $(b\bar{b})$-

Table 6. Predictions for the mass spectra of the yet-to-be observed tt system.

$t\bar{t}$ Bound state nL	m_t(GeV)							
	50	70	90	110	130	150	170	190
$1S$	96·64	135·35	174·10	212·85	251·70	290·40	329·47	368·09
$2S$	99·00	138·65	178·30	218·02	257·70	297·38	337·07	376·753
$3S$	99·50	139·35	179·17	219·02	258·80	298·71	338·58	378·432
$4S$	99·81	139·656	179·51	219·42	259·325	299·22	339·135	379·05

system. Also the Cornell potential, which explains the ${}^1P_1$ level of charmonium spectrum, fails in reproducing the experimental value of $M_{b\bar{b}}(1\,{}^1P_1)$. The QCD-sum rule calculations also predict the centre of gravity of $1\,{}^3P_{0,1,2}$ levels of $(b\bar{b})$-system at 9830 $\pm$ 30 MeV, which is in disagreement with the experimental value of 9900 MeV (Khare 1987). In view of this, the present potential model seems to be encouragingly successful.

We now extend this model with the same potential parameters as given in (14) to predict the S-state masses of the yet to be observed $(t\bar{t})$-system. We take the t-quark mass in the range $50 \leqslant m_t \leqslant 200$ GeV. These results are displayed in table 6. We notice that for $m_t = 50$ GeV, the level spacing $(M_{2S}\text{-}M_{1S}) = 2{\cdot}36$ GeV as against the corresponding value of 2·2 GeV obtained by the Cornell potential (Eichten *et al* 1978, 1980). Correlating the Cornell potential with Hulthen potential at a very short distance, we find that for both $(c\bar{c})$ and $(b\bar{b})$-systems, our potential gives the quark-gluon coupling constant $\alpha_s \simeq \frac{3}{4}V_0 a = 0{\cdot}37$ as against $\alpha_s = 0{\cdot}2$ given by QCD for $(c\bar{c})$-system (Eichten *et al* 1975). However, our value is quite acceptable in the sense that one needs a larger value of $\alpha_s \simeq 0{\cdot}5$ to reproduce the $1S$, $2S$ and other levels of both $(c\bar{c})$ and $(b\bar{b})$ systems simultaneously (De Rujula *et al* 1977; Moorhouse *et al* 1977; Quigg and Rosner 1979; Ono 1982).

4. Conclusion

We conclude that the gross features like the spin-averaged mass spectra and leptonic decay widths as well as the fine-hyperfine structures of the $(c\bar{c})$ and $(b\bar{b})$ systems can be described reasonably well in a flavour-independent manner by a simple Hulthen plus linear potential. Here the Hulthen potential has been considered to have a Lorentz vector character, whereas the confining potential in linear form is taken as an equal mixture of vector and scalar parts. This assignment of mixed Lorentz character to the confining interaction is in line with the phenomenological findings of other workers (Appelquist *et al* 1978; Beavis *et al* 1979). A similar conclusion was also reached in non-relativistic (Barik and Jena 1980, 1981) as well as relativistic (Barik and Jena 1982; Jena and Tripathi 1983) fits of the meson spectra with non-Coulombic power-law potential. Although this model realizes the $1\,{}^1P_1$ level of the $(b\bar{b})$-system in close agreement with the experimental data, it does not show the observed coincidence between the centre of gravity mass of the ${}^3P_J$ levels of and spin-singlet mass of the $(c\bar{c})_{1P}$-state. It is understood that the $1\,{}^1P_1$-masses of $(c\bar{c})$ and $(b\bar{b})$-spectra have been obtained from a very difficult experiment which needs further experimentation. Till then, it may be difficult to understand theoretically the above features in the P-level splittings.

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