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Heat conduction through moist soils at different temperatures

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Abstract. A numerical model has been described to estimate the effective thermal conductivity of moist soils, considering the effective continuous medium approximation and taking all possible interactions. Numerical solutions of the exact formulations are presented. Experimental measurements have been carried out for the thermal conductivity of dune sand at different moisture contents employing the method of unsteady state line source. The predicted and measured values show reasonable agreement.

Keywords. Moisture content; effective thermal conductivity; water vapour diffusion; vapour pressure gradient.

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1. Introduction

An understanding of effective thermal conductivity (ETC) of soils is useful from the point of view of energy saving from thermal energy storage devices (Beniwal *et al* 1985). Much attention has been focussed in this field because of its wide applications in the design and construction of buildings, roads, pipelines etc (De Vries 1958; Alpret *et al* 1976; Eckert and Faghri 1980; Dayan *et al* 1984). The ETC of soil depends on the solid soil material, soil texture, pore size distribution, water content and soil temperature. The heat flow is mainly due to conduction mechanism through solid particles, air and water in the pores and around the particles. In addition, heat flow in moist soils also occurs due to water vapour diffusion.

Many attempts (Maxwell 1904; Powers 1961; Sugawara and Yoshizawa 1961; Parrot and Stuckes 1975; Pande *et al* 1984; Pande and Chaudhary 1984) have been made to estimate the ETC of two-phase systems. Few of them, originally developed for two-phase systems, have also been extended to three-phase systems but with disappointing results. For the case of large variation of porosity, no single theory is applicable to more than one kind of sample. In the present study, possible interactions of continuous and dispersed phases have been considered. The model of Pande *et al* (1984), originally developed for two-phase systems for the prediction of effective thermal conductivity, can be extended to systems like moist soil. The technique met here is that of a real situation in a natural system and as such one may hope for a better prediction.

2. Theory

Using the model developed by Pande *et al* (1984) for two-phase systems in which a medium of homogeneous cubic dispersion of finitely-spaced, spherical and interacting solid particles of phase λ_s has been considered in the continuous phase λ_a (air). In the case of loose materials none of the phases provides continuous matrix because such a multiphase medium has a large porosity range. The equations obtained do not also correspond to a real system. Here one can adopt the effective continuous medium (ECM) approach where equal volumes of solid and void occur in the system. A real system can be obtained by introducing small dispersions (ξ) of solid or gas phase in the ECM. By using the same procedure and the equations developed by Pande and Chaudhary (1984), the relevant equations up to the fourth neighbour are

$$\begin{aligned} \lambda_{en} &= \lambda_{ecm}(1 + 2{\cdot}309\,\xi_s^{2/3}), \\ \lambda_{en} &= \lambda_{ecm}(1 + 3{\cdot}233\,\xi_s^{2/3}), \\ \lambda_{en} &= \lambda_{ecm}(1 + 3{\cdot}489\,\xi_s^{2/3}), \\ \lambda_{en} &= \lambda_{ecm}(1 + 3{\cdot}844\,\xi_s^{2/3}), \end{aligned} \tag{1}$$

for $\psi_s - 0{\cdot}5 = \xi_s > 0$ and for the reverse case

$$\begin{aligned} \lambda_{en} &= \lambda_{ecm}(1 - 1{\cdot}154\,\xi_g^{2/3}), \\ \lambda_{en} &= \lambda_{ecm}(1 - 1{\cdot}616\,\xi_g^{2/3}), \\ \lambda_{en} &= \lambda_{ecm}(1 - 1{\cdot}745\,\xi_g^{2/3}), \\ \lambda_{en} &= \lambda_{ecm}(1 - 1{\cdot}922\,\xi_g^{2/3}), \end{aligned} \tag{2}$$

for $\psi_g - 0{\cdot}5 = \xi_g > 0$, where ξ_s and ξ_g represent small dispersions of solid and gas phase respectively in ECM and ψ_s and ψ_g are volume fractions of solid and gas phase.

Following the approach of Pande and Chaudhary (1984) and taking all possible combinations of these expressions, a set of equations for effective thermal conductivity of ECM would be

$$\begin{aligned} \lambda_{ecm} &= 1{\cdot}092(\lambda_g \lambda_s)^{1/2}, \\ \lambda_{ecm} &= 1{\cdot}054(\lambda_g \lambda_s)^{1/2}, \\ \lambda_{ecm} &= 1{\cdot}019(\lambda_g \lambda_s)^{1/2}, \\ \lambda_{ecm} &= 0{\cdot}896(\lambda_g \lambda_s)^{1/2}, \\ \lambda_{ecm} &= 0{\cdot}478(\lambda_g \lambda_s)^{1/2}, \\ \lambda_{ecm} &= 0{\cdot}445(\lambda_g \lambda_s)^{1/2}, \\ \lambda_{ecm} &= 0{\cdot}420(\lambda_g \lambda_s)^{1/2}, \\ \lambda_{ecm} &= 0{\cdot}315(\lambda_g \lambda_s)^{1/2}, \end{aligned} \tag{3}$$

where λ_s and λ_g are the thermal conductivities of solid and gas phase respectively. We have dropped expressions which are imaginary.

2.1 *Thermal conductivity of moist soil*

When the volume fraction of water is zero, the soil is a two-phase system composed of solid and air. Gradual addition of water in the sample changes the thermal conductivity of a phase other than the solid. This process may be considered similar to the dispersion of water vapour in the continuous air medium. When the volume fraction of water is equal to the air phase, it means that the corresponding volume of air phase is completely replaced by water. The sample then is again a two-phase system composed of solid and water. Between these possibilities we have three phases: solid, air and water. When dispersion of water in the air is small, the thermal conductivity of moist air with in the pore will be (Beniwal *et al* 1985)

$$\lambda_{ma} = \lambda_a\left[1 + 3{\cdot}844\left(\frac{\lambda_w - \lambda_a}{\lambda_w + 2\lambda_a}\right)\psi_{ma}^{2/3}\right], \tag{4}$$

for $0 < \psi_{ma} < 0{\cdot}5$.

At saturation the air is completely replaced and then $\psi_{ma} \rightarrow \psi_w$ and $\lambda_{ma} \rightarrow \lambda_w$. When ψ_{ma} lies between 0·5 and 1·0, the value of ETC of moist air is

$$\lambda_{ma} = \lambda_w\left[1 + 3{\cdot}844\left(\frac{\lambda_a - \lambda_w}{\lambda_a + 2\lambda_w}\right)(1 - \psi_{ma}^{2/3})\right], \tag{5}$$

for $0{\cdot}5 < \psi_{ma} < 1{\cdot}0$.

If ψ_m be the volume fraction of moisture and ψ_a that of air, then ψ_{ma}, the volume fraction of moisture with respect to air, can be expressed as

$$\psi_{ma} = [\psi_m/\psi_a]$$

where $\psi_m = (m/M)\ \psi_a$. Here m and M represent the varying moisture content and moisture content at saturation, in weight percent, respectively. λ_a and λ_w i.e. thermal conductivity of air and water both depends on the temperature. These dependencies (De Vries 1974) are

$$\lambda_w = 0{\cdot}55 + 2{\cdot}34 \times 10^{-3}T - 1{\cdot}1 \times 10^{-5}T^2$$

and

$$\lambda_a = 0{\cdot}0237 + 6{\cdot}41 \times 10^{-5}T \tag{6}$$

respectively. Here T is the temperature expressed in °C. On introducing above, one obtains the full description of the situation.

In soil, because of the large porosities none of the phases (solid, liquid and air) provides a continuous medium, the continuous medium that one may imagine is the ECM formed by equal volume fractions of two phases. By allowing a small dispersion of the solid or liquid or air phase in effective continuous medium one can generate the real system. When a solid phase with $\psi_s > 0{\cdot}5$, we have a small dispersion of solid material which is $\xi_s = \psi_s - 0{\cdot}5$ in the ECM formed by moist air and solid phase. Then the ETC of moist soil will be

$$\lambda_e = \lambda_{ecm}\left[1 + 3{\cdot}844\left(\frac{\lambda_s - \lambda_{ecm}}{\lambda_s + 2\lambda_{ecm}}\right)\xi_s^{2/3}\right]. \tag{7}$$

Also, one may have expression for the case when $\psi_s < 0{\cdot}5$ by replacing $\xi_s^{2/3}$ in (7) by $(0{\cdot}5 - \psi_s)^{2/3}$.

3. Experimental measurements

To test the validity of the expression developed, the ETC values of moist soil were measured by a dynamical method. The ETC of the soil in which a thermal probe was placed is calculated by the relation

$$\lambda_e = \frac{Q}{4\pi(\theta_2 - \theta_1)} \ln(t_2/t_1), \tag{8}$$

where Q is the power per unit length of heat source, θ_2 and θ_1 are two values of temperatures of the probe surface at times t_2 and t_1 respectively.

The experimental set-up consisted mainly of a sample container, a constant temperature bath and power and temperature measuring units. The sample container is a double-walled cylinder (inner diameter 7·5 cm) fabricated from copper sheet (figure 1).

The thermal probe was inserted along the central axis of a sample container having the soil. Water of constant temperature was then circulated through the double-walled sample container from a thermostat through rubber tubes. The circulation of water was continued for 5 to 6 hours to attain a uniform temperature in the whole system. At each established value, the power was imparted to the probe and the temperature-time history of the sample was recorded by a digital microvoltmeter. The moisture content was measured by weighing representative amounts of sample before and after drying in an oven at 110 ± 5°C for 24 hours. The difference between the mass of moist soil and dry soil provided a profile of the mass of liquid in the sample.

4. Results and discussion

The measured ETC values of each soil sample are reported in table 1. The calculated values of ETC of unsaturated and saturated samples at different temperatures are also given in table 1 along with estimated values using other models (Lichtnecker 1926; Brailsford and Major 1964). At different temperatures, the thermal conductivity of air

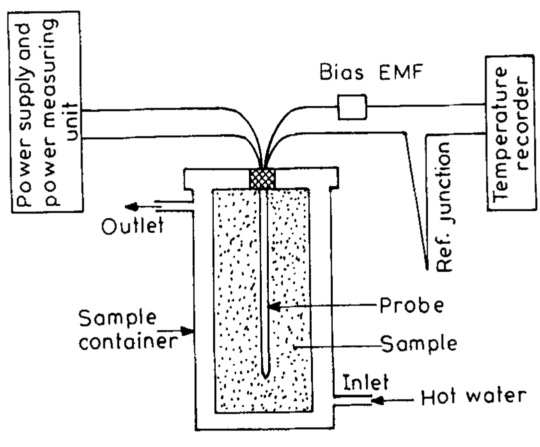

Figure 1. Experimental arrangement for the measurement of λ_e of the sample material.

Table 1. Thermal conductivity of moist soils ($Wm^{-1} k^{-1}$) at different temperatures.

Moisture content by weight	λ_e Experimental	λ_e Present Model	λ_e Lichtnecker's Model	λ_e Brailsford and Major's Model
At 30°C				
24·74	1·959	1·968	1·537	1·986
17·72	1·266	1·135	1·030	1·885
10·40	1·050	0·917	0·679	1·782
6·43	0·806	0·813	0·542	1·727
1·02	0·244	0·259	0·398	1·642
0	0·209	0·217	0·375	1·639
At 40°C				
24·50	1·980	1·951	1·544	2·003
18·12	1·417	1·210	1·077	1·909
13·50	1·123	0·971	0·829	1·843
4·83	0·864	0·799	0·508	1·721
1·03	0·257	0·256	0·416	1·663
0	0·218	0·219	0·387	1·649
At 50°C				
24·19	1·982	1·924	1·545	2·020
22·49	1·699	1·735	1·406	1·994
14·77	1·202	1·006	0·914	1·881
8·14	1·021	0·962	0·631	1·785
2·31	0·594	0·656	0·456	1·703
0·97	0·260	0·264	0·423	1·684
0	0·226	0·228	0·401	1·650
At 60°C				
24·60	2·033	1·960	1·593	2·032
18·47	1·455	1·278	1·133	1·938
10·23	1·205	1·183	0·716	1·815
4·042	0·823	0·763	0·507	1·712
0	0·238	0·230	0·405	1·665

and water was calculated using (6). We have taken the thermal conductivity of solid phase independent of temperature in the range 30–60°C of value 3·35 $Wm^{-1}k^{-1}$ (Singh *et al* 1987). It is seen that the present theory gives values much better than the models of Brailsford and Major or Lichtnecker. The percentage error between the calculated and experimental results using the present model lie was 0–17%. The calculated results are even better in low and high content regions. The large variation is probably due to an uneven distribution of fluid phase in parts of the wet sample at different environment temperatures.

The values of λ_e of sand increase slowly as the percentage of water raised from zero to a certain specific value. A sharp rise is noticed as the percentage of water was further increased. This behaviour can be understood through soil-water interaction. At minor moisture values, the water molecules form films around soil particles. Although these films do not improve the thermal contact between solid particles, the increase in thermal conductivity here is due to adsorbed water. λ_e then increases sharply as the amount of water added fills the wedges in the sample.

Because of water vapour diffusion the value of λ_e of moist sand increases with increase in temperature. The diffusion depends on vapour pressure gradient and diffusion coefficient. Both these parameters increase with increase of temperature.

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