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Masses in structure functions

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Abstract. We derive an approximate formula for the quark mass in ξ-formalism. Using the QCD-based form of structure function, we analyse the SLAC-MIT data to estimate the experimentally allowed ranges of quark mass. The possible variation of our results with QCD cut-off parameter, gluon distribution, hard intrinsic charm component of the proton wavefunction and the higher twist effects are also discussed.

Keywords. Quark mass; structure function; quantum chromodynamics.

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1. Introduction

There are numerous theoretical conjectures about the possible masses of quarks (Gasser and Leutwyler 1982). Admittedly in a theory like quantum chromodynamics (QCD), where quarks are confined, it may not be possible to directly measure quark masses. However the mass parameter that appears in the Lagrangian can be related to other quantities which are experimentally accessible. Sometime ago, it was argued (Chakravarty 1983) that the decay of τ-lepton provides a reasonable basis for the determination of the masses of the up and down quarks. Similarly there were suggestions that the quark masses can be estimated from both the transverse cut-off in e^+e^- annihilation (Choudhury 1976) and the experimental data on charged hadronic multiplicity (Choudhury and Paranjape 1978).

Quark masses also occur in the physics of deep inelastic scattering through the ξ-variable (Georgi and Politzer 1976). However, in the subsequent analysis (Aubert *et al* 1982; Godbole and Roy 1982), only target mass effects were considered neglecting the constituent mass effects. Specifically it was observed (Godbole and Roy 1982) that mass effects are quantitatively significant only for the low Q^2 data of the SLAC-MIT experiment (Bodek *et al* 1979), while for the other high statistics experiments like CERN–Dortmund–Heidelberg–Saclay collaboration (Watschack 1981), European–Muon collaboration (Aubert *et al* 1981; Drees 1981) and Bologna–CERN–Dubna–Munchen-Saclay collaboration (Bollini *et al* 1981; Smadja 1981), such effects are not that significant.

The present work is devoted to the derivation of an approximate formula for the quark mass which occurred in the ξ-formalism (Georgi and Politzer 1976) and its phenomenological application to the SLAC–MIT data (Bodek *et al* 1979). We use the approximate QCD model of $F_2^{ep}(x, Q^2)$ of Buras and Gaemers (1978) for numerical analysis. We also study the possible variation of our results with QCD cut-off parameter Λ, gluon distribution, intrinsic charm component of proton wavefunction (Brodsky *et al* 1980; Roy 1980) and higher twist effects (Abbott *et al* 1980; Aubert *et al* 1981; Bollini *et al* 1981; Godbole and Roy 1982) in the structure function. Further, our result on quark mass is compared with similar studies by others.

In QCD, the running quark mass $m_q(Q^2)$ corresponds to constituent and current quark masses in the limits of small and large Q^2 respectively. The Q^2 region considered in the present work is close to the current quark limit as regards the u, d quark masses (Georgi and Politzer 1976). However, there are suggestions (Georgi and Politzer 1976) that the light quark masses will be indistinguishable from zero in inclusive lepton hadron scattering. One has also to state clearly which particular quark flavour mass one is trying to estimate. Since we have not incorporated flavour symmetry breaking in the formalism our estimate should therefore measure the average current quark mass in the explored Q^2 regime i.e. $m_q(Q^2) \sim [\sum m_f(Q^2)]/f$ where m_f is the current quark mass of flavour f.

In perturbative QCD, at non-asymptotic Q^2 range, one has additional complications due to target mass effects and heavy quark mass effects besides the contribution of higher twist operator. As these effects are not completely understood at present theoretically, we have confined ourselves to higher twist, since its phenomenological structure was discussed earlier by several authors (Abbott *et al* 1980; Aubert *et al* 1981; Bollini *et al* 1981; Godbole and Roy 1982) with reasonable agreement.

Besides the approximate QCD model of Buras and Gaemers (1978) there are several other phenomenological structure functions (Perkin *et al* 1977; Karlinger and Sullivan 1978) in the literature. We study the sensitivity of our results to these different sets of parametrization.

In our estimate, we need the value of Λ, the QCD cut-off parameter as well. The present value of Λ as estimated from the high Q^2 deep inelastic scattering data is around 0·2 GeV (Duke and Roberts 1985; Virchaux 1987) to be compared with the earlier one Λ = 0·1 GeV (Rith 1983). We also test the sensitivity of our result with other values of Λ (Buras and Gaemers 1978; Roy *et al* 1978).

2. Formalism

The structure function with target and quark mass correction has the form (Georgi and Politzer 1976)

$$\bar{F}_2(x,Q^2) = \frac{x^2}{\xi^2}v^3F_2(\xi,Q^2) + \frac{6M^2}{Q^2}x^3v^4\int_\xi^1 \frac{\mathrm{d}x' F_2(x',Q^2)}{x'^2} + 12\frac{M^4}{Q^4}x^4v^5\int_\xi^1 \mathrm{d}x' \int_{x'}^1 \frac{\mathrm{d}x'' F_2(x'',Q^2)}{x''^2}, \tag{1}$$

where

$$\xi = \frac{x\left[1+\left(1+\frac{4m_q^2}{Q^2}\right)^{1/2}\right]}{1+\left(1+\frac{4M^2x^2}{Q^2}\right)^{1/2}}, \tag{2}$$

$$v = \left(1+\frac{4M^2x^2}{Q^2}\right)^{-1/2}. \tag{3}$$

Here $m_q \equiv m_q(Q^2)$ is the momentum-dependent running quark mass, M is the target mass and x and Q^2 are the usual variables.

In order to derive the approximate expression for quark mass, we note that

$$\frac{x^2 v^3}{\xi^2} = 1 + \frac{n_1(x, M^2, m_q^2)}{Q^2} + \frac{n_2(x, M^2, m_q^2)}{Q^4} + O(1/Q^6), \tag{4}$$

$$x^3 v^4 = x^3\left(1 + \frac{l_1(x, M^2)}{Q^2} + O(1/Q^4)\right), \tag{5}$$

$$x^4 v^5 = x^4 + O(1/Q^2), \tag{6}$$

where

$$n_1 = -2(m_q^2 + 2M^2x^2), \tag{7}$$

$$n_2 = 4\left(m_q^4 + 2m_q^2 M^2 x^2 + \frac{9}{2}M^4 x^4\right), \tag{8}$$

$$l_1 = -8M^2x^2. \tag{9}$$

Further as

$$\xi = x + \frac{x}{Q^2}(m_q^2 - x^2M^4) + \frac{x}{Q^4}(-m_q^4 - x^2M^2m_q^2 + 2x^4M^4) + O(1/Q^6), \tag{10}$$

$$F_2(\xi, Q^2) = F_2(x, Q^2)\left[1 + \frac{f(x, Q^2, m_q^2, M^2)}{Q^2} + \frac{g(x, Q^2, m_q^2, M^2)}{Q^4} + O(1/Q^6)\right] \tag{11}$$

where

$$f(x, Q^2, m_q^2, M^2) = -(m_q^2 - x^2M^2)\frac{\tilde{f}(x, Q^2)}{F_2(x, Q^2)}, \tag{12}$$

and

$$g(x, Q^2, m_q^2, M^2) = \frac{\tilde{g}(x, Q^2, m_q^2)}{F_2(x, Q^2)}. \tag{13}$$

The forms of $\tilde{f}$ and $\tilde{g}$ for the approximate QCD structure function (Buras and Gaemers 1978) to be discussed in § 2.1, can be calculated explicitly. Let us define

$$F_2'(\xi, Q^2) = \int_\xi^1 \frac{dx' F_2(x', Q^2)}{x'^2} \tag{14}$$

and

$$F_2''(\xi, Q^2) = \int_\xi^1 dx' \int_x^1 \frac{dx'' F_2(x'', Q^2)}{x''^2}. \tag{15}$$

Let us now assume that both F'_2 and F''_2 can be expressed as following power series:

$$F'_2(\xi, Q^2) = F'_2(x, Q^2)\left[1 + \frac{t_1(x, Q^2, m_q^2, M^2)}{Q^2} + \frac{t_2(x, Q^2, m_q^2, M^2)}{Q^4} + O(1/Q^6)\right], \tag{16}$$

$$F''_2(\xi, Q^2) = F''_2(x, Q^2)\left[1 + \frac{u_1(x, Q^2, m_q^2, M^2)}{Q^2} + \frac{u_2(x, Q^2, m_q^2, M^2}{Q^4} + O(1/Q^6)\right], \tag{17}$$

where t_1, t_2, u_1 and u_2 are some functions of x, Q^2, m_q^2 and M^2 and

$$F'_2(x, Q^2) = \int_x^1 \frac{\mathrm{d}x' F_2(x', Q^2)}{x'^2}, \tag{18}$$

$$F''_2(x, Q^2) = \int_x^1 \mathrm{d}x' \int_{x'}^1 \frac{\mathrm{d}x'' F_2(x'', Q^2)}{x''^2}. \tag{19}$$

We now demonstrate that if we consider terms $O(1/Q^2)$ in ξ formalism, an expression for the quark mass can be obtained which can be estimated phenomenologically. Taking terms $O(1/Q^2)$

$$\bar{F}_2(x, Q^2) = F_2(x, Q^2)\left[1 + \frac{1}{Q^2}\left\{n_1 - (m_q^2 - x^2M^2)\frac{\tilde{f}(x, Q^2)}{F_2(x, Q^2)} + \frac{6M^2x^3F'_2(x, Q^2)}{F_2(x, Q^2)}\right\}\right], \tag{20}$$

using the expressions for m_1 and $\tilde{f}$ as given in (7) and (12) one obtains

$$m_q^2 = \left(\frac{Q^2}{2 + \frac{\tilde{f}(x, Q^2)}{F_2(x, Q^2)}}\right)\left(\frac{F_2(x, Q^2) - \bar{F}_2(x, Q^2)}{F_2(x, Q^2)}\right) - \frac{\left\{M^2x^2\left(4 - \frac{\tilde{f}(x, Q^2)}{F_2(x, Q^2)}\right) - 6M^2x^3\frac{F'_2(x, Q^2)}{F_2(x, Q^2)}\right\}}{2 + \frac{\tilde{f}(x, Q^2)}{F_2(x, Q^2)}}. \tag{21}$$

If we include the higher twist term (Abbott *et al* 1980; Bollini *et al* 1981; Aubert *et al* 1982; Godbole and Roy 1982) with the following form

$$F_2(x, Q^2) = F_2^{\mathrm{QCD}}(x, Q^2)\left[1 + \frac{\mu^2}{(1 - x)Q^2}\right], \tag{22}$$

equation (20) is modified to

$$\bar{F}_2(x,Q^2)=F_2(x,Q^2)\left[1+\frac{1}{Q^2}\left\{n_1-(m_q^2-x^2M^2)\frac{\tilde{f}(x,Q^2)}{F_2(x,Q^2)}+\frac{6M^2x^3F'(x,Q^2)}{F_2(x,Q^2)}+\frac{u^2}{1-x}\right\}\right]. \tag{23}$$

while (21) gets the structure

$$m_q^2=\left(\frac{Q^2}{2+\frac{\tilde{f}(x,Q^2)}{F_2(x,Q^2)}}\right)\left(\frac{F_2(x,Q^2)-\bar{F}_2(x,Q^2)}{F_2(x,Q^2)}\right)-\frac{\left\{M^2x^2\left(4-\frac{\tilde{f}(x,Q^2)}{F_2(x,Q^2)}\right)-\frac{u^2}{1-x}-\frac{6M^2x^3F_2'(x,Q^2)}{F_2(x,Q^2)}\right\}}{2+\frac{\tilde{f}(x,Q^2)}{F_2(x,Q^2)}}. \tag{24}$$

If we include an intrinsic charm component $|uudc\bar{c}\rangle$ in the valence part of the proton wave function with a hard x distribution given as (Brodsky *et al* 1980)

$$C(x)=\bar{C}(x)=18x^2\left[\tfrac{1}{3}(1-x)(1+10x+x^2)-2x(1+x)\ln\frac{1}{x}\right]. \tag{25}$$

The structure function $F_2(x,Q^2)$ acquires an additional piece

$$F_2^{\mathrm{IC}}(x)=\frac{4}{9}x(C(x)+\bar{C}(x)). \tag{26}$$

In this case $F_2(x,Q^2)$, $\tilde{f}(x,Q^2)$ and $F_2'(x,Q^2)$ of (21) and (24) are modified to

$$F_2(x,Q^2)\rightarrow F_2(x,Q^2)+F_2^{\mathrm{IC}}(x), \tag{27}$$

$$\tilde{f}(x,Q^2)\rightarrow\tilde{f}(x,Q^2)-\frac{n(x)}{d(x)}F_2^{\mathrm{IC}}(x), \tag{28}$$

$$F_2'(x,Q^2)\rightarrow F_2'(x,Q^2)+H(x), \tag{29}$$

where

$$n(x)=3\left\{\tfrac{1}{3}(1-x)(1+10x+x^2-2x(1+x)\ln\frac{1}{x}\right\}-x\left\{\tfrac{1}{3}(1-x)(1+10x+x^2)v(x)+2x(1+x)\ln\frac{1}{x}\omega(x)\right\}, \tag{30}$$

$$d(x) = \tfrac{1}{3}(1-x)(1+10x+x^2) - 2x(1+x)\ln\frac{1}{x}, \tag{31}$$

$$H(x) = \int_x^1 \frac{F_2^{IC}(x')\,dx'}{x'}, \tag{32}$$

with

$$v(x) = \frac{1}{1-x} - \frac{2x^2+10x}{x(1+10x+x^2)}, \tag{33}$$

$$\omega(x) = \frac{1}{x} + \frac{1}{1+x}. \tag{34}$$

2.1 *QCD model of structure function*

In order to evaluate numerically equations (21) and (24) we assume the validity of the approximate QCD model of F_2^{ep} as given by Buras and Gaemers (1978). Here the sea contribution is approximated in terms of the first two moments $\xi^1(Q^2)$ and $\xi^2(Q^2)$ and are given as

$$\xi(x,Q^2) = \xi^1(Q^2)\left[\frac{\xi^1(Q^2)}{\xi^2(Q^2)} - 1\right](1-x)\left(\frac{\xi^1(Q^2)}{\xi^2(Q^2)} - 2\right) \tag{35}$$

and identically to ξ_c. The moments $\xi^1(Q^2)$ and $\xi^2(Q^2)$ have structures

$$\xi^1(Q^2) = \tfrac{1}{4}[\tfrac{3}{14} + (3\xi^1 + V^1 + \xi_c^1 - \tfrac{3}{14})L^{-56/75} + (\xi^1 - \xi_c^1 - V^1)L^{-32/75}, \tag{36}$$

$$\xi^2(Q^2) = \tfrac{1}{4}[O_1^2(Q^2) + (\xi^2 - \xi_c^2 - V^2)L^{-2/3}], \tag{37}$$

where the moments on the right side are evaluated at a moderately low Q^2, Q_0^2 and

$$L = \ln(Q^2/\Lambda^2)/\ln(Q_0^2/\Lambda^2), \tag{38}$$

Λ being the QCD cut-off parameter. The operator $O_1^2(Q^2)$ in (37) is given by

$$O_1^2(Q^2) = [0{\cdot}925(3\xi^2 + \xi_c^2 + V^2) + 0{\cdot}144G^2]L^{-0{\cdot}609} + [0{\cdot}075(3\xi^2 + \xi_c^2 + V^2) - 0{\cdot}144G^2]L^{-1{\cdot}386} \tag{39}$$

G^2 being the second gluon moment. The valence part of the QCD model (Buras and Gaemers 1978) is approximately given by

$$xV(x,Q^2) = \frac{1{\cdot}5x^{(0{\cdot}7-0{\cdot}176\ln L)}(1-x)^{2{\cdot}6+0{\cdot}8\ln L}}{B(0{\cdot}7-0{\cdot}176\ln L, 3{\cdot}6+0{\cdot}8\ln L)} \tag{40}$$

where the beta function in the denominator ensures the usual zeroth moment sum rule. Buras and Gaemers (1978) used the following inputs:

$$Q_0^2 = 1{\cdot}8\,\text{GeV}^2;\quad \Lambda = 0{\cdot}3\,\text{GeV};\quad \xi^1 = 0{\cdot}018$$

$$\xi^2 = 0{\cdot}1528 \times 10^{-2}; \quad G^2 = 0{\cdot}0335$$
$$V^1 = 0{\cdot}244; \quad V^2 = 0{\cdot}0785. \tag{41}$$

To check the sensitivity of our results on the parameters, we shall repeat calculations with $\Lambda = 0{\cdot}1$ GeV, $0{\cdot}2$ GeV and $0{\cdot}5$ GeV and $G^2 = 0{\cdot}057$. We note that $\Lambda = 0{\cdot}5$ GeV represents the maximal value of scale-breaking parameters allowed by the electron and muon data (Roy *et al* 1978) while $\Lambda = 0{\cdot}2$ GeV is the best fit obtained in recent experiments (Duke and Roberts 1985; Virchaux 1987) which supersedes the earlier value $\Lambda = 0{\cdot}1$ (Rith 1983). In analysing the effects of higher twist and intrinsic charm, equations (22) to (34), we choose both $\Lambda = 0{\cdot}1$ and $0{\cdot}2$ GeV and $\mu^2 = 0{\cdot}2\,\text{GeV}^2$ (Godbole and Roy 1982). We have also chosen $\xi_c^1 = \xi_c^2 = 0$.

2.2 *Low Q^2 analysis*

The prescription for the structure function given above is valid down to $Q^2 = 1{\cdot}8\,\text{GeV}^2$ (Roy *et al* 1978). Below this, it has been parameterized by Stein *et al* (1975) as

$$F_2^{ep}(x', Q^2) = F_2^{ep}(x') f(Q^2), \tag{42}$$

where x' is the variable given by Bloom and Gilman (1970) as

$$1/x' = (1/x) + (M^2/Q^2), \tag{43}$$

The function $f(Q^2)$ defined in (42) can be expressed in terms of the elastic form factor using the dipole approximation (Stein *et al* 1975)

$$f(Q^2) = 1 - \left(\frac{1 + 7{\cdot}8Q^2/M^2}{1 + Q^2/M^2}\right)\left(1 + \frac{Q^2}{0{\cdot}71}\right)^{-4{\cdot}0}, \tag{44}$$

which is ≈ 1 for $Q^2 \geqslant 1{\cdot}8\,\text{GeV}^2$. The scaling function $F_2^{ep}(x)$ is given as (Barger and Phillips 1974)

$$F_2^{ep}(x') = 0{\cdot}173x'^{(1/2)}(1 - x'^2)^3 + 0{\cdot}123x'^{(1/2)}(1 - x'^2)^5$$
$$+ 0{\cdot}575x'^{(1/2)}(1 - x'^2)^7 + 0{\cdot}184(1 - x')^9. \tag{45}$$

Assuming that ξ formalism makes sense at such low Q^2 with $O(1/Q^2)$ approximation, we use (42)–(45) to estimate the quark masses occurring in (24).

Including the higher twist term of (22) in (42), one has

$$F_2^{ep}(x', Q^2) = F_2^{ep}(x') f(Q^2)\left(1 + \frac{\mu^2}{(1 - x)Q^2}\right). \tag{46}$$

We note that in the present approximation, the quark mass acquires explicit x and Q^2 dependences (equations (21) and (24)).

For comparison we note that in QCD (Truong 1982), the running quark mass $m_q(Q^2)$ is related to its scale-invariant version $\hat{m}$ by the following Q^2 dependent expression

$$m_q(Q^2) = \frac{\hat{m}}{(\frac{1}{2}\ln(Q^2/\Lambda^2))^{-r_1/\beta_1}}, \tag{47}$$

where $r_1 = 2$ for three colours and

$$\beta_1 = -(11/2) + (n/3), \tag{48}$$

n being the number of flavours. Our approximate formula ((21) and (24)) has an x dependence as well, besides such Q^2 dependence.

2.3 *Phenomenological structure functions*

In order to study the sensitivity of our results to different sets of parametrization of F_2^{ep} other than Buras and Gaemers (1978), we use the models of some other authors as well.

(a) *PSS models:* Perkins *et al* (1977) have shown that scale-breaking in electron (muon) scattering on both proton and isoscalar target can be described in terms of single form factor:

$$F_2(x, Q^2) = F_2(x, Q_0^2)\left(\frac{Q^2}{Q_0^2}\right)^{0\cdot25-x}, \tag{49}$$

where Q_0^2 is 3 GeV^2. This has two variants (Roy *et al* 1978; Choudhury and Misra 1987);

(i) PSS (I): Here the valence quarks have the structures

$$V_{u,d}(x, Q^2) = V_{u,d}(x, Q_1^2(x))\left(\frac{Q^2}{Q_1^2(x)}\right)^{0\cdot25-x} \tag{50}$$

with

$$Q_1^2(x) \sim 13\cdot3x, \tag{51}$$

(ii) PSS (II): Here

$$V_{u,d}(x, Q^2) = V_{u,d}(x, Q_1^2(x))\left(\frac{Q^2}{Q_1^2(x)}\right)^{0\cdot25-x}. \tag{52}$$

which mimics the QCD model of Buras and Gaemers (1978).

(b) KS models: Karlinger and Sullivan (1978) considered logarithmic and power law phenomenological structure functions as defined below:

(i) KS (I): Here

$$F_2^{ep}(x, Q^2) = \left(1 + b(x)\frac{\ln(Q^2/Q_0^2)}{\ln(Q_0^2/\Lambda^2)}\right)F_2(x, Q_0^2) \tag{53}$$

$Q_0^2 = 3\,\text{GeV}^2$, $\Lambda = 0\cdot4\,\text{GeV}^2$, $b(x) = b_0[1-(x/x_0)^\alpha]$, $x_0^{-1} = 4\cdot489$, $b_0 = 0\cdot804$ and $\alpha = 0\cdot6343$. It is to be noted that (53) is to be used recursively for high Q^2 range.

(ii) KS (II): Here

$$F_2^{ep}(x, Q^2) = (Q^2/Q_0^2)^{f(x)}F_2(x, Q_0^2) \tag{54}$$

with

$$Q_0^2 = 3\,\text{GeV}^2, \quad f(x) = a + bx + \varepsilon/x, \quad a = 0\cdot21324,$$

$$b = -0\cdot97955 \quad \text{and} \quad \varepsilon = 5\cdot9 \times 10^{-4}.$$

3. Results and discussion

We now report on the estimation of quark masses using the formalism of § 2. To that end we use the SLAC–MIT data (Bodek *et al* 1979) in the Q^2 range of $1 < Q^2 < 16\,\text{GeV}^2$. As discussed earlier, for $Q^2 > 2\,\text{GeV}^2$, we use the approximate QCD formula of the structure function as given by Buras and Gaemers (1978) and compare the sensitivity with other phenomenological forms (Perkins *et al* 1977; Karlinger and Sullivan 1978). For $Q^2 < 2\,\text{GeV}^2$ we use the one given by Stein *et al* (1975) and Barger and Phillips (1974). In our analysis we also study the sensitivity of our result with QCD cut-off parameter (Λ), gluon distribution (G^2) and intrinsic charm (IC). We have made detailed numerical analysis of (21) and (24) in the entire (x, Q^2) range of SLAC–MIT data and the following features are observed:

(i) As Λ is increased, quark mass is decreased.
(ii) Inclusion of intrinsic charm (IC) in the structure function increase the value of quark mass.
(iii) Higher twist (HT) seems to increase the quark mass further suggesting the following inequality:

$$m_q(\text{with IC}) < m_q(\text{with HT}) < m_q(\text{with IC and HT}). \tag{55}$$

(iv) The average value of the quark mass with $\Lambda = 0{\cdot}1$ and $0{\cdot}2$ GeV for the (x, Q^2) range under study is found to be $m_q \sim 0{\cdot}526$ GeV and $m_q \sim 0{\cdot}402$ GeV respectively with IC and HT.
(v) In low Q^2 region ($Q^2 < 2$ GeV) where higher twist effect is dominant, $m_q \leqslant 0{\cdot}267$ GeV.

Table 1. Quark masses (m_q) in GeV estimated from SLAC–MIT data (Bodek *et al* 1979) at $Q^2 = 4{\cdot}0\,\text{GeV}^2$ using QCD structure function (Buras and Gaemers 1978) and including intrinsic charm (IC) and higher twist (HT) effect with $\Lambda = 0{\cdot}1$ GeV and $0{\cdot}2$ GeV and $G^2 = 0{\cdot}0335$.

	m_q in GeV					
	With HT		With IC		With IC and HT	
x	$\Lambda = 0{\cdot}1$ GeV	$\Lambda = 0{\cdot}2$ GeV	$\Lambda = 0{\cdot}1$ GeV	$\Lambda = 0{\cdot}2$ GeV	$\Lambda = 0{\cdot}1$ GeV	$\Lambda = 0{\cdot}2$ GeV
0·20	$0{\cdot}441 \pm 0{\cdot}085$	$0{\cdot}401 \pm 0{\cdot}060$ / $0{\cdot}070$	$0{\cdot}319 \pm 0{\cdot}103$ / $0{\cdot}158$	$0{\cdot}312 \pm 0{\cdot}084$ / $0{\cdot}095$	$0{\cdot}455 \pm 0{\cdot}077$ / $0{\cdot}093$	$0{\cdot}415 \pm 0{\cdot}065$ / $0{\cdot}087$
0·25	$0{\cdot}407 \pm 0{\cdot}078$ / $0{\cdot}096$	$0{\cdot}382 \pm 0{\cdot}058$ / $0{\cdot}084$	$0{\cdot}278 \pm 0{\cdot}088$ / $0{\cdot}133$	$0{\cdot}215 \pm 0{\cdot}090$ / $0{\cdot}095$	$0{\cdot}416 \pm 0{\cdot}063$ / $0{\cdot}074$	$0{\cdot}398 \pm 0{\cdot}058$ / $0{\cdot}064$
0·33	$0{\cdot}391 \pm 0{\cdot}040$ / $0{\cdot}045$	$0{\cdot}371 \pm 0{\cdot}038$ / $0{\cdot}040$	$0{\cdot}281 \pm 0{\cdot}032$ / $0{\cdot}036$	$0{\cdot}228 \pm 0{\cdot}035$ / $0{\cdot}037$	$0{\cdot}388 \pm 0{\cdot}024$ / $0{\cdot}025$	$0{\cdot}355 \pm 0{\cdot}023$ / $0{\cdot}024$
0·40	$0{\cdot}358 \pm 0{\cdot}034$ / $0{\cdot}037$	$0{\cdot}322 \pm 0{\cdot}032$ / $0{\cdot}036$	$0{\cdot}301 \pm 0{\cdot}015$ / $0{\cdot}016$	$0{\cdot}249 \pm 0{\cdot}017$ / $0{\cdot}019$	$0{\cdot}368 \pm 0{\cdot}012$ / $0{\cdot}013$	$0{\cdot}336 \pm 0{\cdot}011$ / $0{\cdot}013$
0·50	$0{\cdot}348 \pm 0{\cdot}028$ / $0{\cdot}031$	$0{\cdot}354 \pm 0{\cdot}026$ / $0{\cdot}030$	$0{\cdot}433 \pm 0{\cdot}003$ / $0{\cdot}004$	$0{\cdot}387 \pm 0{\cdot}004$ / $0{\cdot}005$	$0{\cdot}455 \pm 0{\cdot}003$ / $0{\cdot}004$	$0{\cdot}401 \pm 0{\cdot}028$ / $0{\cdot}038$
0·60	$0{\cdot}472 \pm 0{\cdot}018$ / $0{\cdot}019$	$0{\cdot}412 \pm 0{\cdot}017$ / $0{\cdot}019$	$0{\cdot}559 \pm 0{\cdot}0009$ / $0{\cdot}0010$	$0{\cdot}499 \pm 0{\cdot}001$ / $0{\cdot}002$	$0{\cdot}567 \pm 0{\cdot}0009$ / $0{\cdot}0010$	$0{\cdot}507 \pm 0{\cdot}001$ / $0{\cdot}002$

In table 1 we show the estimation of quark mass for a typical $Q^2 = 4\,\text{GeV}^2$ with $\Lambda = 0{\cdot}1$ and $0{\cdot}2$. It includes both IC and HT contribution. In table 2 we also show the prediction of PSS (I), PSS (II), KS (I) and KS (II) parametrization. We observe that the quark mass decreases in these models as compared with Buras and Gaemers (1978). For high x regime ($x \geqslant 0{\cdot}33$), KS models cannot even accommodate any positive quark mass.

In figure 1 (a to i) we show the relationship between m_q^2 and Λ graphically for typical $Q^2 = 2{\cdot}0, 2{\cdot}5$ and $3{\cdot}0\,\text{GeV}^2$. It is seen that for fixed (x, Q^2), a solution, $m_q = 0$ is obtained only for a particular value of $\Lambda = \Lambda_0$ (say). For other values of Λ, $m_q \neq 0$. We note that the physical region of $m_q^2 - \Lambda$ plane is always positive. Negative regions are shown in figures 1 (a–i) only to facilitate the evaluation of Λ_0 corresponding to $m_q^2 = 0$. From these figures one can read any desired m_q and its corresponding Λ. We further note that Λ is invariably less than $0{\cdot}5\,\text{GeV}$ for $m_q^2 \geqslant 0$.

Let us now discuss if our analysis can be compared with similar analysis by other workers. In the present work, we have estimated the value of running quark mass as in the ξ formalism (Georgi and Politzer 1976). It has both x and Q^2 dependences (equations (21) and (24)).

For low Q^2 we note that the running quark mass is identified with constituent quark mass while for high Q^2 with the current one. For u, d quarks, $Q^2 = 3$–$4\,\text{GeV}^2$ is expected to probe its current value while for s and c quark (if present in the nucleon due to the sea or the IC component), $Q^2 = 3$–$4\,\text{GeV}^2$ may not be large enough to probe its current limit. Therefore, we ought to interpret m_q of table 1 or 2 and figure 1 (a–e) as

$$m_q \sim \frac{\sum m_f(Q^2)}{f}\bigg|\, Q^2 = 4\,\text{GeV}^2, \tag{56}$$

where f is the number of flavours.

We note from table 1 that for $\Lambda = 0{\cdot}2$, $m_q = 0{\cdot}4\,\text{GeV}$ with average $\langle x \rangle \sim 0{\cdot}33$. For

Table 2. Masses m_q in GeV estimated from SLAC–MIT data (Bodek *et al* 1979) at $Q^2 = 4{\cdot}0\,\text{GeV}^2$ using phenomenological structure function PSS(I), PSS(II), KS(I) and KS(II).

	m_q in GeV			
x	PSS I	PSS II	KS I	KS II
0·20	$0 < m_q < 0{\cdot}375$	$0 < m_q < 0{\cdot}309$	$0 < m_q < 0{\cdot}373$	$0 < m_q < 0{\cdot}348$
0·25	$0 < m_q < 0{\cdot}319$	$0 < m_q < 0{\cdot}338$	$0 < m_q < 0{\cdot}284$	$0 < m_q < 0{\cdot}285$
0·33	$0 < m_q < 0{\cdot}226$	$0{\cdot}306 \pm 0{\cdot}043$ $0{\cdot}050$	$0 < m_q < 0{\cdot}089$	No solution for $m_q \geqslant 0$
0·40	$0{\cdot}185 \pm 0{\cdot}054$ $0{\cdot}080$	$0{\cdot}229 \pm 0{\cdot}041$ $0{\cdot}051$	No solution for $m_q \geqslant 0$	-do-
0·50	$0{\cdot}138 \pm 0{\cdot}044$ $0{\cdot}069$	$0{\cdot}160 \pm 0{\cdot}044$ $0{\cdot}062$	-do-	-do-
0·60	$0{\cdot}301 \pm 0{\cdot}018$ $0{\cdot}019$	$0{\cdot}199 \pm 0{\cdot}029$ $0{\cdot}034$	-do-	$0 < m_q < 0{\cdot}143$

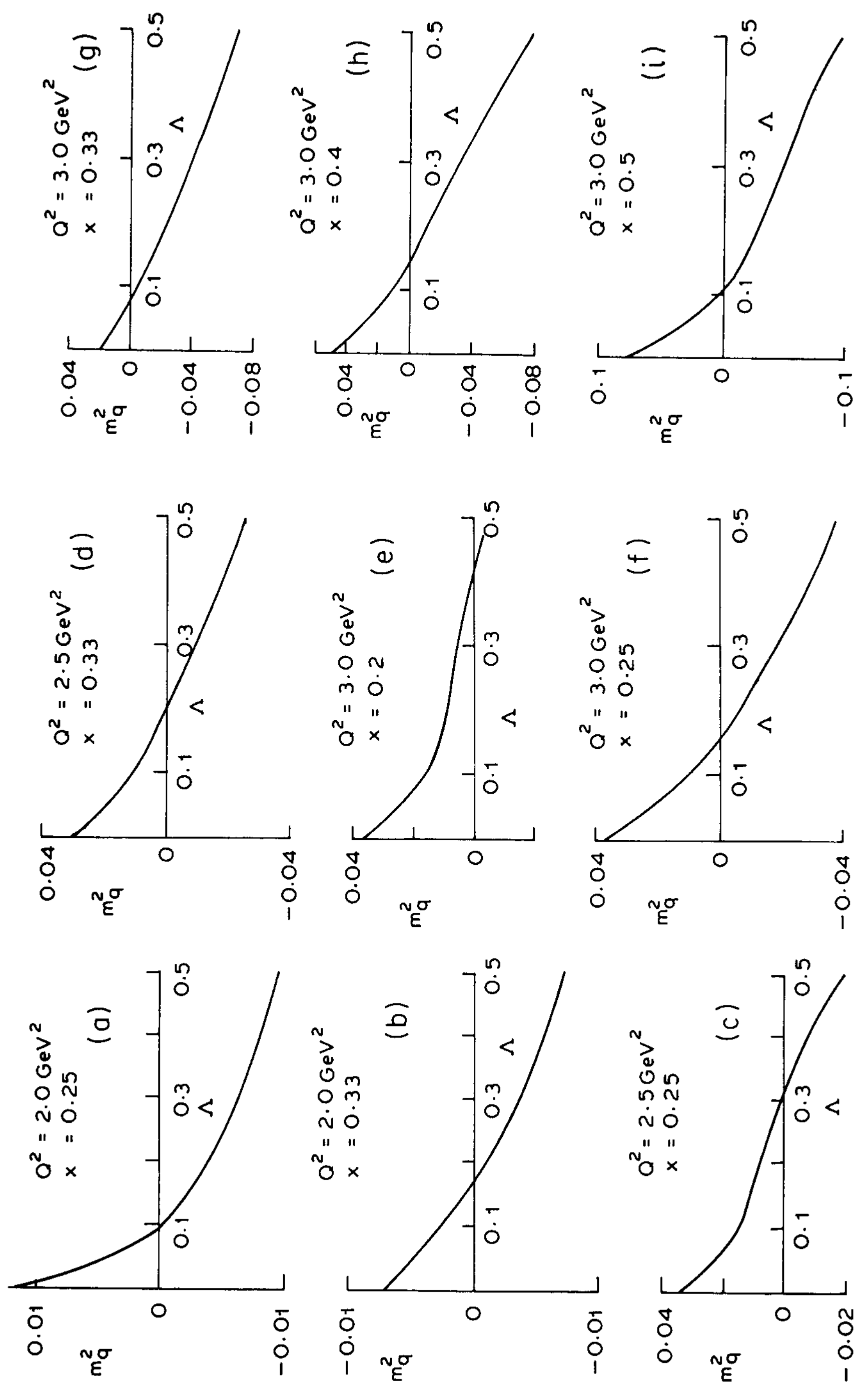

Figure 1. **a–i** Theoretical $m_q^2 - \Lambda$ curves for some typical (x, Q^2) data points.

PSS(I), PSS(II), KS(I) and KS(II) models, it is reduced to 0·2, 0·2, 0·17 and 0·16 GeV respectively. Thus our results are sensitive to different sets of parametrization, although qualitative features are not altered substantially. Our numerical values of the quark mass are higher than those obtained by other authors (Leutwyler 1974; Testa 1975; Choudhury and Paranjape 1978; Truong 1982). For example, while the QCD phenomenology (Choudhury and Paranjape 1978) with universal multiplicity hypothesis (Brodsky and Gunion 1976) gives a value of $m_q \sim 0{\cdot}1$ GeV, Testa (1975), on the other hand, obtains light quark mass $m_u = m_d = 0{\cdot}011$ GeV. However, the null plane dynamics (Leutwyler 1974) yields $m_u = m_d = 0{\cdot}054$ GeV and $m_s = 0{\cdot}125$–$0{\cdot}15$ GeV. A more recent estimate (Truong 1982) with QCD sum rule method (Shifman *et al* 1979; Narison 1987) gives $m_u = 0{\cdot}012$ and $m_d = 0{\cdot}022$ GeV.

We note that these authors estimate the scale-invariant masses $\hat{m}$ of (47) rather the running quark mass itself. Putting $Q^2 = 4\,\text{GeV}^2$, $\Lambda = 0{\cdot}2$ GeV, $r_1 = 2$ (three colour) and $\beta_1 = -25/6$ (four flavours) in (47), one has

$$m(Q^2)|_{Q^2=4\,\text{GeV}^2} \sim \tfrac{2}{3}\hat{m} \tag{57}$$

relating the running mass to its scale-invariant version in QCD. Thus our estimate of $m_q \sim 0{\cdot}4$–$0{\cdot}16$ GeV always exceeds similar estimation for u, d quarks. A plausible interpretation of our result is through (56) so that it measures in average sense the masses of s and c quarks as well. However, for s and c quarks $Q^2 = 4\,\text{GeV}^2$ might be low enough to be nearer to the constituent limits rather than the current ones. Hence the present analysis cannot clearly separate the current and constituent limits of various quark flavours as proper flavour symmetry breaking is not considered in this formalism. We therefore conclude that our analysis measures the average running quark mass for $Q^2 \sim 4\,\text{GeV}^2$ in the exact flavour symmetry. The possibility of estimating particular quark flavour mass in both current and constituent limits using the above formalism is an interesting one and warrants serious consideration. This possibility together with the study of stability of our results with terms $O(1/Q^4)$ in the quark mass expansion are currently under investigation.

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