Pramāṇa – J. Phys., Vol. 31, No. 6, December 1988, pp. 469–478. © Printed in India.

Magnetic symmetry and dyons

J M S RANA, O P S NEGI and B S RAJPUT
Department of Physics, Kumaun University, Nainital 263002, India

MS received 6 June 1988

Abstract. A self-consistent theory of dyons in Abelian and non-Abelian limits has been formulated in terms of an extra magnetic symmetry and topological magnetic charge. It has been shown that the restricted gauge potential describes the fields of dyons in terms of two regular (time-like) potentials only when recourse is made to the duality of topological (magnetic) and isocolour (electric) charges. Choosing a suitable Lagrangian density for the system of dyons in non-Abelian gauge theory, the field equations, energy-momentum tensor, Hamiltonian and momentum densities have also been derived and the conservation of the four-linear momentum and the total angular momentum has been demonstrated.

Keywords. Magnetic symmetry; monopoles; dyons; topological charge; non-Abelian gauge theory.

PACS Nos 11·30; 14·80

1. Introduction

The question of existence of monopoles ('t Hooft 1974; Polyakov 1974) has now become a subject of utmost interest and enormous potential importance in connection with quark confinement problem in quantum chromodynamics (Peshkin 1978; 't Hooft 1979), possible condensation of vacuum ('t Hooft 1979, 1981), its role as catalyst in proton decay (Rubakov 1981; Callan 1983), the possible explanation of CP violation in terms of non-zero vacuum angle of world in magnetic gauge space (Witten 1979), the role of monopole in current grand unified theories (Dokos *et al* 1980; Daniel *et al* 1980a) and the unification programme of electromagnetic and gravito-Heavisidean fields (Rajput 1982; Gross and Perry 1983). The magnetic condensation of vacuum guarantees the absolute colour confinement through dual Meissner effect and the related problem deals with the multiple structure of non-Abelian gauge theory of monopoles (Witten 1983). So far, this problem has not been taken seriously except for some efforts made by Cho in a series of papers (Cho 1980, 1981) to analyse magnetic symmetry of restricted quantum chromodynamics. Daniel *et al* (1980b) demonstrated a non-perturbative theory of monopoles in standard SU(5) model of grand unified theories and suggested that electric quark confinement in quantum chromodynamics is caused by squeezing of chromoelectric flux, in analogy with the Meissner effect in superconductivity. Despite the enormous potential importance of monopoles and the fact that the formalism necessary to describe them has been clumsy and not manifestly covariant, Rajput and coworkers (Rajput and Om

Prakash 1976, 1978; Rajput and Joshi 1979, 1981; Rajput 1982; Rajput and Bhakuni 1982; Rajput *et al* 1983) have constructed a self-consistent quantum field theory of pointlike as well as extended dyons, particles carrying simultaneous electric and magnetic charges. The existence of such particles (dyons) has also been demonstrated by Julia and Zee (1975) by extending the models of 't Hooft (1974) and Polyakov (1974). Furthermore, Witten (1979) has shown that if θ-angle of the world is non-zero, the monopoles must have integral or half integral charges (i.e. monopoles are necessarily dyons). Pantaleon (1983) examined the recent results of Fairbank *et al* (1981) about the reported evidence of objects with fractional electric charge and that of Cabrera (1982) about an event interpreted as monopole and showed that the situation is puzzling if both these results are true since it conflicts with either Dirac's quantization condition or the colour force confining quarks or exact gauge symmetry as $SU(3)_c \times U(2)$. If monopoles are taken as dyons such difficulties are automatically removed. Keeping these facts in view, the study of dyons is as essential as that of the monopoles in grand unified theories.

In the present paper, we have attempted to formulate the topological concept of magnetic charge in terms of magnetic symmetry gauge group and then extended it to the theory of dyons in Abelian and non-Abelian limits. It has been shown that in the magnetic gauge space the gauge field is made of two parts, the "electric" part which is not restricted by magnetic symmetry and the "magnetic" part which is completely determined by the magnetic symmetry. The magnetic potential of magnetic symmetry suffers from Dirac's (1931) string singularity and it is therefore described in terms of spacelike potential. As such, it could not describe the fields associated with dyons. Singular potential may be replaced by a regular one when recourse is made to the duality of topological (magnetic) and isoelectric charges. Hence the restricted potential describes the gauge field strength, field equations, electric and magnetic fields and equation of motion for dyons. It has also been shown that by incorporating the generalized charge of dyons, the theory presented here yields the results similar to those derived earlier (Rajput and Om Prakash 1976, 1978; Rajput and Joshi 1981).

Choosing a suitable Lagrangian density of dyons in terms of magnetic symmetry in non-Abelian gauge theory, the field equations, energy momentum tensor, Hamiltonian and momentum densities have been derived for the system of dyons in a magnetic gauge space. It has also been shown that the total linear and angular momentum operators commute with the Hamiltonian of dyonic system and that the total angular momentum is made of three parts, the orbital, spin and isotopic spin ones.

2. Magnetic symmetry and topological charges

Non-Abelian gauge theory can be viewed as Einstein's theory of gravitation in a higher-dimensional unified space P (Cho and Freund 1975) consisting of the four-dimensional external space-time M and the n-dimensional internal space G. Let us consider the metric $g_{AB}(A,\ B = 1,\ 2,\ 3 \ldots 4+n)$ in this $(4+n)$-dimensional unified space P by introducing n-dimensional isometry group G with its Killing vector fields spanning the internal space. More precisely, one can choose the n linearly independent and complete Killing vector fields, ξ_i $(i = 1, 2, \ldots n)$ as (Cho 1982a, b);

$$\mathscr{L}\xi_i g_{AB} = 0$$

where $\mathscr{L}\xi_i$ is the Lie derivative along the direction of ξ_i. These n-Killing vector fields satisfy the following canonical commutation relations of the isometry semi-simple group G:

$$[\xi_i, \xi_j] = f_{ij}{}^k \xi_k. \tag{2}$$

The isometry leads to identify P as a principal fibre bundle $P(M, G)$ with $M = P/G$ as the base manifold and G as the structure group. Since G acts on the right side of P, the isometry group G may be referred to as the right isometry (Cho 1975, 1982a). It has been conjectured that the dynamics of magnetic monopole is effectively described by a gauge theory based on magnetic gauge group (magnetic symmetry) which has the topological meaning. Thus, with this analysis we can define the magnetic symmetry (Cho 1980, 1981, 1982) as an additional internal isometry H having some additional Killing vector fields of generalized gauge theory. These additional Killing vectors are purely internal ones and hence commute with the already existing fields ξ_i of G. Let the additional Killing vector fields be m_a ($a = 1, 2 \ldots k$, $k = \dim. H$) where H is the Cartan's subgroup of G. Then by definition we have,

$$\left.\begin{aligned} & m_a = m_a^i(x)\xi_i, \quad (i = 1, 2, 3) \\ & [\xi_i, m_a] = 0, \\ \text{and} \quad & [m_a, m_b] = -f_{ab}^{(H)c} m_c, \\ & \mathscr{L}_{ma} g_{AB} = 0, \end{aligned}\right\} \tag{3}$$

where $\mathscr{L}_{m_a}$ is the Lie derivative along the direction of magnetic symmetry m_a. Here again the isometry group H commutes with the right isometry G and hence called the left isometry (Cho 1975, 1982b). The topological magnetic charge associated with monopoles corresponds to the elements of second homotopy group $\pi_2\,(G/H)$. Here we consider G as SU(2) Yang-Mill's group of isotopic spin and H as the global gauge group of electromagnetic interaction U(1). Thus,

$$\pi_2(G/H) \to \pi_2(\mathrm{SU}(2)/\mathrm{U}(1)) \tag{4}$$

is determined by the second homotopy directly leading to the magnetic symmetry gauge group of monopoles in terms of $\hat{m}$. Thus, monopoles are described in terms of the topological charges. The magnetic symmetry $\hat{m}$ given by (3) may also be represented in terms of the following column matrix;

$$\hat{m} = \begin{pmatrix} m^1 \\ m^2 \\ m^3 \\ \vdots \\ m^n \end{pmatrix}, \tag{5}$$

while for SU(2) gauge degrees of freedom one can rotate the magnetic vector $\hat{m}$ to a fixed time-independent direction (say, the third direction of isotopic spin) by a gauge transformation;

$$\hat{m} \to \xi_3 = U\hat{m} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}. \tag{6}$$

The magnetic symmetry obviously imposes a strong constraint on the connection and hence may be regarded as a symmetry of gauge potential. Thus, this gauge symmetry defined by (3) restricts not only the internal metric

$$\phi_{ij} = g_{AB}\xi_i^A \xi_j^B \tag{7}$$

but also the gauge potential. As such we have,

$$D_\mu \hat{m} = \partial_\mu \hat{m} + g\mathbf{B}_\mu \times \hat{m} = 0, \tag{8}$$

where $\mathbf{B}_\mu$ is the restricted gauge potential of group G and g is the structure constant. This condition also implies that for an internal symmetry group SU(N) the magnetic symmetry may be described by a single adjoint representation scalar multiplet $\hat{m}(x)$ with

$$\hat{m}^2 = \text{constant (or 1 in general)}, \tag{9}$$

whose little group is H at every space-time point. For SU(2), the restricted potential $\mathbf{B}_\mu$ may be given by,

$$\mathbf{B}_\mu = A_\mu \hat{m} - \frac{1}{g}\hat{m} \times \partial_\mu \hat{m}, \tag{10}$$

where $A_\mu = \hat{m}.\, \mathbf{B}_\mu$, the Abelian component of $\mathbf{B}_\mu$, is not restricted by magnetic symmetry condition (8). The restricted gauge potential has two parts, the unrestricted part A_μ and the other part completely defined by magnetic symmetry. We may identify the unrestricted part A_μ as the electric one and the restricted part as the magnetic one. In other words, the gauge potential consists of poles corresponding to electric and magnetic charges in non-Abelian gauge theory like Julia-Zee dyons (Julia and Zee 1975). The gauge field strength $\mathbf{G}_{\mu\nu}$ corresponding to the gauge potential $\mathbf{B}_\mu$ may be given as (Cho 1980a);

$$\begin{aligned}\mathbf{G}_{\mu\nu} &= \partial_\mu \mathbf{B}_\nu - \partial_\nu \mathbf{B}_\mu + g\mathbf{B}_\mu \times \mathbf{B}_\nu \\ &= (F_{\mu\nu} + H_{\mu\nu})\hat{m} = \mathbf{F}_{\mu\nu} + \mathbf{H}_{\mu\nu},\end{aligned} \tag{11}$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu,$$

and

$$H_{\mu\nu} = -\frac{1}{g}\hat{m}\cdot(\partial_\mu \hat{m} \times \partial_\nu \hat{m}) \tag{12}$$

are respectively the electric and magnetic constituents of gauge field strength. The magnetic gauge field strength requires an extra magnetic or topological potential. For SU(2) Yang-Mills theory, $\xi \to (0, 0, 1)$ defines the colour triplets and hence the restricted potential has a dual structure. It describes not only the colour electric charges but also the colour magnetic ones. The magnetic counterpart of potential has been defined in a series of papers by Cho (1980, 1981, 1982) as;

$$\mathbf{B}_\mu = (A_\mu + C_\mu^*)\hat{m}, \tag{13}$$

where C_μ^* is the dual magnetic potential having singularity. Thus, the magnetic counterpart of restricted gauge theory suffers from the string singularities, i.e.

$$H_{\mu\nu} = \partial_\mu C_\nu^* - \partial_\nu C_\mu^* \tag{14}$$

contains the Dirac's string singularity and as such, we could not take $\mathbf{G}_{\mu\nu}$ and $\mathbf{B}_\mu$ respectively as gauge field strength and gauge potential of dyon. Thus, the theory of topological charges of magnetic monopoles still suffers from the Dirac's string singularity. The form of C_μ^* may be obtained in terms of magnetic symmetry;

$$\hat{m} = \begin{pmatrix} \sin\alpha\cos\beta \\ \sin\alpha\sin\beta \\ \cos\alpha \end{pmatrix} \tag{15}$$

as

$$C_\mu^* = -\frac{1}{g}\cos\alpha\,\partial_\mu\beta, \tag{16}$$

where α and β are the Eulerian angles of the rotation in isotopic spin space. This potential describes the space-like behaviour while A_μ has the time-like structure.

3. Abelian gauge theory of dyons

To overcome the above mentioned Dirac string singularity of magnetic potential C_μ^*, let us first define the dual dynamics of colour isocharges. By definition in Abelian gauge, we have;

$$G_{\mu\nu} = \partial_\nu B_\mu - \partial_\mu B_\nu = F_{\mu\nu} + H_{\mu\nu}. \tag{17}$$

Let us introduce the electric and magnetic four-currents J_μ and K_μ respectively as the sources of electric and magnetic charges. Then we get,

$$G_{\mu\nu,\nu} = J_\mu = F_{\mu\nu,\nu}$$

and

$$G_{\mu\nu,\nu}^* = H_{\mu\nu,\nu}^* = K_\mu, \tag{18}$$

where $G_{\mu\nu}^* = \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}G^{\rho\sigma}$ is the dual of $G_{\mu\nu}$. Here K_μ may be referred to as the monopole topological current while $F_{\mu\nu}$ and $H_{\mu\nu}$ describe the pointlike behaviour and not the extended one. The topological current of the monopole carries the well-known string singularity and the space-like behaviour of magnetic potential C_μ^* makes it the current carried by a charged particle having space-like momenta and hence one cannot add two potentials of different kinds. Thus, we must modify the definition of potential $\mathbf{B}_\mu$ in terms of two similar types of potentials having time-like behaviour. On the other hand, if there is the existence of magnetic monopoles in our space-time world, we must have to take their potential time-like one. Furthermore, we remove the string singularity with the help of duality transformations

$$(\mathbf{E} \to \mathbf{H}, \mathbf{H} \to -\mathbf{E}), \tag{19}$$

where $\mathbf{E}$ and $\mathbf{H}$ are the electric and magnetic fields of our everyday experience. Thus, $H_{\mu\nu}^*$ can be described in terms of the regular potential C_μ i.e. the dual magnetic potential, as follows;

$$H^*_{\mu\nu} = \partial_\mu C_\nu - \partial_\nu C_\mu, \tag{20}$$

which does not contain any irregular string singularity. As such, we can replace C^*_μ by C_μ to describe the monopoles by time-like potential and the resulting theory becomes the theory of dyons. Then we get the following sets of Maxwell's equations;

$$\begin{aligned} G_{\mu\nu,\nu} &= F_{\mu\nu,\nu} = J_\mu, \\ G^d_{\mu\nu,\nu} &= H^*_{\mu\nu,\nu} = K_\mu, \end{aligned} \tag{21}$$

which reduces to the following form in terms of electric and magnetic potentials;

$$\begin{aligned} \partial^\nu(\partial_\nu A_\mu - \partial_\mu A_\nu) &= J_\mu \\ \partial^\nu(\partial_\nu C_\mu - \partial_\mu C_\nu) &= K_\mu. \end{aligned} \tag{22}$$

Furthermore, in terms of the dual potential the magnetic symmetry could be regarded as the ordinary gauge symmetry of potential C_μ. The gauge field strength of dyons in terms of two potentials may also be written as follows;

$$G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + \varepsilon_{\mu\nu\rho\sigma}\partial^\rho C^\sigma \tag{23}$$

giving rise to the following expressions (Fryberger 1985; Han and Biedenhan 1971) for electric and magnetic fields of dyons;

$$\begin{aligned} \mathbf{E} &= -\nabla\phi - (\partial\mathbf{A}/\partial t) - \nabla\times\mathbf{C}, \\ \mathbf{H} &= -\nabla\psi - (\partial\mathbf{C}/\partial t) + \nabla\times\mathbf{A}, \end{aligned} \tag{24}$$

where we have used the following electric and magnetic four-potentials;

$$\{A^\mu\} = \{\phi, \mathbf{A}\} \quad \text{and} \quad \{C^\mu\} = \{\psi, \mathbf{C}\}. \tag{25}$$

These expressions of electromagnetic fields associated with dyons lead to the following form of Maxwell's equations.

$$\begin{aligned} \nabla\cdot\mathbf{E} &= J_0, \\ \nabla\cdot\mathbf{H} &= K_0, \\ \nabla\times\mathbf{H} &= \mathbf{j} + (\partial\mathbf{E}/\partial t), \\ \nabla\times\mathbf{E} &= -\mathbf{k} - (\partial\mathbf{H}/\partial t), \end{aligned} \tag{26}$$

which can be combined into equations (21) and (22). Equations (21), (22) and (24) are invariant under duality transformations of electric and magnetic counterparts. The topological charge of monopole is inversely proportional to the corresponding electric charge as given by Cho (1980, 1981, 1982) and thus the dyon-dyon system leads to the following chirality quantization condition;

$$\mu_{ij} = e_i g_j - e_j g_i = n, \tag{27}$$

where n has integral and half-integral values but its half-integral values are forbidden by the requirement of locality (Rajput and Om Prakash 1978; Dirac 1931).

The Lorentz force equation of motion for a dyon moving with the four-velocity u^ν in a gauge field of $\mathbf{B}_\mu$, may be written as

$$f_\mu = QG_{\mu\nu}u^\nu, \tag{28}$$

where Q is the charge of dyon. If we consider the dyonic charge as a complex quantity with its electric and magnetic constituents as real and imaginary parts i.e.,

$$Q = e - ig, \tag{29}$$

then we may construct the electrodynamics of dyons similar to that described by Rajput and coworkers (Rajput and Om Prakash 1976, 1978; Rajput and Joshi 1979, 1981; Rajput 1982; Rajput and Bhakuni 1982; Rajput *et al* 1983) by replacing the potential, current, field and gauge field strength by generalized ones in terms of complex quantities. One can also construct the angular momentum operator for dyon-dyon system in terms of the topological structure of magnetic charge. The generalized field tensor is then written as

$$\mathscr{G}_{\mu\nu} = G_{\mu\nu} - iG^*_{\mu\nu} \tag{30}$$

and the corresponding field equations are given by;

$$\mathscr{G}_{\mu\nu,\nu} = J_\mu,$$
$$\mathscr{G}^*_{\mu\nu,\nu} = 0, \tag{31}$$

where $J_\mu = J_\mu - iK_\mu$ is the generalized current associated with generalized charge of dyons.

4. Non-Abelian gauge theory of dyons in terms of magnetic symmetry

In order to formulate a non-Abelian gauge theory of dyons in terms of magnetic symmetry in a restricted connection space π_2 (G/H), we start from the gauge field strength $\mathbf{G}_{\mu\nu}$ given by (11). In this gauge field strength we substitute the singular potential C^*_μ by the regular dual potential C_μ with

$$\begin{aligned} H_{\mu\nu} &= \tfrac{1}{2}\varepsilon_{\mu\nu\rho\sigma}\hat{m}\cdot(\partial\rho\hat{m} \times \partial\sigma\hat{m}) \\ &= \partial_\mu C_\nu - \partial_\nu C_\mu \end{aligned} \tag{32}$$

and then it does not contain string singularities. The corresponding gauge field strength of restricted potential $\mathbf{B}_\mu$, given by

$$\mathbf{G}_{\mu\nu} = (F_{\mu\nu} + H_{\mu\nu})\hat{m},$$

describes the gauge field strength of dyons in terms of non-Abelian magnetic symmetry $\hat{m}$. It satisfies the following identity;

$$[D_\mu, D_\nu] = g\mathbf{G}_{\mu\nu} \times \hat{m}, \tag{33}$$

where the nonvanishing components of $\mathbf{G}_{\mu\nu}$ satisfying the magnetic symmetry requirement (8) are the components of little group H of magnetic symmetry $\hat{m}$.

The Lagrangian density of the fields associated with dyons may be written as follows in terms of non-Abelian magnetic gauge symmetry and Higg's field;

$$L = -\tfrac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} + \tfrac{1}{2}D_\mu\phi_i D^\mu\phi_i - V(\phi), \tag{34}$$

where

$$G^a_{\mu\nu} = B^a_{\mu,\nu} - B^a_{\nu,\mu} + g\varepsilon_{abc}B^b_\mu B^c_\nu$$

$$D_\mu\phi_i = \partial_\mu\phi_i + g\mathbf{B}_\mu \times \boldsymbol{\phi}_i \tag{35}$$

and

$$V(\phi) = -(m^2/2)\boldsymbol{\phi}^2 + (\lambda/4)(\phi^2)^2$$

with $m^2 > 0$ leading to the extended structure of dyons in non-Abelian magnetic gauge symmetry. The Lagrangian density given by (34) yields the following field equations with respect to an independent variation of B_μ and ϕ;

$$D^\nu G^a_{\mu\nu} = -\{(D_\mu\phi_i)g \times \phi\} = 0, \tag{36}$$

and

$$D^\nu(D_\mu\phi_i) = -m^2\phi_i + \lambda(\boldsymbol{\phi})^2\phi_i \text{ (Higg's current)}. \tag{37}$$

These equations show that the Higg's field has nothing to do with the covariant derivative of gauge field strength but gives rise to the vanishing four-current and generates the dyonic current as the bosonic one in terms of Higg's potential and the mass of Julia-Zee (1975) dyon. The energy-momentum tensor may be written in the following form from Lagrangian density given by (34);

$$T^{\mu\nu} = D^\mu\phi_i D_\mu\phi_i + \tfrac{1}{2}G^a_{\mu\gamma}G^{a\mu\gamma} - g^{\mu\nu}L(\mu,\nu,\gamma = 0,1,2,3), \tag{38}$$

which gives rise to the following expressions for Hamiltonian and momentum densities;

$$\begin{aligned} T^{00} = {} & (D^0\phi_i)(D_0\phi_i) + \tfrac{1}{2}G^{a0\gamma}G^a_{0\gamma} + \tfrac{1}{4}G^a_{\rho\sigma}G^{a\rho\sigma} \\ & -\tfrac{1}{2}D\rho\phi_i D^\rho\phi_i + V(\phi), \end{aligned} \tag{39}$$

$$T^{0j} = (D^0\phi_i)(D_0\phi_i) + \tfrac{1}{2}G^a_{0j}G^{a0j}. \tag{40}$$

These Hamiltonian and momentum densities, after integrating with respect to volume, yield the corresponding Hamiltonian and momentum operators which satisfy the following commutation relation;

$$[\hat{H}, \mathbf{P}] = 0 \tag{41}$$

or in general

$$[P^\mu, P^\nu] = 0. \tag{42}$$

As such, the four-momentum is conserved for a system of Julia-Zee (1975) dyon in terms of the non-Abelian magnetic symmetry. Similarly, we can construct the angular momentum operator for dyon as;

$$\begin{aligned} \mathbf{J} &= \mathbf{r} \times \mathbf{p} \\ &= \mathbf{L} + \mathbf{S} + \mathbf{I}, \end{aligned}$$

where **L** is the orbital part, **S** is the ordinary spin and **I** is the isotopic spin in restricted gauge space π_2 (SU(2)/U(1)). This angular momentum commutes with the Hamiltonian, i.e.

$$[\hat{H}, \mathbf{J}] = 0,$$

leading to the conservation of angular momentum operator for dyons in restricted chromodynamics where electric and magnetic parts of dyons play the role of isocolour charges.

5. Discussion and conclusion

From the foregoing analysis it is clear that the restricted potential $\mathbf{B}_\mu$ given by (10) has a dual structure. It describes not only the colour electric charges of non-Abelian gauge theory but also the colour magnetic ones. Magnetic symmetry $\hat{m}$ restricts the unified connection space to the second homotopic space $\pi_2\,(G/H)$ but it could not restrict the colour electric potential. In other words, the topological concept of magnetic monopoles as the constituent of dyons arises only due to the appearence of an additional internal symmetry i.e. the magnetic symmetry. At this moment one should be careful to emphasize two points; firstly, the magnetic symmetry may be used to specify the global topology of the gauge fields yielding magnetic charges in addition to electric charges. Secondly, the magnetic symmetry inevitably chooses the colour direction by selecting the colour electric potentials of the Cartesian's subgroup. The colour direction allows us to circumvent the disturbing Schielder's theorem (Schielder 1981) to define a colour charge without violating the full gauge invariance. Equation (35) leads to the current generalized by Higg's field and shows that the dyons have the mass leading to their extended structure in non-Abelian gauge theory.

Furthermore, in terms of the dual potential, the magnetic symmetry could naturally be regarded as an ordinary Abelian gauge symmetry of magnetic potential. In other words, we have made here the magnetic symmetry a genuine Noether's symmetry of Lagrangian and thus the topological charge can indeed be viewed as the dual object of Noether charge. Moreover, it has been shown by Polyakov (1978) that at a high temperature quarks are liberated due to the condensation of chromoelectric vortices. This would lead to the squeezing of chromomagnetic flux and therefore to confinement of monopoles carrying $\mathrm{SU}(3)^c_M$ quantum numbers. In this case the restricted chromodynamics (RCD) would explain the confinement of the colour in quantum chromodynamics. In strong coupling limit the dynamical breaking of magnetic symmetry could indeed occur which would guarantee quark confinement in quantun chromodynamics. In weak coupling limit, however, the magnetic symmetry is not likely to be broken since in this limit the magnetic coupling may become too strong to allow monopole and dyon condensation for physical vacuum. Then not only the quarks but also the monopoles (dyons) become unavoidable as physical states.

In our forthcoming paper we shall extend this study of magnetic symmetry (dyons) to the SU(5) gauge group of strong and electroweak symmetry.

Acknowledgement

One of us (JMSR) is thankful to the Department of Science and Technology, New Delhi for financial assistance.

References

Cabrera B 1982 *Phys. Rev. Lett.* **48** 1378
Callan C G 1983 *Nucl. Phys.* **B212** 391
Cho Y M 1975 *J. Math. Phys.* **16** 2029
Cho Y M and Freund P G O 1975 *Phys. Rev.* **D12** 1711
Cho Y M 1980a *Phys. Rev.* **D21** 1080
Cho Y M 1980b *Phys. Rev. Lett.* **44** 1115
Cho Y M 1981a *Phys. Rev. Lett.* **46** 302
Cho Y M 1981b *Phys. Rev.* **D23** 2415
Cho Y M 1982a Reports, SNU-HEPT 83-04, MZ-TH/82-08, MZ-TH/82-05
Cho Y M 1982b CERN Report, TH-3414; Mainz Report, MZ-TH/82-0
Daniel M, Lazarides G and Safi Q 1980a *Phys. Lett.* **B916** 72
Daniel M, Lazarides G and Safi Q 1980b *Nucl. Phys.* **B170** 156
Dirac P A M 1931 *Proc. R. Soc.* (*London*) **A133** 60
Dokos C P and Tomaras T 1980 *Phys. Rev.* **D21** 2940
Fairbank W, Rue G La and Phillips J 1981 *Phys. Rev. Lett.* **47** 967
Fryberger D 1985 *IEEE Trans. Mag.* **21** 84
Gross D J and Perry M J 1983 *Nucl. Phys.* **B226** 29
Han M Y and Biedenhan L C 1971 *Nuovo Cimento* **A2** 544
Julia B and Zee A 1975 *Phys. Rev.* **D11** 2227
Pantaleon J 1983 *Nucl. Phys.* **B219** 367
Peshkin M 1978 *Ann. Phys.* **113** 122
Polyakov A M 1974 *JETP Lett.* **20** 194
Polyakov A M 1978 *Phys. Lett.* **B72** 477
Rajput B S 1982 *Lett. Nuovo Cimento* **35** 205
Rajput B S and Bhakuni D S 1982 *Lett. Nuovo Cimento* **34** 509
Rajput B S and Bhakuni D S 1983 *Lett. Nuovo Cimento* **36** 499
Rajput B S and Joshi D C 1979 *Pramana – J. Phys.* **13** 639
Rajput B S and Joshi D C 1981 *Hadronic J.* **4** 1805
Rajput B S, Kumar S R and Negi O P S 1983 *Indian J. Pure Appl. Phys.* **21** 638, 25
Rajput B S and Om Prakash 1978 *Indian J. Pure Appl. Phys.* **16** 593
Rajput B S and Om Prakash 1976 *Indian J. Phys.* **50** 929
Rubakov V 1981 *JEPT Lett.* **33** 645
Schielder S 1981 *Nuovo Cimento* **A63** 137
't Hooft G 1974 *Nucl. Phys.* **B79** 276
't Hooft G 1978 *Nucl. Phys.* **B138** 1
't Hooft G 1979 *Nucl. Phys.* **B153** 141
't Hooft G 1981 *Nucl. Phys.* **B190** 45
Witten E 1983 *Nucl. Phys.* **B223** 422, 433
Witten E 1979 *Phys. Lett.* **B86** 283
Zwanziger D 1971 *Phys. Rev.* **D3** 880