On the measurement of electron tunnelling time

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Abstract. A suggestion to experimentally measure the electron tunnelling time by observing the tunnelling current cut-off as a function of the magnetic field intensity in semiconductor pn tunnel junctions, when they are placed in a crossed electric and magnetic field configuration, has been made in this paper. A simple and a rigorous quantum mechanical analysis to justify the above proposition have been presented. An order of agreement between the tunnelling time values derived from the published experimental data and our theoretical prediction has been noticed.

Keywords. Tunnelling time; magneto-tunnelling effect; magnetic field intensity.

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1. Introduction

The electron tunnelling time measures the time during which an electron undergoes quantum scattering by a potential barrier (Roy 1986). We feel that an investigation of the magneto-tunnelling effect in semiconductor pn tunnel junctions placed in crossed electric and magnetic fields may provide a possible clue to its measurement (Ghosh 1988).

Experimentally it has been established (Calawa *et al* 1960; Lax 1960; Chynoweth *et al* 1960; Rediker and Calawa 1961; Esaki and Haering 1962; Butcher *et al* 1961) that the tunnelling current across semiconductor pn tunnel junctions decreases with increasing magnetic field intensities at liquid helium temperatures. Such a decrease has been detected both for longitudinal and transverse magnetic field directions. But this effect has been found to be more pronounced when the magnetic field is normal to the direction of the current flow. Theoretical explanations to such dependances have already been developed on conventional lines where it is presumed that the tunnelling phenomenon propagates with the speed of light (Calawa *et al* 1960; Haering and Adams 1961; Haering and Muller 1961; Argyres 1962; Aronov and Pikus 1967). Here, we undertake to re-examine such a magneto-tunnelling effect on the basis of our new theory (Roy 1986).

2. A possible way to measure tunnelling time

Let us consider an electron to be initially at rest at the origin. When a magnetic field B_z and an electric field $-E_x$ are applied on it along Z and X axes respectively, the resulting electron trajectory is known to be a cycloid propagating along Y-axis (Ghosh 1988). After some time, when the electron acquires a finite velocity under the influence of the electric field, its equation of motion may be written as,

$$\left.\begin{aligned} \mathrm{d}V_x/\mathrm{d}t &= a - \omega_c V_y, \\ \mathrm{d}V_y/\mathrm{d}t &= \omega_c V_x \\ \text{and} \qquad \mathrm{d}V_z/\mathrm{d}t &= 0, \end{aligned}\right\} \tag{1}$$

where $a = qE_x/m$ and $\omega_c = qB_z/m$. We have regarded the electron charge as negative in the above considerations. On solving (1) and after applying boundary conditions appropriately, we get,

$$V_x = \frac{a}{\omega_c}\cdot\sin(\omega_c t) \tag{2}$$

and

$$V_y = \frac{a}{\omega_c}[1 - \cos(\omega_c t)]. \tag{3}$$

Equation (2) is particularly important because it clearly states that crossed electric and magnetic field vectors impose on the X-component of the electron velocity an oscillation of angular frequency ω_c.

Let us next imagine a potential barrier extending along X-axis with its surface oriented along the Y-direction to be available for tunnelling studies. As before, an electric field is applied along negative X-axis and a magnetic field along the Z-direction. In such a case, by virtue of (2), we know that the X-component of electron wave vector becomes a harmonic function of time, given by,

$$k_x = k_{x0}\cdot\sin(\omega_c t), \tag{4}$$

where

$$k_{x0} = ma/\hbar\omega_c. \tag{5}$$

In order that electron tunnelling is detected, the average electron wave vector over the tunnelling period τ must not disappear. Incidentally, the former works out to be,

$$\bar{k}_x = \frac{\int_0^\tau k_x\cdot\mathrm{d}t}{\tau} = \left[\frac{k_{x0}\omega_c\tau}{2}\right]\cdot\frac{\sin^2(\omega_c\tau/2)}{(\omega_c\tau/2)^2}, \tag{6}$$

where $\bar{k}_{\max} = (k_{x0}\omega_c\tau/2)$. Thus, according to (6), when the magnetic flux density is small, the average electron wave vector is $\bar{k}_{\max}$. As the flux density increases in magnitude, $\bar{k}_x$ starts falling rapidly unless the current is cut off. This occurs when,

$$(\omega_c\tau/2) = \pi \quad \text{or} \quad \tau = 2\pi/\omega_c = T_c. \tag{7}$$

It has been mentioned earlier that such a behaviour has been noticed experimentally. Therefore, if this current cut-off as a function of the magnetic field in a crossed electric and magnetic field is realizable in practice, it would be possible to measure the electron tunnelling time τ. Such values may thereafter be compared with the theoretically predicted ones.

From (6) and (7) the condition for tunnelling to occur may, therefore, be set as,

$$\tau \leqslant T_c, \tag{8}$$

where $T_c = 2\pi/\omega_c$ and $\tau = h/2(V_0 - E)$. With these values, (8) reduces to,

$$\chi_{\text{eff}}^2 = \chi_0^2 - (qB_z/\hbar) \geqslant 0, \tag{9}$$

where $\chi_0^2 = (2m/\hbar^2)(V_0 - E)$. The tunnelling according to (9), would be observed to cease when $\chi_{\text{eff}}^2 = 0$. The effect of increasing magnetic flux density may thus be incorporated by changing the barrier penetration constant from χ_0 to χ_{eff} in appropriate tunnelling equations.

3. Rigorous analysis of magneto-tunnelling effect

The non-relativistic hamiltonian of a charged particle moving in an electromagnetic field may be expressed as (Ghosh 1988),

$$H = -\frac{\hbar^2}{2m}\nabla^2 + \frac{i\hbar q}{m}[A\cdot\nabla + \tfrac{1}{2}\nabla X A] + \frac{q^2 A^2}{2m} + q\phi + V(x,t), \tag{10}$$

where A and ϕ are vector and scalar potentials respectively. $V(x,t)$ denotes the potential energy of electrons due to sources other than electromagnetic. Now, for a magnetic field directed along Z-axis, we may write,

$$A = \tfrac{1}{2}(B_z X r). \tag{11}$$

For simplicity we may choose $A_x = 0$, $A_y = x\cdot B_z$ and $A_z = 0$. Taking $\phi = 0$, (10) reduces to,

$$H = -\frac{\hbar^2}{2m}\nabla^2 + \frac{i\hbar q}{m} x B_z \frac{\partial}{\partial y} + \frac{q^2 B_z^2 x^2}{2m} + V(x,t). \tag{12}$$

Next substituting,

$$V(x,t) = V_0 \pm \Delta E \tag{13}$$

for tunnelling (Roy and Ghosh 1987), where V_0 = maximum barrier height and $\Delta E \sim h/\tau$, we get,

$$H = -\frac{\hbar^2}{2m}\nabla^2 + \frac{i\hbar q}{m} x B_z \frac{\partial}{\partial y} + \frac{q^2 B_z^2 x^2}{2m} + V_0 \pm \Delta E. \tag{14}$$

The time-independent or the space part of the wave equation may now be expressed as,

$$\nabla^2\Phi + \frac{2m}{\hbar^2}\left[E - \frac{q^2 B_z^2}{2m}x'^2 + \frac{p_y^2}{2m} - V_0\right]\Phi = 0, \tag{15}$$

where

$$x' = x - (p_y/qB_z). \tag{16}$$

The solution of (15) may now be expressed as,

$$\Phi(x, y, z) = X(x)\exp\left[\frac{i}{\hbar}(p_y \cdot y + p_z \cdot z)\right]. \tag{17}$$

On substituting (17) in (15), we get,

$$\frac{d^2X(x)}{dx^2} + \frac{2m}{\hbar^2}\left[E' - \frac{q^2B_z^2x'^2}{2m} - V_0\right]X(x) = 0, \tag{18}$$

where

$$E' = E - (p_z^2/2m). \tag{19}$$

For the case of tunnelling we may rewrite (18) as follows:

$$\frac{d^2X(x)}{dx^2} - \frac{2m}{\hbar^2}\left[V' + \frac{q^2B_z^2}{2m}x'^2\right]X(x) = 0, \tag{20}$$

where $V' = (V_0 - E)$. To reduce (20) into a dimensionless form, we choose,

$$\xi = \beta x', \tag{21}$$

where β is a suitable constant. On combining (20) and (21), we find,

$$\frac{d^2X(\xi)}{d\xi^2} - \left[\frac{2mV'}{\hbar^2\beta'^2} + \frac{q^2B_z^2\xi^2}{\hbar^2\beta'^4}\right]X(\xi) = 0. \tag{22}$$

Now β' is so chosen that,

$$(q^2/\hbar^2\beta'^4) = 1 \quad \text{or} \quad \beta' = (q/\hbar)^{\frac{1}{2}}. \tag{23}$$

We also define,

$$\lambda = \frac{2mV'}{\hbar^2\beta'^2} = \frac{2mV'}{\hbar q} = \frac{\hbar\chi_0^2}{q}. \tag{24}$$

Incorporating (23) and (24) in (22), we find,

$$\frac{d^2X(\xi)}{d\xi^2} - [\lambda + B_z^2 \cdot \xi^2] \cdot X(\xi) = 0. \tag{25}$$

The solution of (25) may be written as,

$$X(\xi) = u(\xi)\exp\left[-\frac{B_z\xi^2}{2}\right], \tag{26}$$

where $u(\xi)$ satisfies the following differential equation:

$$\frac{d^2u}{d\xi^2} - 2B_z\xi\frac{du}{d\xi} - (B_z + \lambda)u = 0. \tag{27}$$

Solving (27) by the method of series solutions (Ghosh 1988), we get,

$$u = a\left[1 + \frac{B_z(\lambda' + 1)}{2!}\xi^2 + \frac{B_z^2(\lambda' + 1)(\lambda' + 5)}{4!}\xi^4 + \cdots\right]$$
$$\pm b\left[\xi + \frac{B_z(\lambda' + 3)}{3!}\xi^3 + \frac{B_z^2(\lambda' + 3)(\lambda' + 7)}{5!}\xi^5 + \cdots\right] \tag{28}$$

where

$$\lambda' = \lambda/B_z. \tag{29}$$

Since, in the absence of B_z, (26) must reduce to the characteristic decaying and growing waves of exponential types, a and b in (28) must be suitably chosen. Accordingly, we write,

$$a = 1 \quad \text{and} \quad b = \{B_z(\lambda' + 1)\}^{\frac{1}{2}} = (\lambda + B_z)^{\frac{1}{2}}. \tag{30}$$

Again for normal values of B_z ($\sim 10\,\mathrm{T}$) and $V'(= 1\,\mathrm{eV})$, λ' in (29) turns out to be 10^3, which is considerably larger compared to unity. With this approximation (28) reduces to,

$$u(\xi) = \left[1 \pm [B_z(\lambda' + 1)]^{\frac{1}{2}}\cdot\xi + \frac{\{[B_z(\lambda' + 1)]^{\frac{1}{2}}\cdot\xi\}^2}{2!}\right.$$
$$\left.\pm \frac{\{[B_z(\lambda' + 1)]^{\frac{1}{2}}\cdot\xi\}^3}{3!} + \cdots\right]. \tag{31}$$

That is either

$$u(\xi) \simeq \alpha \exp[-[B_z(\lambda' + 1)]^{\frac{1}{2}}\cdot\xi] \tag{32}$$

or

$$u(\xi) \simeq \beta \exp[+[B_z(\lambda' + 1)]^{\frac{1}{2}}\cdot\xi] \tag{33}$$

depending on whether we choose negative or positive sign in (31). Using (32) and (33) in (26) we get,

$$X(\xi) = [\alpha \exp\{-[B_z(\lambda' + 1)]^{\frac{1}{2}}\cdot\xi\}$$
$$+ \beta \exp\{[B_z(\lambda' + 1)]^{\frac{1}{2}}\cdot\xi\}]\exp\left(-\frac{B_z\xi^2}{2}\right). \tag{34}$$

Again for $B_z = 10\,\mathrm{T}$ and $x \sim 10\,\mathrm{Å}$, $\frac{1}{2}(B_z\xi^2) \simeq 10^{-2}$. In that case, (34) further simplifies to,

$$X(\xi) \simeq \alpha \exp[-\xi(\lambda + B_z)^{\frac{1}{2}}] + \beta \exp[+\xi(\lambda + B_z)^{\frac{1}{2}}]. \tag{35}$$

On substituting ξ in (35) and on combining it with the time-part of the wavefunction, one gets,

$$\psi(x, t) = a_l(t)\alpha \exp\left[-\left(\frac{q}{\hbar}(\lambda + B_z)\right)^{\frac{1}{2}}\cdot x\right]\exp(-i\omega_l t)$$
$$+ a_r(t)\beta \exp\left[+\left(\frac{q}{\hbar}(\lambda + B_z)\right)^{\frac{1}{2}}\cdot x\right]\exp(-i\omega_r t). \tag{36}$$

It is interesting to notice that (36) reduces to the characteristic barrier wavefunction in the standard form when $B_z = 0$. The effective barrier penetration constant may now be identified as,

$$\chi_{\text{eff}} = \left\{\frac{q}{\hbar}(B_z + \lambda)\right\}^{\frac{1}{2}}. \quad (37)$$

The tunnel effect cannot be detected if χ_{eff} reduces to zero. In that case, it follows from (37) that,

$$\omega_c \simeq 1/\tau. \quad (38)$$

This result is identical to (7). Expanding (37) and taking care of the negative charge on the electron, we get,

$$\chi_{\text{eff}}^2 = \chi_0^2 - (qB_z/\hbar). \quad (39)$$

This is once again identical to (9).

4. Discussion

The published literature clearly demonstrates (Calawa *et al* 1960; Chynoweth *et al* 1960; Lax 1960; Rediker and Calawa 1961; Butcher *et al* 1961; Esaki and Haering 1962; Burstein and Lundqvist 1969; Duke 1969) that the tunnelling current across degenerate pn junctions decreases with increase in the magnetic flux density B_z in a crossed electric and magnetic field configuration at liquid helium temperatures. By noting the tunnelling current cut-off at a particular value of the magnetic flux density, the magnitude of the tunnelling time may be derived immediately. In table 1 we present a few experimental data which clearly show that the measured tunnelling time values agree fairly closely with our predictions.

The basis of our tunnelling time calculations in the earlier table has been the finite interaction of the electron wave with the potential barrier. When this interaction time τ becomes comparable to the cyclotron period T_c or becomes larger than this, the electron barrier interaction becomes far from complete and eventually tunnelling ceases.

The conventional explanation of the experimentally observed tunnelling current cut-off is however based on the conventional presumption that tunnelling propagates with

Table 1. Calculation of tunnelling time.

Material	E_g(eV)(0°K)	$h/2E_g = \tau_{\text{th}}$ (s)	m^*/m	$B_z(T)$	$2\pi/\omega_c = \tau_{\text{exp}}$ (s)	$\tau_{\text{exp}}/\tau_{\text{th}}$
PbTe	0·165	$1{\cdot}11 \times 10^{-14}$	0·022	9·8	$8{\cdot}05 \times 10^{-14}$	7·25
InSb	0·235	$0{\cdot}88 \times 10^{-14}$	0·016	18·0	$3{\cdot}17 \times 10^{-14}$	3·60
InAs	0·430	$0{\cdot}48 \times 10^{-14}$	0·025	8·8	$10{\cdot}12 \times 10^{-14}$	21·08
Ge	0·785	$0{\cdot}26 \times 10^{-14}$	0·078	11·0	$25{\cdot}18 \times 10^{14}$	96·61
GaSb	0·810	$0{\cdot}25 \times 10^{-14}$	0·047	6·0	$28{\cdot}55 \times 10^{-14}$	114·20

the speed of light. Tunnelling taking place across a device placed in a crossed electric and magnetic field configuration is sought to be observed from another frame of reference moving relative to the first one with an appropriate rectilinear velocity so that the effective magnetic field in that frame reduces to zero. The corresponding electric field in that particular frame is then obtained by employing relativistic transformation equations. The phenomenon then becomes amenable for analysis by conventional tunnelling physics. The reason for agreement of the conventional theory with experiment has been due to the ultra-high speed of the tunnelling process.

The ideas as presented in this paper for the measurement of tunnelling time is radically different from those of others. The latter invariably presumes either wave or particle aspect of matter during tunnelling as suggested by the conventional theory and never the quantum scattering of the electron wave with the potential barrier. We, therefore, did not feel it necessary to present a review of those ideas here.

Our concept of a single electron tunnelling across a potential barrier based on the interaction picture may be summarized as follows: Before the electron is incident upon a barrier we identify its momentum (or energy) exactly. Consequently, the electron propagates as a wave having an infinite uncertainty in the position space. The moment the wave falls upon a potential barrier, it tries to localize the electron within it owing to the finite dimensions of the latter. Consequently, the electron wave undergoes quantum scattering both in the position as well as in the momentum spaces. The extent of such scatterings are governed by the height $(V_0 - E)$ and the width W of the potential barrier. By virtue of Heisenberg's uncertainty relations we immediately obtain the following condition for the optimum quantum scattering to take place (Ghosh 1988):

$$(V_0 - E)\cdot W^2 = h^2/2m. \tag{40}$$

Thus according to (40), the barrier height and width are not independent of one another for quantum scattering.

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