Pramāṇa – J. Phys., Vol. 31, No. 5, November 1988, pp. 357–363. © Printed in India.

Triple differential cross-sections for the fast electron/positron impact ionization of hydrogen and helium in the Glauber approximation

SUSHMA SAXENA and M K SRIVASTAVA
Department of Physics, University of Roorkee, Roorkee 247 667, India

MS received 9 May 1988; revised 8 August 1988

Abstract. Triple differential cross-section for electron and positron impact ionization of hydrogen and helium are calculated by using the Glauber approximation along with post-collision interaction effects which are estimated classically. The present results are compared with the recent absolute data of Ehrhardt, Jung and coworkers for the electron impact case. The positron impact case is found to lead to a larger binary to recoil peak maxima ratio (compared to the electron impact case) which further increases when post-collision interaction effects are included.

Keywords. Ionization; electron impact; positron impact; Glauber approximation; post-collision interaction.

PACS No. 34·80

1. Introduction

The recent availability of absolute triple differential cross-section (TDCS) data for the electron impact ionization of hydrogen and helium has provided an attractive spawning ground to test various models. The TDCS data are known to be very sensitive to the model used to describe the process specially in the case of asymmetric geometry. In this case for a fast incident electron (energy E_0, momentum $\mathbf{k}_0$), carrying most of the energy E_a (momentum $\mathbf{k}_a$) after scattering through a small angle θ_a, the angular distribution of a slow 'ejected' electron (energy E_b, momentum $\mathbf{k}_b$) shows two peaks: a peak (binary peak) near the momentum transfer direction and another subsidiary one (recoil peak) near the opposite direction. The calculations concentrate mainly on the magnitude and the angular positions of the binary and recoil peak intensities.

The calculation in the first Born approximation leads to, at variance with experimental data, (i) an angular distribution which is symmetric about the momentum transfer direction, (ii) a too large binary peak maximum and (iii) a too small recoil peak maximum. The Glauber approximation is found to lead to a substantial reduction in the size of the binary peak and some enhancement in the size of the recoil peak. Both these features (although not sufficiently so) are in accord with experimental data. However, the angular distribution continues to remain symmetrical about the direction of momentum transfer. Klar and Franz (1986) have recently shown that if the effects of the post-collision interaction (PCI) are included, the binary and recoil peak positions shift to larger angles bringing them in agreement with experiment. The calculations have been carried out in the first Born (B1-PCI) and Glauber (G-PCI) approximations for the ionization of hydrogen and helium for the electron impact case (Klar and Franz 1986; Klar *et al* 1986, 1987; Roy and Ray 1987). The G-PCI results are found to be quite good with respect to peak positions and peak intensities in the case of hydrogen (Klar *et al* 1987) and lead to very good values of binary to recoil peak intensity ratio in the case of helium (Roy and Ray 1987).

In this paper we apply the G-PCI approach to calculate TDCS for the electron/positron impact ionization of hydrogen and helium. The electron results are compared with the recent absolute data of Ehrhardt *et al* (1985) for hydrogen and of Jung *et al* (1985) for helium. The aim is to explore G-PCI approach as a simple alternative to treatments based on the second Born (Byron *et al* 1980, 1982; Ehrhardt *et al* 1982) and modified Glauber approximations (Baliyan and Srivastava 1985, 1986a, b) which though lead to good results are computationally quite involved. The electron and positron cases differ in the interference between the projectile-nucleus and projectile-electron interactions and a comparison of their TDCS is expected to be quite interesting. Such comparisons have been carried out both for hydrogen and helium by using the Coulomb-projected Born (Basu *et al* 1985; Ghosh *et al* 1985a, b; Mandal *et al* 1986), the second Born (Joachain 1984) and the modified Glauber (Saxena and Srivastava 1987) approximations. The first Born approximation leads to identical results for positron impact and electron impact ionization. The Glauber approximation, because of the inclusion of higher order terms, leads by itself to different results in the two cases. The post-collision interaction effects involving energy exchange between the two outgoing particles and deviation in their trajectories are also different in the two cases.

2. Calculations

The TDCS is calculated by the relation

$$\frac{d^3\sigma}{d\Omega_a\, d\Omega_b\, dE_b} = \frac{k_a k_b}{k_0} |f_G|^2, \tag{1}$$

where f_G is the scattering amplitude in the Glauber approximation. In the case of hydrogen, the electron/positron-target interaction is given by

$$V_H(\mathbf{r}_0, \mathbf{r}_1) = \mp \frac{1}{\mathbf{r}_0} \pm \frac{1}{|\mathbf{r}_0 - \mathbf{r}_1|}, \tag{2}$$

where the upper sign refers to the electron impact case while the lower one to the positron impact case, $\mathbf{r}_0$ and $\mathbf{r}_1$ are respectively the position vectors of the incident electron (positron) and the bound electron. The amplitude f_G is evaluated by following the method of Roy *et al* (1981). In the case of helium the projectile-target interaction is given by

$$V_{\mathrm{He}}(\mathbf{r}_0, \mathbf{r}_1, \mathbf{r}_2) = \mp \frac{2}{\mathbf{r}_0} \pm \frac{1}{|\mathbf{r}_0 - \mathbf{r}_1|} \pm \frac{1}{|\mathbf{r}_0 - \mathbf{r}_2|}, \tag{3}$$

where $\mathbf{r}_1$ and $\mathbf{r}_2$ are the position vectors of the two target electrons. To evaluate the amplitude f_G we have followed the method of Roy *et al* (1983). For the final state of

$He^+ + e^-$ subsystem we take the symmetrized product of the He^+ ground state wavefunction for the bound electron with the continuum wavefunction for the ejected electron. The latter is taken to be a Coulomb wave (with $Z = 1$) orthogonalized to the ground state orbital. For the ground state of helium we take the analytical fit to the Hartree-Fock wavefunction given by Byron and Joachain (1966).

In the case of incident positrons there is in general a possibility of the ejected electron being attached to the outgoing scattered positron forming positronium which later breaks up but in the present case where the scattered positron is of very high energy and the ejected electron is of very low energy, the positronium formation can be ignored.

The PCI effects have been estimated classically by following the procedure of Popov and Benayoun (1981), Popov and Erokhin (1983), Klar and Franz (1986), and Klar *et al* (1986) and take account of the post-collision energy and the momentum exchange between the two outgoing particles. They are incorporated in the calculations by evaluating the scattering amplitude at the initial values (at the boundary of the reaction zone) $E_a(0)$, $E_b(0)$, $\theta_a(0)$ and $\theta_b(0)$ for obtaining the cross-section at E_a, E_b, θ_a and θ_b which are the energies and the angles at the position of the detectors. The former can be obtained by using the following relations:

$$\theta_i(0) = \theta_i - \sin(\theta_a + \theta_b) \int_0^{\infty} r_a r_b r_{ab}^{-3}\, dt \int_t^{\infty} [r_i(t')]^{-2}\, dt', \quad i = a, b, \tag{4}$$

$$E_a(0) \simeq E_a + \left.\frac{1}{r_a}\right|_{t=0} - \int_0^{\infty} \dot{r}_a r_{ab}^{-3} [r_a - r_b \cos(\theta_a + \theta_b)]\, dt, \tag{5}$$

$$E_b(0) \simeq E_b - \int_0^{\infty} \dot{r}_b r_{ab}^{-3} [r_b - r_a \cos(\theta_a + \theta_b)]\, dt. \tag{6}$$

At the incident energies considered here, the PCI effects are not dominant and equations (4) to (6) may be solved by assuming straight-line trajectories

$$E_a = \tfrac{1}{2}\dot{r}_a^2 \tag{7}$$

$$E_b = \tfrac{1}{2}\dot{r}_b^2 - (1/r_b), \tag{8}$$

with initial values $r_a(0) \equiv r_{0a}$ and $r_b(0) \equiv r_{0b}$. Details are given in Klar and Franz (1986) and Klar *et al* (1986). We have chosen the boundary values r_{0a} and r_{0b} to be 2·5 Å and 0·529 Å for hydrogen and 3·5 Å and 0·2254 Å for helium respectively. The choice for r_{0b} corresponds to the position where the target electron is most likely to be found.

3. Results and discussion

We have calculated TDCS in the Glauber approximation for electron and positron impact ionization of hydrogen with and without PCI at $E_0 = 250\,\text{eV}$ for $\theta_a = 8°$, $E_b = 5{\cdot}0\,\text{eV}$ corresponding to the electron impact absolute data of Ehrhardt *et al* (1985). Klar *et al* (1987) have also performed similar calculations for the electron impact case. However, our choice, $r_{0a} = 2{\cdot}5$ Å and $r_{0b} = 0{\cdot}529$ Å for the PCI parameters are different from theirs ($r_{0a} = 1{\cdot}27$ Å and $r_{0b} = 0{\cdot}21$ Å). Figure 1 displays our results.

In the case of electron impact, the incorporation of PCI effects, which include repulsion between the two outgoing electrons, shifts the binary and the recoil peaks to

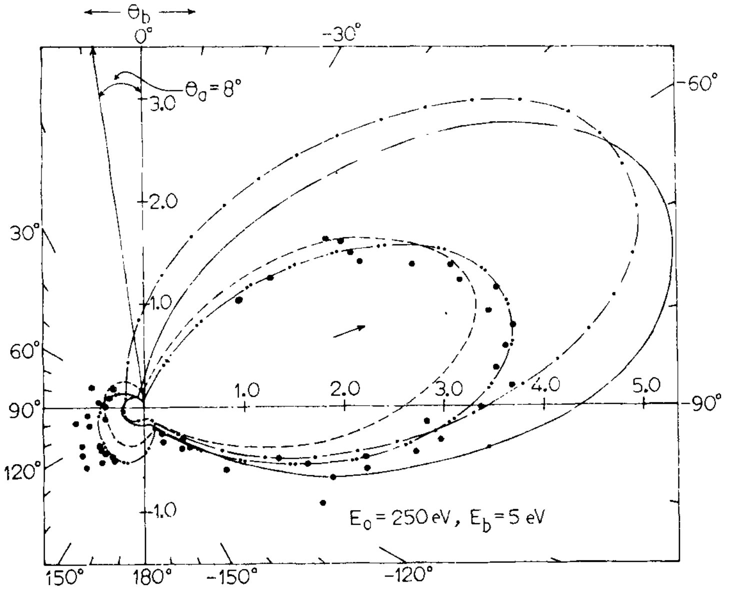

Figure 1. Triple differential cross-section for electron and positron impact ionization of atomic hydrogen in units of $10^{-22}\,\mathrm{m}^2\,\mathrm{Sr}^{-2}\,\mathrm{eV}^{-1}$, for $E_0 = 250\,\mathrm{eV}$, $E_b = 5\,\mathrm{eV}$, $\theta_a = 8°$. The curves are: G-PCI (for positron) —·––; *G* (for positron) —; G-PCI (for electron) —··—··—; *G* (for electron)– – –. The experimental data are the absolute measurements of Ehrhardt *et al* (1985). The arrow indicates the direction of momentum transfer **k**.

larger angles of ejection. In the positron impact case this displacement is opposite to that of the electron case as expected. Furthermore with PCI, the size of the recoil peak decreases in the positron impact case, with the result that the binary-to-recoil peak maxima ratio (I_b/I_r) becomes larger. The G-PCI results for electrons show a good agreement with the experimental data. One would expect G-PCI approach to lead to good results in the positron impact case also, although there are no experimental data to check it.

3.2 *Helium*

Calculations have been carried out at an incident energy $E_0 = 600\,\mathrm{eV}$ for (i) $\theta_a = 4°$, $E_b = 2{\cdot}5\,\mathrm{eV}$, (ii) $\theta_a = 10°$, $E_b = 2{\cdot}5\,\mathrm{eV}$, (iii) $\theta_a = 4°$, $E_b = 10{\cdot}0\,\mathrm{eV}$ and (iv) $\theta_a = 10°$, $E_b = 10{\cdot}0\,\mathrm{eV}$, corresponding to electron impact absolute TDCS data of Jung *et al* (1985). Roy and Ray (1987) have also reported similar calculations for the electron impact case at $E_0 = 600\,\mathrm{eV}$, $E_b = 5\,\mathrm{eV}$ and $\theta_a = 4°$ and 10°. Figures 2 (a–d) present our TDCS results.

The binary peak for the electron case continues to be overestimated even when PCI effects are taken into account. However, they lead to an enhancement in the corresponding recoil peak. The electron G-PCI results show good agreement with the data in the recoil peak region at $E_b = 2{\cdot}5\,\mathrm{eV}$ (figures 2 (a, b)). At $E_b = 10{\cdot}0\,\mathrm{eV}$, the recoil peak is still underestimated. The Glauber (*G*) results for the positron show larger (smaller) binary (recoil) peak compared to the electron case. When PCI effects are

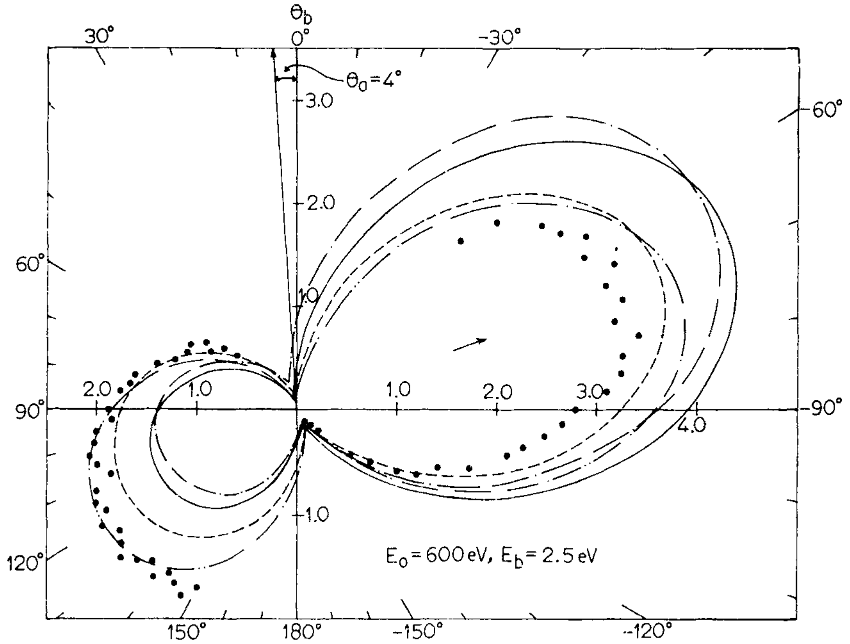

Figure 2a. Triple differential cross-section for electron and positron impact ionization of helium in units of $10^{-22}\,\mathrm{m}^2\,\mathrm{Sr}^{-2}\,\mathrm{eV}^{-1}$, for $E_0 = 600\,\mathrm{eV}$, $E_b = 2{\cdot}5\,\mathrm{eV}$, $\theta_a = 4°$. The curves are: G-PCI (for positron) —·—; *G* (for positron) —; G-PCI (for electron) —··—··—; *G* (for electron) ———. The experimental data are the absolute measurements of Jung *et al* (1985). The arrow indicates the direction of momentum transfer **k**.

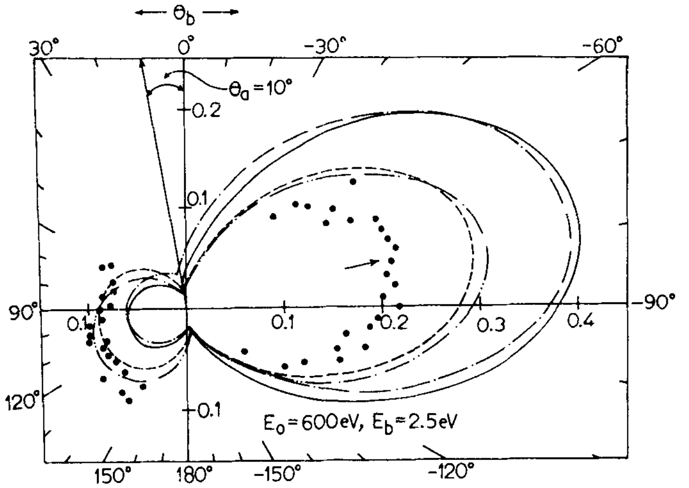

Figure 2b. Same as in figure 2*a*, but for $E_0 = 600\,\mathrm{eV}$, $E_b = 2{\cdot}5\,\mathrm{eV}$, $\theta_a = 10°$.

introduced (i) both the peaks shift to smaller angles θ_b of ejection, (ii) the binary peak remains almost unchanged in magnitude and (iii) the recoil peak gets reduced.

To summarize the Glauber approximation for the positron impact ionization leads to a larger binary peak and a smaller recoil peak compared to the electron case. The

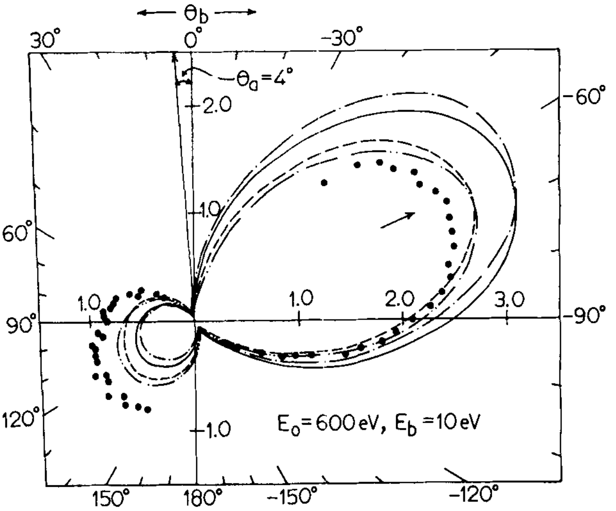

Figure 2c. Same as in figure 2a, but for $E_0 = 600\,\text{eV}$, $E_b = 10\,\text{eV}$, $\theta_a = 4°$.

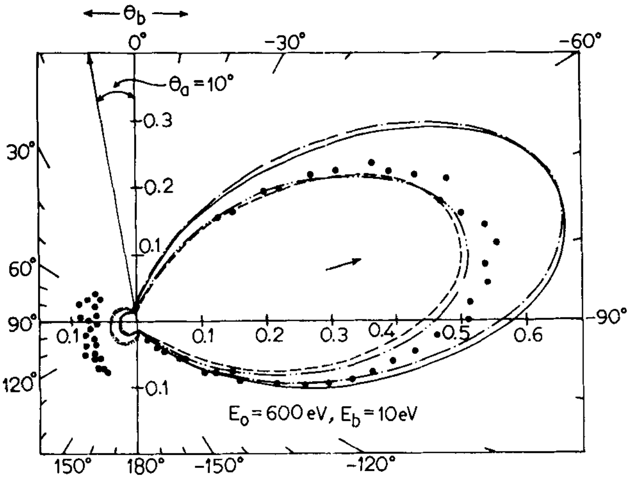

Figure 2d. Same as in figure 2a, but for $E_0 = 600\,\text{eV}$, $E_b = 10\,\text{eV}$, $\theta_a = 10°$.

changes produced by the incorporation of PCI effects are opposite to each other in the two cases. In the case of positron impact they lead to a shift of the peaks to smaller angles and a reduction in the recoil peak size. The binary-to-recoil peak maxima ratio is larger in the positron case and increases further when PCI effects are introduced. There are no experimental data for positron-induced ionization; however the fair agreement observed in the electron case indicates that similar should be the case with positrons.

Acknowledgements

This work was supported by the Indian Space Research Organisation. One of us (SS) would like to thank the University Grants Commission for the award of a research fellowship.

References

Baliyan K S and Srivastava M K 1985 *Phys. Rev.* **A32** 3098

Baliyan K S and Srivastava M K 1986a *Phys. Rev.* **A33** 2155

Baliyan K S and Srivastava M K 1986b *J. Phys.* **B19** 3603

Basu M, Mazumdar P S and Ghosh A S 1985 *J. Phys.* **B18** 369

Byron F W Jr and Joachain C J 1966 *Phys. Rev.* **146** 1

Byron F W Jr, Joachain C J and Piraux B 1980 *J. Phys.* **B13** L673

Byron F W Jr, Joachain C J and Piraux B 1982 *J. Phys.* **B15** L293

Ehrhardt H, Fischer M, Jung K, Byron F W Jr, Joachain C J and Piraux B 1982 *Phys. Rev. Lett.* **48** 1807

Ehrhardt H, Knoth G, Schlemmer P and Jung K 1985 *Phys. Lett.* **A110** 92

Ghosh A S, Mauzumdar P S and Basu M 1985a *J. Phys.* **B18** 1881

Ghosh A S, Mauzumdar P S and Basu M 1985b *Can. J. Phys.* **63** 62

Joachain C J 1984 *Positron scattering in gases* (eds) J W Humberston and M R C McDowell (New York: Plenum)

Jung K, Müller-Fiedler R, Schlemmer P, Ehrhardt H and Klar H 1985 *J. Phys.* **B18** 2955

Klar H and Franz A 1986 *Phys. Rev.* **A33** 2103

Klar H, Franz A and Tenhagen H 1986 *Z. Phys.* **D1** 373

Klar H, Roy A C, Schlemmer P, Jung K and Ehrhardt H 1987 *J. Phys.* **B20** 821

Mandal P, Roy K and Sil N C 1986 *Phys. Rev.* **A33** 756

Popov Yu V and Benayoun J J 1981 *J. Phys.* **B14** 3513

Popov Yu V and Erokhin V F 1983 *Phys. Lett.* **A97** 280

Roy A C, Das A K and Sil N C 1981 *Phys. Rev.* **A23** 1662

Roy A C, Das A K and Sil N C 1983 *Phys. Rev.* **A28** 181

Roy A C and Ray H 1987 in *Electronic and atomic collisions*, Abstracts of contributed papers, XV I.C.P.E.A.C. (eds) J Geddes, H B Gilbody, A E Kingston, C J Latimar and H J R Walters (Brighton, UK) p. 242

Saxena S and Srivastava M K 1987 *Can. J. Phys.* **65** 458