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Electron-sodium elastic scattering in the presence of strong non-resonant laser field

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Abstract. The elastic differential cross-section for $\bar{e}$-Na scattering in the presence of non-resonant laser field is studied for the exchange of $\ell = 0, 1, 2$ photons. The undressed contribution is evaluated within the framework of the eikonal Born series approximation and the effect of exchange is taken into account via the Ochkur approximation. The sodium atom has been treated in the frozen core approximation with special attention to the effect of the dressing of the target by the laser field. The 'dressing' of the target leads to quite an increase in the cross-section over the 'undressed' value near the forward direction for the exchange of one or two photons.

Keywords. Elastic scattering; electron impact; sodium; laser field; dressing effect; laser polarization effects.

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1. Introduction

The electron-atom scattering in a strong laser field has been studied during the last few years. From the theoretical point of view this problem is interesting because it introduces new parameters in the scattering: the photon energy $\hbar\omega$, the laser intensity (or electric field intensity) and its polarization. Special attention has been devoted to the low-frequency behaviour of the scattering phenomena starting with the work of Kroll and Watson (1973). In the presence of a strong electromagnetic field, multiphoton processes become important and a proper account of the modification or 'dressing' of atomic states due to the field is needed. The dressing effects are of order ω^2 (ω being the frequency of the laser field) and can, therefore, be neglected in the low frequency approximation in the case when the electromagnetic field is not strong. Byron and Joachain (1984) have shown that for a strong field the dressing of the target cannot be ignored even for the low frequencies. They found that it leads to a dramatically large increase in the cross-section for electron-atom elastic scattering at small scattering angles and with the transfer of $\ell(\neq 0)$ photons and that the size of this effect depends on the dipole polarizability of the target. This study has recently been extended so as to be valid (i) to all orders in the external field projectile interaction, (ii) without limitation on the laser frequency and (iii) for an arbitrary number of exchanged photons. It has been applied to the elastic scattering of electrons by hydrogen and helium. Another treatment, based on first order perturbation in the external field for both the target and the projectile, has recently been proposed by Dubois *et al* (1986). Mandal *et al* (1986) have looked at the effect of target 'dressing' on the exchange amplitude.

We investigate here elastic $\bar{e}$-Na scattering using the dressed wavefunction of the target atom corresponding to the absorption of zero, one and two photons. The term elastic means that the atomic states before and after the collision remain unaltered even though the energy of the scattered electron may change due to the emission or absorption of photons. This case is particularly attractive both for experimental and theoretical studies. In extending the analysis from hydrogen to an alkali atom the additional difficulties arise from the many-electron character of the alkali atom, but the frozen core approximation which reduces it to an effective one-electron model atom allows us to circumvent these difficulties.

The theory based on the work of Byron *et al* (1987) and the details of the calculation are briefly given in the next section. Section 3 contains the results and their discussion.

2. Theory and calculation

Consider the elastic scattering of electrons with a one-electron target in the presence of a strong laser field which is assumed to be homogeneous, linearly polarized and in single-mode and is treated in the dipole approximation. We have, working in the Coulomb gauge,

$$\mathbf{E}(t) = \mathbf{E}_L \sin \omega t, \quad \mathbf{A}(t) = \mathbf{A}_0 \cos \omega t \tag{1}$$

with $\mathbf{A}_0 = c\mathbf{E}_L/\omega$. The laser field strength is chosen such that

$$\mathbf{E}_L \ll e/a_0^2. \tag{2}$$

The Hamiltonian of the electron-atom system in the presence of the laser field can be written as

$$H = H_f + H_t + V, \tag{3}$$

where V is the electron-atom interaction. H_f and H_t are respectively the free electron and the target Hamiltonians in the presence of the laser field, satisfying

$$i\hbar \frac{\partial \phi}{\partial t} = H_f \phi, \tag{4}$$

$$i\hbar \frac{\partial \psi}{\partial t} = H_t \psi, \tag{5}$$

where $H_f = 1/2m(\mathbf{P} + e/c\mathbf{A})^2$ and $\mathbf{P}$ is the momentum operator. The solution (Volkov solution) of (4) is

$$\phi_{\mathbf{k}_i}(\mathbf{r}_0, t) = (2\pi)^{-3/2} \exp\left[i(\mathbf{k}_i \cdot \mathbf{r}_0 - \mathbf{k}_i \cdot \boldsymbol{\alpha}_0 \sin \omega t - E_k t/\hbar)\right], \tag{6}$$

where $\mathbf{k}_i$ is the momentum vector of the free electron (coordinate $\mathbf{r}_0$), $E_{k_i} = \hbar^2 k_i^2/2m$ and $\boldsymbol{\alpha}_0 = e\mathbf{E}_L/m\omega^2$.

In the presence of the laser, the ground state wavefunction may be expressed in terms of the wavefunctions of the unperturbed atom:

$$\Phi_0(t) = \sum_n a_n(t)\Phi_n^0(t) \tag{7}$$

$$\Phi_n^0(t) = \phi_n \exp(-i\omega_n t/\hbar). \tag{8}$$

Using the first order time-dependent perturbation theory and removing the low frequency approximation of Byron and Joachain (1984), the dressed ground state wavefunction of the target can be written (Francken and Joachin) as

$$\begin{aligned}\phi_0(\mathbf{r}, t) &= \exp(-i\omega_0 t/\hbar)\exp(-i\mathbf{a}\cdot\mathbf{r}) \\ &\times [\phi_0(\mathbf{r}) - \sin\omega t \sum_n \frac{\omega_{n0} M_{np,0}}{\hbar(\omega_{n0}^2 - \omega^2)}\phi_{np}(r) \\ &- i\cos\omega t \sum_n \frac{\omega M_{np,0}}{\hbar(\omega_{n0}^2 - \omega^2)}\phi_{np}(r)],\end{aligned} \tag{9}$$

where $\mathbf{r}$ is the target electron coordinate, ϕ_{np} is the nth undressed p-state, $\hbar\,\omega_{n0} = \omega_n - \omega_0$ is the corresponding excitation energy, the summation includes the continuum states, $\mathbf{a} = e\mathbf{A}/\hbar c$ and $M_{np,0}$ is defined as

$$M_{np,0} = \mathbf{E}_L \cdot \langle \phi_{np} | e\mathbf{r} | \phi_0 \rangle. \tag{10}$$

The second and third terms in (9) represent the dressing effect. This dressed state is used in place of the unperturbed atomic state in the scattering calculation. The elastic scattering amplitude in the first Born approximation corresponding to a transfer of ℓ photons is given by Francken and Joachain (1987) as

$$\begin{aligned}f^{B1,\ell}(\Delta) &= J_\ell(\Delta\cdot\boldsymbol{\alpha}_0) f^{B1}(\Delta) - iJ_\ell'(\Delta\cdot\boldsymbol{\alpha}_0) \\ &- \sum_n \frac{\omega_{n0}(f^{B1}_{0,np}(\Delta) M_{np,0} + M_{0,np} f^{B1}_{np,0}(\Delta))}{\omega_{n0}^2 - \omega^2},\end{aligned} \tag{11}$$

where J_ℓ is Bessel function of order ℓ, J_ℓ' is its derivative, f^{B1}, $f^{B1}_{np,0}$ and $f^{B1}_{0,np}$ are respectively the first Born amplitude corresponding to the scattering $0 \to 0$, $0 \to np$ and $np \to 0$ in the absence of the laser field, $\Delta = \mathbf{k}_i - \mathbf{k}_f$ is the momentum transfer and $\mathbf{k}_i$ and $\mathbf{k}_f$ are the initial and final wave vector of the scattering electron. They are related by the energy conservation condition

$$\frac{\hbar^2 k_i^2}{2m} = \frac{\hbar^2 k_f^2}{2m} \pm \ell\hbar\omega. \tag{12}$$

Following Byron and Joachain (1984) we use f^{EBS} (the EBS direct elastic amplitude) in place of f^{B1} in the first term of (11), the second term in (11) is evaluated by replacing the excitation energies $\hbar\omega_{n0}$ by an average excitation energy $\hbar\omega$ (taken equal to 4·7418 eV) and using the closure approximation. With all these changes the direct amplitude is given by

$$f^{\ell}(\Delta) = J_{\ell}(\Delta\cdot\alpha_0) f^{\mathrm{EBS}} + \frac{4\bar{\omega}}{\Delta^2(\bar{\omega}^2 - \omega^2)} J_{\ell}'(\Delta\cdot\alpha_0) \times \mathbf{E}_L\cdot\nabla_{\Delta}\langle\phi_0|\exp(i\Delta\cdot\mathbf{r})|\phi_0\rangle. \tag{13}$$

The exchange amplitude is approximated by

$$g^{\ell}(\Delta) = J_{\ell}(\Delta\cdot\alpha_0) g_{\mathrm{Och}}(\Delta), \tag{14}$$

where g_{Och} is the Ochkur (1963) elastic scattering amplitude. Finally the differential cross-section is given by

$$\frac{d\sigma^{\ell}}{d\Omega} = \frac{k_f}{k_i}\left[\tfrac{1}{4}|f^{\ell} + g^{\ell}|^2 + \tfrac{3}{4}|f^{\ell} - g^{\ell}|^2\right]. \tag{15}$$

The sodium atom is treated as a hydrogenic system in the frozen core approximation using the wavefunction of Daniele (1980):

$$\phi_0(r) \equiv \phi_{3s}(r) = [(b_1 + b_2 r + b_3 r^2)\exp(-\beta r)]\frac{1}{\sqrt{4\pi}}, \tag{16}$$

where

$b_1 = 0{\cdot}23416$ $\quad$ $b_2 = -1{\cdot}33799,$

$b_3 = 1{\cdot}18303$ $\quad$ $\beta = 1{\cdot}16915.$

The electric field $\mathbf{E}_L$ of the laser is taken to be equal to 0·002 a.u. and its direction parallel to the momentum transfer direction, parallel to the incident electron direction and perpendicular to the incident electron direction. The laser frequency is chosen to be such that $\hbar\omega = 0{\cdot}3657$ eV (which corresponds to CO_2 laser frequency) and the incident electron energy is taken to be 100 eV and 200 eV.

3. Results

Figures 1(a–c) show the differential cross-sections for the elastic $\bar{e}$-Na scattering with the transfer of 0, 1 and 2 photons over an angular range $\theta \leqslant 8^{\circ}$ at an incident electron energy of 100 eV. The target dressing is expected to modify the cross-section significantly only at small scattering angles. The laser electric field is taken to be parallel to Δ. The results with dressed target state are compared with those without dressing. The field-free results are also shown. The presence of laser reduces the cross-section and leads to a structure in the angular dependence. The $\ell = 0$ results without dressing and the field-free results (figure 1a) are identical at $\theta = 0{\cdot}5^{\circ}$, thereafter $\ell = 0$ results decrease quite sharply and show a minimum at $\theta \simeq 4{\cdot}5^{\circ}$. The corresponding results with dressed target state show an enhancement at small θ and exhibit a similar structure as the 'undressed results'. In the case of $\ell = 1$ and 2 (figures 1b and 1c) the dressing of the target leads to spectacular increase in the cross-section near the forward direction as pointed out by Byron and Joachain (1984) and Byron *et al* (1987). The location of minima moves to larger angles with increasing number of exchanged photons. The

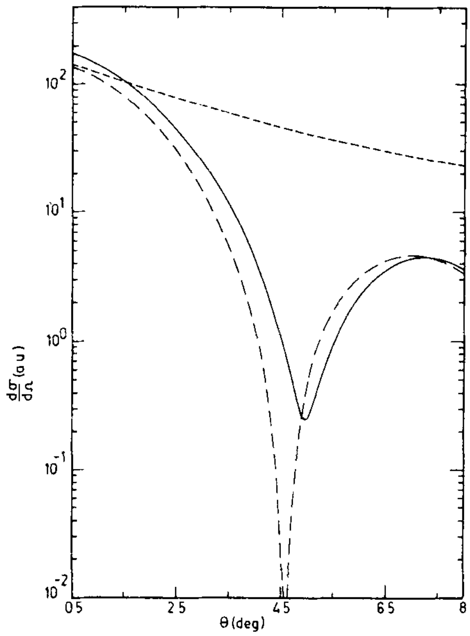

Figure 1a. Differential cross-sections in a.u. for the elastic $\bar{e}$-Na scattering with transfer of no photons at an incident electron energy 100 eV in the presence of laser field with $\mathbf{E}_L \parallel \mathbf{\Delta}$. Results with dressing —; Results without dressing —·—; Field-free results——.

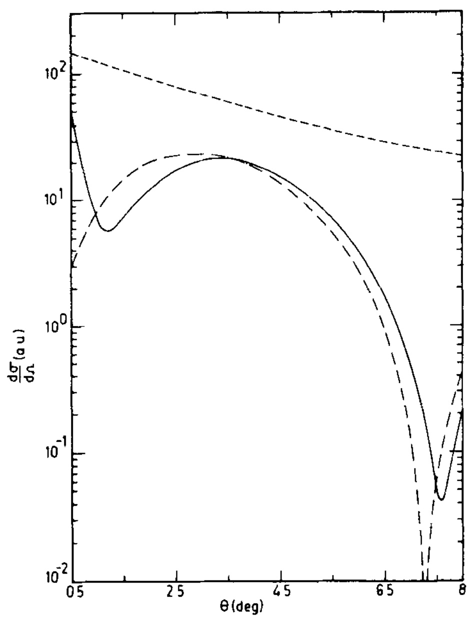

Figure 1b. Same as figure 1a but for $\ell = 1$.

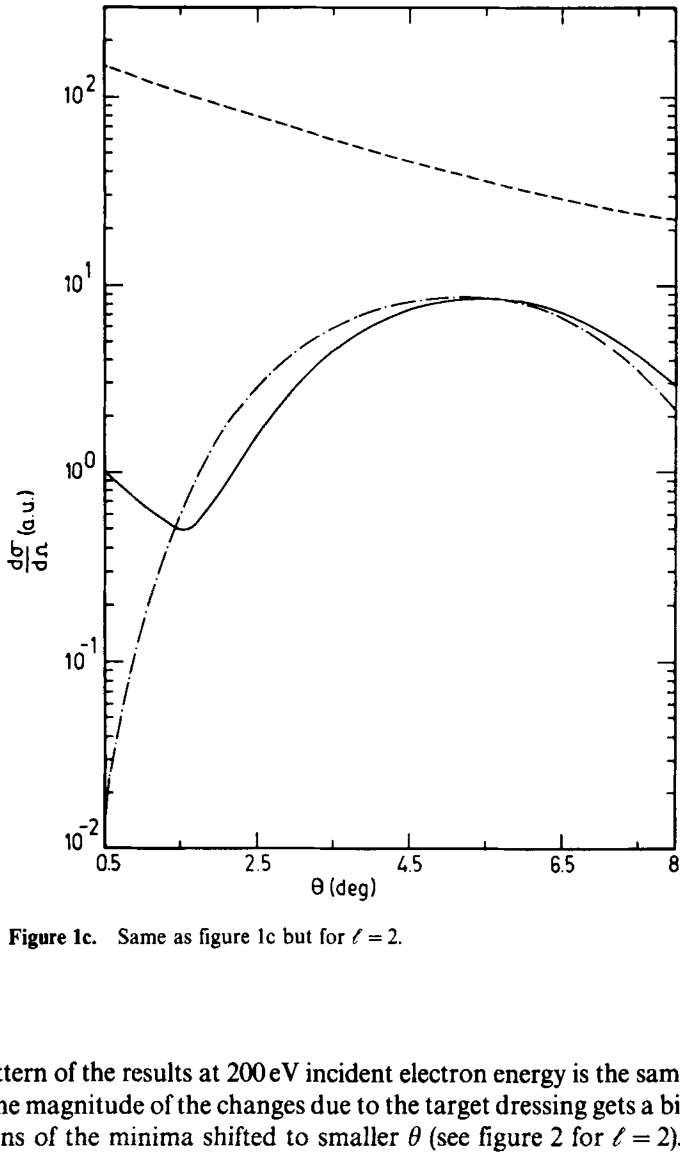

Figure 1c. Same as figure 1c but for $\ell = 2$.

general pattern of the results at 200 eV incident electron energy is the same as at 100 eV; however, the magnitude of the changes due to the target dressing gets a bit reduced and the positions of the minima shifted to smaller θ (see figure 2 for $\ell = 2$).

Similar results are observed when the laser electric field is kept fixed at right angles to $\mathbf{k}_i$.

Figures 3 (a and b) show our results at 200 eV incident energy for the case when the electric field is parallel to $\mathbf{k}_i$. The $\ell = 0$ laser assisted cross-sections and the field-free results are almost the same in this case over the angular range considered here. For one- and two-photon absorption, spectacular changes are again observed. The dressing effects lead to rise in the cross-section near the forward direction.

Figure 4 shows the variation of the cross-section with the direction of the laser electric field for $\ell = 1$ at a fixed small scattering angle $\theta = 1°$ and incident energy 200 eV. The cross-section drops to a very low value (the undressed ones actually drop to zero) when the electric field is perpendicular to $\mathbf{\Delta}$. The relative magnitude of the dressed and undressed results, however, depends on the value of ℓ and the scattering angle.

The above reported features are expected to be measurable without much difficulty.

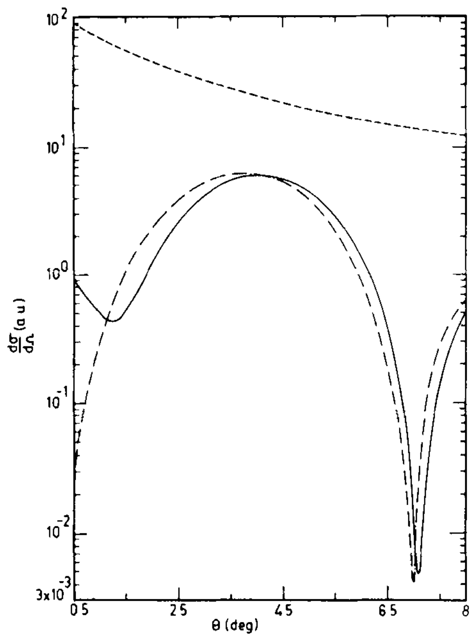

Figure 2. Differential cross-sections in a.u. for the elastic $\tilde{e}$-Na scattering with transfer of two photons at an incident electron energy 200 eV in the presence of laser field with $\mathbf{E}_L \parallel \mathbf{\Delta}$. Results with dressing —; Results without dressing —·—; Field-free results——.

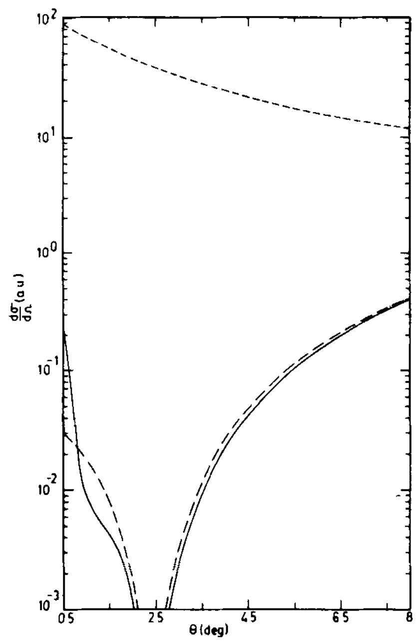

Figure 3a. Differential cross-sections in a.u. for the elastic $\tilde{e}$-Na scattering with transfer of one photon at an incident electron energy 200 eV in the presence of laser field with $\mathbf{E}_L \parallel \mathbf{k}_i$. Results with dressing —; Results without dressing —·—; Field-free result——.

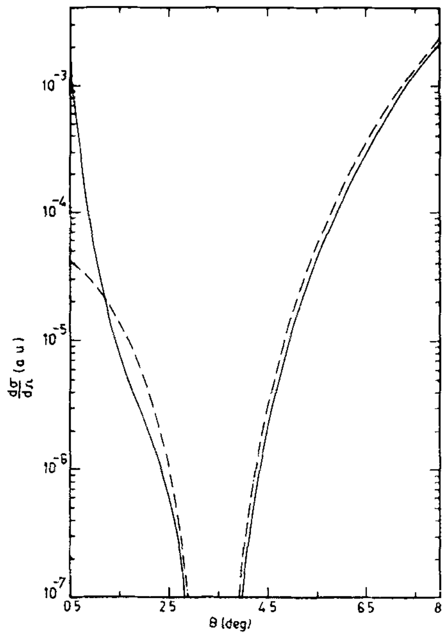

Figure 3b. Same as figure 3a but for $\ell = 2$.

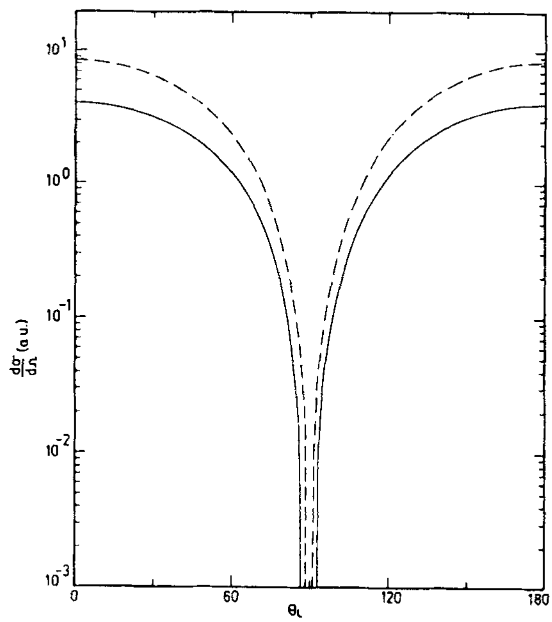

Figure 4. Variation of the differential cross section with the direction of $\mathbf{E}_L$ for one photon absorption at a fixed small scattering angle $\theta = 1°$ at incident electron energy 200 eV.

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References

Byron F W Jr, Francken P and Joachain C J 1987 *J. Phys.* **B20** 5487
Byron F W Jr and Joachain C J 1984 *J. Phys.* **B17** L295
Dubois A, Maquet A and Jetzke S 1986 *Phys. Rev.* **A34** 1888
Daniele R 1980 *J. Chem. Phys.* **72** 1276
Francken P and Joachain C J 1987 *Phys. Rev.* **A35** 1590
Kroll N M and Watson K M 1973 *Phys. Rev.* **A8** 804
Mandal S K, Basu M and Ghosh A S 1986 *J. Phys.* **B19** 3333
Ochkur V I 1963 *Zh. Eksp. Teor. Fiz.* **45** 734