Pramāṇa - J. Phys., Vol. 31, No. 4, October 1988, pp. 241-251. © Printed in India.

# Collective octupole vibration and proton single-particle levels in zirconium nuclei

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MS received 21 May 1988

**Abstract.** Self-energy correction to the shell model single-particle motion, arising from the excitation of octupole vibration in the intermediate state, accounts quite well for the energy shifts of the $2p_{1/2}$ and $1g_{9/2}$ proton orbits in zirconium nuclei.

**Keywords.** Nuclear structure; collective model; single-particle model; zirconium nuclei; energy shifts.

**PACS Nos 21·60; 21·65; 21·10**

## 1. Introduction

Nuclei possess both single-particle and collective aspects. The shell model with its associated energy gaps and single-particle states is very essential to our understanding of the microscopic structure of nuclei. Collective vibrational and rotational states with enhanced r ltipole transitions (Bohr and Mottelson 1975) and with properties vastly different from single-particle excitations are a regular feature of almost all nuclei, barring only the very lightest of nuclei. One needs both these degrees of freedom, single-particle and collective, and the interplay between them for a proper understanding of the nuclear properties (Bohr and Mottelson 1975).

The interaction between a single-particle (or a quasiparticle) and a collective mode is quite important and leads to some remarkable models. Interaction between particles and collective rotation gives the rotation-particle coupling (Bohr and Mottelson 1975) and particle-rotor models (Stephens 1975). The interaction between quasiparticles and quadrupole vibration leads to the quasiparticle-vibration coupling model (Kisslinger and Sorensen 1963) which describes adequately the spectra and electromagnetic transitions in many odd mass nuclei. Virtual excitation of particle-hole configurations and vibrational modes can also lead to modifications of the properties of single-particle states (Bertsch and Kuo 1968; Hamamoto 1974, 1975; Hamamoto and Siemens 1976) such as shifts in single-particle energies.

The spectrum of nuclear vibrations is very rich with various multipole and spin and isospin modes (Satchler 1977). In this work we treat the effect of the low energy octupole vibration on the proton single-particle states in zirconium nuclei and show that the virtual excitation of an octupole vibration gives energy shifts which agree with the experimental trend of single-particle energy systematics in this region. Specifically we treat the effect of octupole vibration on the energies of $2p_{1/2}$ and $1g_{9/2}$ proton levels and their systematics as a function of neutron number. It is seen in this work that there is a systematic lowering of the $2p_{1/2}$ proton orbit relative to the $1g_{9/2}$ orbit as a result of the coupling of octupole vibration to single-particle states.

The paper is organised as follows. In § 2 we analyse the energies of the $5^-$ states in zirconium nuclei and the $(1/2)^-$ and $(9/2)^+$ states in niobium nuclei and deduce the corresponding trends in the $2p_{1/2}$ and $1g_{9/2}$ proton levels. The many-body formalism (random phase approximation) and the expression for the energy shifts are given in § 3. In § 4.1 the shell model space and the iterative procedure to calculate the energy shifts are described. In § 4.2 we present the results of calculation of energy shifts for the proton single-particle levels due to the interaction with octupole vibration. The concluding summary is given in § 5. A preliminary report of this work was earlier reported by us (Praharaj 1979).

## 2. Energy systematics of $5^-$ states in zirconium nuclei and of $1/2^-$ and $9/2^+$ states in niobium nuclei

The $2p_{1/2}$ and the $1g_{9/2}$ are the two active proton orbits in zirconium and the neighbouring nuclei. Although the $fp$ proton shell is sometimes thought to be closed for zirconium nuclei, in reality the $2p_{1/2}$ proton orbit is not completely occupied and there is significant excitation onto the $1g_{9/2}$ proton level. Cohen *et al* (1964) studied the low energy spectra of zirconium nuclei using effective interaction matrix elements and effective single-particle energies in a limited model space. They found that the $^{90}$Zr ground state is a linear combination of $(p_{1/2}^2)_{0^+}$ and $(g_{9/2}^2)_{0^+}$ configurations with

$$|0^+\rangle = a(p_{1/2}^2)_{0^+} - (1-a^2)^{1/2}(g_{9/2}^2)_{0^+}, \tag{1}$$

where $a \approx 0.8$. Thus the main component of the ground state is the $(p_{1/2}^2)_{0^+}$ proton configuration with some admixture from $(g_{9/2}^2)_{0^+}$ protons.

The energy of the $5^-$ state in zirconium nuclei is a measure of the excitation energy of the $(g_{9/2}p_{1/2})_{5^-}$ proton configuration relative to the $|0^+\rangle$ ground state. Going from $^{90}$Zr to $^{96}$Zr, with the neutron number increased from 50 to 56, the excitation energy of the $5^-$ state goes on increasing (see figure 1a). This implies a lowering of the $(p_{1/2}^2)$ proton configuration relative to $(g_{9/2}p_{1/2})_{5^-}$ and hence of the $2p_{1/2}$ proton orbit relative to the $1g_{9/2}$.

A similar trend is also found for the energy difference $E_{(1/2)^-} - E_{(9/2)^+}$ of the $(1/2)^-$ and $(9/2)^+$ states of niobium nuclei as one goes from $^{91}$Nb to $^{97}$Nb (see Figure 1b). The active proton configurations for these two states of niobium are the three-proton configurations $(g_{9/2}^2p_{1/2})_{(1/2)^-}$ and $(g_{9/2}p_{1/2}^2)_{(9/2)^+}$ respectively. In figure 1b the relative energy $E_{(1/2)^-} - E_{(9/2)^+}$ goes on increasing as we go from $N = 50$ to $N = 56$, again implying a raising of the $g_{9/2}$ level relative to $p_{1/2}$ as the neutron number is increased.

In going from $N = 50$ to $N = 56$ it is mostly the $2d_{5/2}$ neutron level that is being filled. Since $2d_{5/2}$ and $1g_{9/2}$ are of the same parity and belong to the same oscillator shell, there is good spatial overlap between the $2d_{5/2}$ neutrons and the $1g_{9/2}$ proton levels. The attractive quadrupole-quadrupole interaction between protons and neutrons $(-\chi_{pn}Q_p \cdot Q_n)$ is sizeable for the neutrons and protons occupying these two orbits. Such attractive interaction vanishes for the $2p_{1/2}$ protons, since a $p_{1/2}$ state does not have a quadrupole moment, i.e., $\langle p_{1/2}\|Q_2\|p_{1/2}\rangle = 0$. One thus expects the $1g_{9/2}$ proton level to go down in energy relative to the $2p_{1/2}$ proton level as the $2d_{5/2}$ neutron level

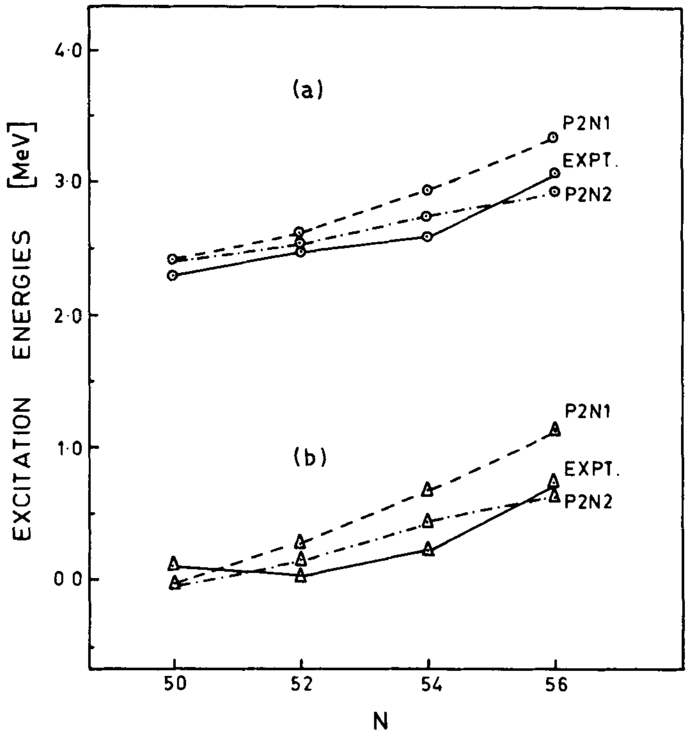


**Figure 1.** (**a**) Excitation energy of $5^-$ state in zirconium nuclei as a function of neutron number. The experimental numbers (Nuclear Data Group 1973) are drawn as solid line. Calculations with two sets of starting single-particle energies are shown as dashed lines. (**b**) The energy difference $E_{(1/2)^-} - E_{(9/2)^+}$ in Niobium nuclei.

becomes more and more occupied, in sharp contrast to the conclusions from the experimental energy systematics in Zr and Nb nuclei pointed out in the last two paragraphs (which requires that the $1g_{9/2}$ proton level should go up in energy relative to the $2p_{1/2}$ level).

In subsequent sections of this work we show that the coupling of the low energy octupole vibration to the shell model single-particle motion can cause the systematic relative lowering of the $2p_{1/2}$ proton level as seen in the excitation energy of the $5^-$ state of Zr and the energies of $(1/2)^-$ and $(9/2)^+$ states of Nb nuclei.

## 3. Collective octupole vibration and the expression for energy shift of shell model states

We consider the self-energy correction to the single-particle energy due to the virtual excitation of lowlying octupole vibration as shown in figure 2b. We treat the energies of the valence proton states in the shell model potential (such as in a Woods-Saxon potential or from Hartree-Fock theory) as the zeroth order energies and evaluate the energy shift due to the coupling with octupole vibration in second order perturbation theory (Fetter and Walecka 1971; Thouless 1972; Brown 1972). We assume an

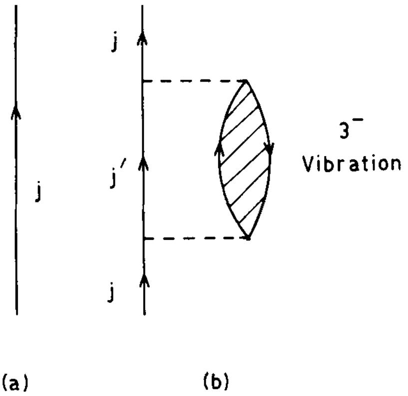


**Figure 2.** (a) Zeroth order single-particle motion. (b) Second order self-energy correction to single-particle motion due to excitation of octupole vibration in the intermediate state.

octupole-octupole interaction (Veje 1966; Bohr and Mottelson 1975)

$$H_{\text{int}} = -\chi_3 Q_3 \cdot Q_3 = -\chi_3 \sum_u Q_{3u} Q_{3-u} (-1)^u. \tag{2}$$

The energy shift of the single-particle level $j$, fully or partially occupied in the ground state is given by the standard rules of perturbation theory (Fetter and Walecka 1971; Brown 1972; Thouless 1972):

$$\Delta\varepsilon_j = \frac{-\chi_3^2}{7(2j+1)} \sum_{j'} \frac{[U_{j'} V_j \langle j' \| Q_3 \| j \rangle \langle \psi_{3^-} \| Q_3 \| 0^+ \rangle]^2}{\varepsilon_{j'} - \varepsilon_j + \mathscr{E}_3}. \tag{3}$$

Here $\varepsilon_j$ is the energy of the single-particle level $j$, $\mathscr{E}_3$ is the energy of the $3^-$ vibration and $\langle \psi_{3^-} \| Q_3 \| 0^+ \rangle$ is the reduced matrix element of the octupole operator $Q_{3u}$

$$Q_{3u} = \sum_i r_i^3 Y_{3u}(\Omega_i) \tag{4}$$

between the ground state and the $3^-$ state. $V_j$ and $U_j$ are respectively the occupation and unoccupation amplitudes of single-particle level $j$. The summation in (3) is over all the active protons and neutrons which are unoccupied, either partially or wholly (note the factor $U_{j'}^2$, in (3)). Coherent excitation in interacting $3^-$ particle-hole configurations leads to the lowering of the energy of the isoscalar $3^-$ vibration and an enhancement of the octupole transition strength $\langle \psi_{3^-} \| Q_3 \| 0^+ \rangle$ (Veje 1966).

Experimentally the excitation energies $\mathscr{E}_3$ of the octupole states in $^{90,92,94,96}$Zr are 2·746, 2·34, 2·057 and 1·905 MeV respectively (Nuclear Data Group 1973). These energies and octupole transition strengths are determined by solving the RPA equation (Veje 1966; Bohr and Mottelson 1975). Analogous equations can be found in the work of Kisslinger and Sorensen (1963) for the quadrupole vibration:

$$2\chi_3 \sum_\alpha \frac{E_\alpha q_3^2(\alpha)}{E_\alpha^2 - \mathscr{E}_3^2} = 1. \tag{5}$$

Here $\alpha$ denotes the particle-hole configuration $j_1, j_2$, which couple to $J^\pi = 3^-$. $E_\alpha$ is the excitation energy for such particle-hole configuration and $q_3$ is the reduced matrix element of the octupole operator.

$$q_3(j_1j_2) = \frac{1}{\sqrt{7}}\langle j_1 \| r^3 Y_3 \| j_2 \rangle \times (-1)^{l_2}(U_{j_1}V_{j_2} + U_{j_2}V_{j_1}). \tag{6}$$

In equations (3) and (6), $j$ is used as a short-hand notation for the single-particle indices $n_{lj}$. We note that for partially occupied levels $j_1$ and $j_2$, the excitation energy $E_\alpha$ of the particle hole configuration $\alpha \equiv j_1 j_2$ is the sum of the energies of the two quasiparticles:

$$E_\alpha = E_{j_1} + E_{j_2}. \tag{7a}$$

For an occupied level $j_1$ and a completely empty level $j_2$

$$E_\alpha = \varepsilon_{j_2} - \varepsilon_{j_1}. \tag{7b}$$

For the octupole transition matrix element we have (Kisslinger and Sorensen 1963; Veje 1966)

$$\langle 3^- \| Q_3 \| 0^+ \rangle = \sqrt{7}\sum_\alpha q_3(\alpha)(Y_\alpha + Z_\alpha) \tag{8}$$

where $Y_\alpha$ and $Z_\alpha$ are the forward-going and backward going amplitudes respectively of RPA:

$$Y_\alpha = -\chi_3 C \frac{q_3(\alpha)}{\mathscr{E}_3 - E_\alpha}, \tag{9a}$$

$$Z_\alpha = +\chi_3 C \frac{q_3(\alpha)}{\mathscr{E}_3 + E_\alpha}, \tag{9b}$$

and

$$C = \left[4\chi_3^2 \mathscr{E}_3 \sum_\alpha \frac{E_\alpha q_3^2(\alpha)}{(\mathscr{E}_3^2 - E_\alpha^2)^2}\right]^{-1/2}. \tag{10}$$

Taking the energy of the $3^-$ vibration $\mathscr{E}_3$ as known from experiment we use equation (5) to get the octupole-octupole force strength and, using (8), (9) and (10), we calculate the reduced octupole matrix element $\langle 3^- \| Q_3 \| 0^+ \rangle$. Equation (3) is then used to get the energy shift $\Delta\varepsilon_j$.

We restrict ourselves to the low energy octupole vibration and do not include the high energy octupole states involving nucleon excitation across three major shells. The reason for keeping only the lowest octupole vibration in (3) is the following: the lowest $3^-$ vibration is very collective in Zr nuclei (Veje 1966) and although its contribution to the energy-weighted sum rule is not so large because of the energy-weight, it is the most dominant term in the present calculation having inverse energy-weight in (3).

## 4. Calculation of energy shifts

### 4.1 *Calculation of energy shifts of valence protons*

The expressions developed in § 3 are used to calculate the energy shifts of $2p_{1/2}$ and $1g_{9/2}$ proton levels. Figure 2a represents the zeroth order energy, i.e., the single-particle energy in a potential well, such as a Woods-Saxon well, given by the mean-field theory (Hartree-Fock theory) (Vautherin and Brink 1972). Figure 2b represents the lowest order self-energy contribution to the energy shift (Fetter and Walecka 1971; Brown 1972; Thouless 1972) due to the coupling with octupole vibration. The octupole vibration is treated in RPA as described in § 3.

The energy shifts $\Delta\varepsilon_{2p_{1/2}}$ and $\Delta\varepsilon_{1g_{9/2}}$ of $2p_{1/2}$ and $1g_{9/2}$ proton single-particles, as given in equation (3) are calculated for the even mass zirconium nuclei $N = 50$ to $N = 56$. The shell model space for the protons and the neutrons is depicted in figure 3. Two sets of single-particle energies each for protons and neutrons are used in our calculation. These single-particle energies are given in table 1. These represent reasonable sets of values in this mass region and combine the single-particle energies of levels in a Woods-Saxon well (Bertsch 1972), from the Hartree-Fock study of Vautherin and Brink (1972) and the location of experimental single-particle states in $^{89}$Sr (for protons) and $^{91}$Zr (for neutrons) (Nuclear Data Group 1973).

The $1f_{7/2}$, $2p_{3/2}$, $1f_{5/2}$ proton levels are assumed to be filled and the proton levels $2d_{5/2}$ and above are assumed to be empty. The $2p_{1/2}$ and $1g_{9/2}$ are the two active valence proton levels. The $1g_{9/2}$ and $2p_{1/2}$ energy difference for protons is taken as 0·914 MeV for $^{90}$Zr by Cohen *et al* (1964). In our calculation we have used two values for this energy difference (see table 1): ($P1$) 0·9 MeV and ($P2$) 1·3 MeV. The latter value is a little nearer to the value of 2 MeV for single-particles in the Woods-Saxon well (Bertsch 1972).

For the neutrons, the $1g_{9/2}$ level and all other levels below are completely filled. The $2d_{5/2}$ neutron level is gradually being filled in going from $^{90}$Zr to $^{96}$Zr and its

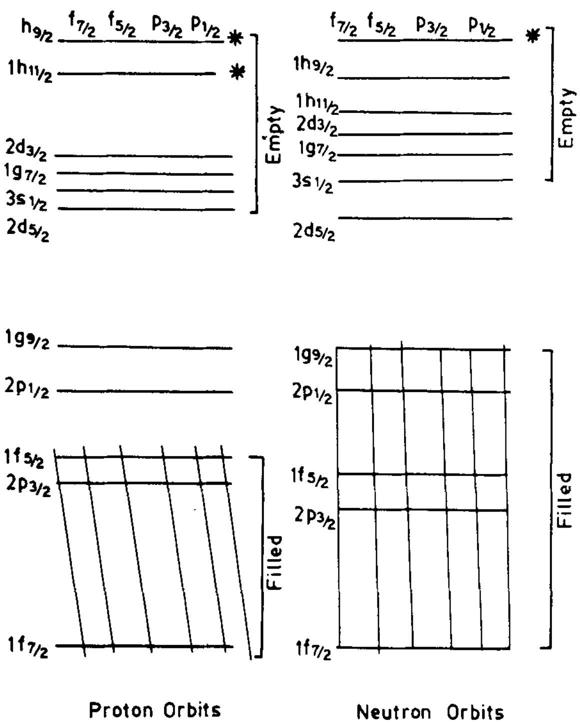


**Figure 3.** Shell model space for protons and neutrons in the zirconium region. The unbound levels are denoted by asterisks.

**Table 1.** Two sets of starting single-particle energies (in MeV) for protons and neutrons. The unbound levels are taken at large excitation energy (see Bertsch 1972; Vautherin and Brink 1972; Yoshida 1962).

| | Set | | | |
|---|---|---|---|---|
| | $P_1$ | $P_2$ | $N_1$ | $N_2$ |
| $1f_{7/2}$ | −8·6 | −8·6 | −11·1 | −11·1 |
| $2p_{3/2}$ | −1·6 | −1·6 | −4·1 | −4·1 |
| $1f_{5/2}$ | −2·0 | −2·0 | −4·5 | −4·5 |
| $2p_{1/2}$ | 0·0 | 0·0 | −2·5 | −2·5 |
| $1g_{9/2}$ | 0·9 | 1·3 | 0·0 | 0·0 |
| $2d_{5/2}$ | 6·9 | 7·3 | 3·4 | 3·4 |
| $3S_{1/2}$ | 8·4 | 8·8 | 6·1 | 5·1 |
| $1g_{7/2}$ | 8·9 | 9·3 | 6·4 | 6·0 |
| $2d_{3/2}$ | 9·9 | 10·3 | 7·1 | 6·1 |
| $1h_{11/2}$ | 36·0 | 36·0 | 7·5 | 6·2 |
| $1h_{9/2}$ | 37·15 | 37·15 | 8·5 | 8·5 |
| $2f_{7/2}$ | 37·15 | 37·15 | 11·15 | 11·15 |
| $2f_{5/2}$ | 37·15 | 37·15 | 11·15 | 11·15 |
| $3p_{3/2}$ | 37·15 | 37·15 | 11·15 | 11·15 |
| $3p_{1/2}$ | 37·15 | 37·15 | 11·15 | 11·15 |

occupation probabilities are $(N-50)/6$ where $N$ is the neutron number. Thus we have

$$V_{d5/2} = [\tfrac{1}{6}(N-50)]^{1/2}. \tag{11}$$

Two sets of neutron single particles are used (see table 1). The neutron energies of set $N_2$ in table 1 contain a few corrections to those of the set $N_1$ so as to agree with the single-neutron energies in the zirconium region as given by Yoshida (1962). In all the single-particle sets of table 1, the unbound levels are taken to be at large positive energies. The energies of the ground and the other lowlying configurations in Zr and Nb nuclei are determined by the interaction among the valence nucleons. However in the calculation for the collective octupole states, nucleons from completely filled orbits are excited to the empty orbits to form particle-hole configurations.

In the calculation for the even zirconium nuclei $^{90-96}$Zr, the energies $\mathscr{E}_3$ of the lowest octupole vibrations are taken from experiment (as given by Bernstein (1969) and equation (5) is used to calculate the octupole-octupole force strength $\chi_3$. This is the opposite of what is usually done in the RPA, where the force strength $\chi_3$ is given and the octupole energy $\mathscr{E}_3$ is calculated from equation (5). Thus we use an "inverse RPA" method to get the force strength $\chi_3$ from the known octupole energy $\mathscr{E}_3$. The value of $\chi_3$ thus obtained is used in equations (9) and (10) to get the forward-going and backward-going RPA amplitudes $Y$ and $Z$ and hence the octupole transition matrix element from equation (8). These are then used in equation (3) to calculate the energy shifts. These energy shifts are now incorporated in the single-particle energies which are again used in equations (5), (8), (9) and (10) to calculate the properties of octupole states. This process is iterated till convergence is obtained. Ten iterations are sufficient to get the convergence.

*4.2 Results of calculation of energy shifts*

The energy of a single-particle state $j$ partially or wholly occupied is affected by the octupole vibration (equation (3)) and its energy is lowered because the denominator in each of the terms in the summation in equation (3) is a positive number. The nucleon in state $j$ goes to an unoccupied state $j'$ in the intermediate state and excites a $3^-$ vibration (figure 2b). However the most significant lowering in energy occurs for the valence protons and neutrons, since the $\varepsilon_{j'} - \varepsilon_j$ are relatively small for the valence nucleons. The energy shifts are comparatively less important for the deeper bound states such as the $1f_{7/2}$, $2p_{3/2}$ and $1f_{5/2}$ states because the excitation energies $\varepsilon_{j'} - \varepsilon_j$ for such states in equation (3) are large (a deeply bound state has to be excited to an empty state above by octupole coupling) and hence the contributions to the energy shift in equation (3) for such deeply bound orbits get damped.

The important octupole couplings among the single-particle states are depicted in figure 4a. The most significant lowering in energy occurs for the $2p_{1/2}$ proton level, because of the octupole coupling with the $2d_{5/2}$ level (see figure 4b). This becomes clear if we examine in table 1 the excitation energies involved in the energy denominator of equation (3) for the various octupole transitions.

The energy shifts of the $2p_{1/2}$ and $1g_{9/2}$ protons are particularly relevant for our discussion since the lowlying states of Zr and Nb nuclei are very sensitive to the energy of the $2p_{1/2}$ proton state relative to the $1g_{9/2}$ proton state. The results for the energy shifts of the $2p_{1/2}$ and $1g_{9/2}$ protons calculated by the iterative procedure outlined in §4.1 are given in table 2 for the four sets of proton and neutron starting energy combinations $(P_1N_1, P_1N_2, P_2N_1, P_2N_2)$ of table 1. The $2p_{1/2}$ and $1g_{9/2}$ proton single-particles thus obtained are used in shell model calculation of the lowlying states of Zr and Nb nuclei with $(p_{1/2}^2)$, $(g_{9/2}^2)$, $(p_{1/2}g_{9/2})$, $(p_{1/2}^2g_{9/2})$ and $(p_{1/2}g_{9/2}^2)$ configurations. For the effective interaction matrix elements, we take those of Cohen *et al* (1964). The excitation energies of the $5^-$ states of even mass Zr nuclei and the energy difference $E_{1/2}^- - E_{9/2}^+$ of the niobium nuclei obtained in such shell model calculations are also listed in table 2.

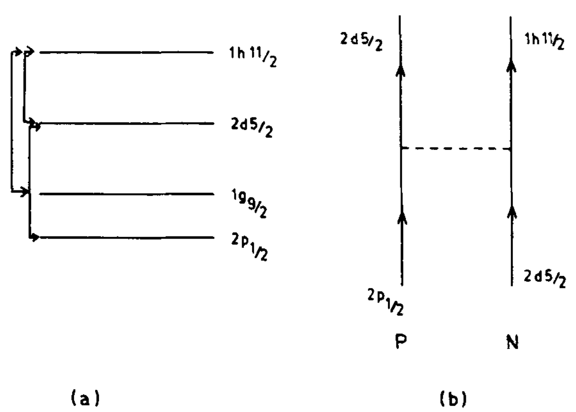


**Figure 4.** (**a**) Possible octupole couplings among single-particle states in the zirconium region. Orbits with large octupole matrix elements are connected by arrows. (**b**) The interaction among $2p_{1/2}$ protons and $2d_{5/2}$ neutrons caused by the octupole-octupole interaction.

**Table 2.** Shifts in $2p_{1/2}$ and $1g_{9/2}$ proton single-particle levels are listed in (a, b) in MeV. $5^-$ excitation energy of Zr nuclei is listed in (c) in MeV. $E_{1/2^-} - E_{9/2^+}$ of Nb nuclei is listed in (d) in MeV. These quantities are given for the four sets of single-particle energies ($P_1N_1$, $P_1N_2$, $P_2N_1$ and $P_2N_2$) of table 1. The experimental numbers for (c) and (d) are from Nuclear Data Group (1973).

| | $n = 50$ | $n = 52$ | $n = 54$ | $n = 56$ |
|---|---|---|---|---|
| $P_1N_1$ (a) | −0·021 | −0·024 | −0·4 | −0·88 |
| (b) | −0·008 | −0·008 | −0·14 | −0·32 |
| (c) | 2·24 | 2·24 | 2·34 | 2·53 |
| (d) | −0·41 | −0·4 | −0·16 | 0·14 |
| $P_1N_2$ (a) | −0·021 | −0·021 | −0·21 | −0·4 |
| (b) | −0·008 | −0·007 | −0·073 | −0·14 |
| (c) | 2·24 | 2·24 | 2·28 | 2·34 |
| (d) | −0·41 | −0·41 | −0·28 | −0·16 |
| $P_2N_1$ (a) | −0·019 | −0·49 | −1·14 | −1·96 |
| (b) | −0·007 | −0·185 | −0·45 | −0·81 |
| (c) | 2·43 | 2·63 | 2·95 | 3·35 |
| (d) | −0·008 | 0·28 | 0·67 | 1·14 |
| $P_2N_2$ (a) | −0·19 | −0·30 | −0·76 | −1·12 |
| (b) | −0·007 | −0·11 | −0·29 | −0·43 |
| (c) | 2·43 | 2·55 | 2·76 | 2·94 |
| (d) | −0·008 | 0·17 | 0·45 | 0·66 |
| EXPT (c) | 2·32 | 2·48 | 2·6 | 3·07 |
| (d) | 0·1 | 0·03 | 0·23 | 0·74 |

For all the four sets of starting single-particle energies used in the calculation we find in table 2 that the $2p_{1/2}$ proton level is lowered in energy relative to the $1g_{9/2}$ level as the neutron number increases. This leads to a systematic increase in the excitation energy of the $5^-$ state in the zirconium nuclei and of the $E_{(1/2)^-} - E_{(9/2)^+}$ in the niobium nuclei with increase of neutron number from 50 to 56. These trends are plotted in figures 1a and 1b respectively and are compared with the experimental numbers. These calculations reproduce the experimental trends reasonably well.

We mention here about the physical mechanism for the relative lowering of the $2p_{1/2}$ proton level (in comparison with the $1g_{9/2}$ level) as more and more neutrons are put into the $2d_{5/2}$ orbit. As the $2d_{5/2}$ neutron level gets occupied, the $2d_{5/2} \rightarrow 1h_{11/2}$ neutron octupole transition amplitude gets larger (see equation (6)) and hence the energy of octupole vibration gets lowered. The dominant octupole-octupole interaction of a $2p_{1/2}$ proton and a $2d_{5/2}$ neutron is depicted in figure 4b, where the proton makes a transition to the $2d_{5/2}$ level and the neutron makes a transition to the $1h_{11/2}$ level. Such a process is absent for $^{90}Zr$ where the $2d_{5/2}$ neutron level is not occupied at all. The process gains in importance as more and more neutrons fill the $2d_{5/2}$ shell. This explains the lowering in energy of the $2p_{1/2}$ proton state as the neutron number increases from $N = 50$ to $N = 56$.

We note from the results presented in table 2 that all the four sets of starting single-particle energies raise excitation energy of the $5^-$ state. They also raise the energy of the $(1/2)^-$ state of niobium nuclei relative to the $(9/2)^+$ state. These are consistent with the experimental trend. However the experimental trend is best reproduced by the parameter sets $P_2N_1$ and $P_2N_2$, corresponding nearly to the $2p_{1/2}$ and $1g_{9/2}$ proton single-particle energies in a Woods-Saxon well (Bertsch 1972).

## 5. Concluding remarks

A dominant effect in the structure of nuclei is the interplay of the single-particle and the collective degrees of freedom (Bohr and Mottelson 1975). As seen in the present work an adequate treatment of both the single-particle and the collective degrees of freedom and the interaction between them is essential to gain an understanding of the energy systematics of the lowlying states in the zirconium and niobium nuclei.

Collective octupole vibration couples with the valence nucleons to modify their self-energies and causes shifts in their energies. Such shifts are particularly significant for the valence proton orbits in the zirconium region. The shell structure in this mass region is such that the $2p_{1/2}$ proton level is appreciably lowered in energy as a result of coupling with the octupole vibration. This energy shift becomes more and more pronounced as the $2d_{5/2}$ neutron level gets filled up. The $1g_{9/2}$ proton level is comparatively less affected by the coupling to the octupole vibration. Experimental systematics of the valence proton energies are reasonably well explained by the energy shifts caused by such coupling with the octupole vibration.

## Acknowledgements

The author is grateful to Professor S P Pandya for sending him a copy of Dr C J Veje's article. This work was partially supported by a grant from the Department of Science and Technology, Government of India.

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