

Regge pole plus cut model for proton-antiproton elastic scattering at collider and tevatron energies

FAZAL-E-ALEEM and MOHAMMAD SALEEM

Centre for High Energy Physics, University of Punjab, Lahore 54590, Pakistan

MS received 1 February 1988; revised 26 April 1988

Abstract. The Regge pole plus cut model has been used to explain the data at the collider energies $\sqrt{s} = 546$ and 630 GeV and the most recent differential cross-section results at $\sqrt{s} = 1.8$ TeV. Predictions of the model at 1.8 and 40 TeV are compared with those of Bourrely *et al.*

Keywords. Proton-antiproton elastic scattering; Regge pole plus cut model; differential cross-section; total cross-section.

PACS Nos 13·85; 13·75

1. Introduction

The high energy proton-proton and antiproton-proton elastic scattering has been extensively studied at the CERN, ISR and the CERN $S\bar{p}pS$ collider. During the last 16 years the measurements of total and elastic differential cross-sections, σ_T and $d\sigma/dt$, the ratio ρ of the real and imaginary parts of the forward scattering amplitude, the local slope parameter B and the ratio σ_{el}/σ_T of integrated to total cross-sections at ISR energies extended to collider and tevatron energies have opened a new era in particle physics. The unexpected increase of the total cross-section σ_T through ISR to collider energies, the appearance of the first dip in $d\sigma/dt$ at ISR energies, the inward movement of the pp dip position with increase in ISR energy, the rising ρ , the significant difference in pp and $\bar{p}p$ elastic differential cross-sections in the vicinity of dip at $\sqrt{s} = 53$ GeV/c, the definite increase in the ratio σ_{el}/σ_T for pp from ISR to collider energies and the transformation of a shallow dip at $\sqrt{s} = 53$ GeV into a shoulder at $\sqrt{s} = 546$ GeV and $\sqrt{s} = 630$ GeV for $\bar{p}p$ elastic scattering make the problem very fascinating. The advent of quantum chromodynamics, usually called QCD, came as a ray of hope for a comprehensive solution of such problems.

QCD is now recognized as the widely accepted theory of strong interactions. It is formulated in close analogy to QED, the highly successful theory of the interactions of photons with electrons. The coupling constant in QED is the fine structure constant α which is $1/137$. The perturbative calculations can therefore always be performed as the higher order terms become smaller and smaller at a rapid pace and the expansion series in QED converges at an early stage. On the other hand, the effective coupling constant for QCD, which unlike QED, is a non-Abelian gauge theory and whose quanta, gluons, carry colours, is given by

$$\alpha(s) = 1/b \ln(-t/\Lambda^2),$$

where b depends upon the group structure. For $SU_c(3)$:

$$b = (1/12\pi)(33 - 2N_f).$$

The effective coupling constant $\alpha(s)$ varies with the momentum a quark transfers to the gluon it emits. At small momentum transfer squared, $-t$, it is about unity. The expansion techniques therefore break down because higher order terms cannot be ignored; the series does not converge. However, for $-t \rightarrow \infty$, the coupling constant goes to zero. Therefore for large values of $-t$, the perturbation techniques may be used even in QCD. It will therefore be very interesting to find out the limits of applicability of perturbative QCD.

In exclusive reactions, the perturbative hard-scattering mechanism and the non-perturbative confinement component compete with each other. QCD tells us that with increasing momentum transfer the non-perturbative component falls faster than the perturbative component at least by a factor of $-t$. Then for large $-t$, the perturbative component will dominate and the process is calculable. Conventionally, this dominance occurs when $-t$ is larger than a few $(\text{GeV}/c)^2$. However, every large $-t$ gluon exchange required by the perturbative mechanism of a given exclusive reaction suppresses the perturbative amplitude by a factor of $\alpha(s)$ (Isgur and Smith 1984). On the other hand, in the non-perturbative component, $\alpha(s)$ is of order unity and therefore no such suppression is produced. Thus, in an exclusive reaction requiring the exchange of n hard gluons, the requirement for perturbative dominance becomes: $(\alpha_s)^n Q^2$, or equivalently $10^{-n} Q^2$ must be larger than a few $(\text{GeV}/c)^2$. This means that asymptopia is simply out of reach in any present or foreseeable exclusive experiment: even at the largest accessible momentum transfers, the non-perturbative confinement component will remain dominant.

Although some physicists do not agree with this interpretation, they also admit (Brodsky and Lepage 1981; Brodsky and Ji 1985) that owing to the technical difficulty of summation over a very large number of Feynman diagrams involved in pp or $\bar{p}p$ elastic scattering, it is not practicable to make calculations for these processes using QCD. The above discussion shows that at present the QCD-based analysis of two-body exclusive reactions is elusive.

As mathematical tools are not available to obtain a QCD based solution of $p(\bar{p})p$ elastic scattering, attempts are to be made for the construction of heuristic models which can explain the experimental data for these processes.

In this paper, we shall consider the pole plus cut model for $\bar{p}p$ elastic scattering at collider energies.

2. Experimental measurements

Let us first describe the experimental measurements at collider energies.

Bozzo *et al* (1984a) presented the results of the measurement of low momentum transfer proton-antiproton elastic scattering at the CERN $S\bar{p}pS$ collider at a centre-of-mass energy $\sqrt{s} = 546 \text{ GeV}$. The differential cross-section was measured in the range $0.03 < -t < 0.32 (\text{GeV}/c)^2$ and is shown in figure 1. The absolute normalization has been obtained through the simultaneous measurement of the total cross-section (Bozzo *et al* 1984b) and implies an overall error of 5%.

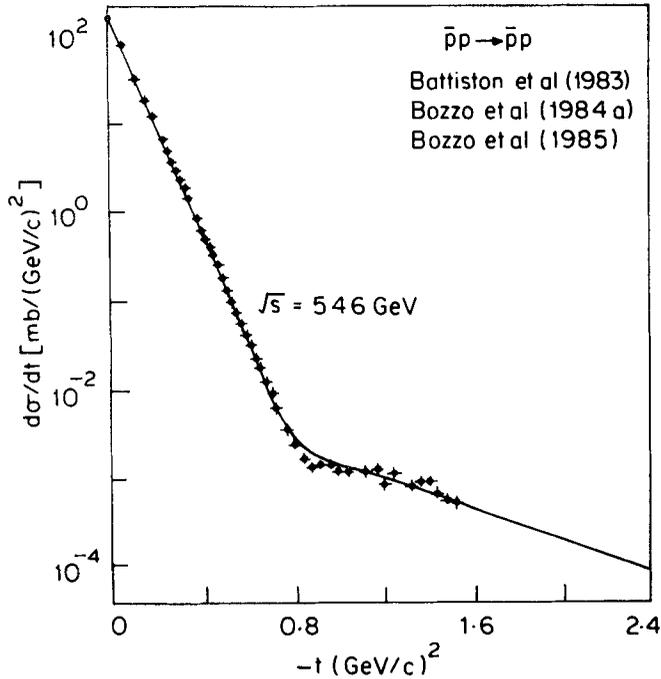


Figure 1. Differential cross-section for $\bar{p}p$ elastic scattering at $\sqrt{s} = 546$ GeV. The curve represents prediction of the model described in the text.

In the region $0.03 < -t < 0.15$ (GeV/c)², the distribution can be well described by a simple exponential $\exp(Bt)$ with a constant slope parameter B . The result of a least-square fit is $B = 15.2 \pm 0.2$ (GeV/c)⁻². For comparison with lower energy data, a fit has also been done in the interval $0.03 < -t < 0.10$ (GeV/c)². The result $B = 15.3 \pm 0.3$ (GeV/c)⁻² confirms that within the given statistics there is no significant quadratic dependence on t in the exponential when fitted in the region $0.03 < -t < 0.15$ (GeV/c)². This value, 15.3 ± 0.3 (GeV/c)⁻², for the slope parameter B at small $-t$ is lower than that obtained from earlier measurements (Battiston *et al* 1982a, 1983) with low statistics (about a thousand events as compared to 10^5 events in Bozzo *et al* 1984a) and is shown in figure 2. An exponential fit to the data in the region $0.15 \leq -t \leq 0.32$ (GeV/c)² gives $B = 14.2 \pm 0.4$ (GeV/c)⁻². It was noticed that the change of slope occurred at $-t = 0.14 \pm 0.02$ (GeV/c)².

Bozzo *et al* (1984b) also reported the results of $\bar{p}p$ total cross-section σ_T and the ratio of the elastic to total cross-section, σ_{el}/σ_T at the CERN $S\bar{p}pS$ collider at the CM energy $\sqrt{s} = 546$ GeV. The method used (Battiston *et al* 1982b) is based on the simultaneous measurement of small angle elastic scattering and of the total inelastic rate and by using a luminosity independent technique. They obtained for the total cross-section the value $\sigma_T = 61.9 \pm 1.5$ mb. An additional systematic error of 1% is due to the uncertainty of the beam momentum. The integrated cross-section σ_{el} was determined as 13.3 ± 0.4 mb while the ratio of the elastic to the total cross-section was found to be $\sigma_{el}/\sigma_T = 0.215 \pm 0.005$. Earlier results with low statistics (Battiston *et al* 1982b; Arnison *et al* 1983) are compatible with this measurement.

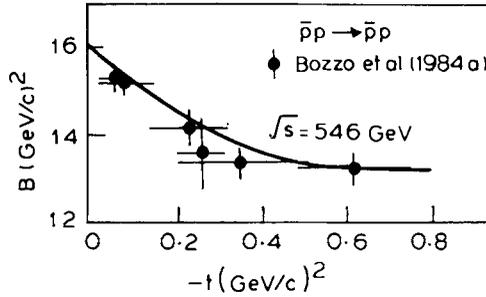


Figure 2. Collider data at $\sqrt{s} = 546$ GeV on the slope parameter B are plotted versus $-t$. The curve represents prediction of the pole plus cut model described in the text.

The total cross-section was also derived by using the machine luminosity. The measurement of the luminosity has an estimated systematic error of 10%, which reflects in a 5% error on σ_T ; the contribution of the error from the elastic rate is in this case negligible. The result obtained was $\sigma_T = 61 \pm 3$ mb, which agrees with the more accurate value from the luminosity independent method.

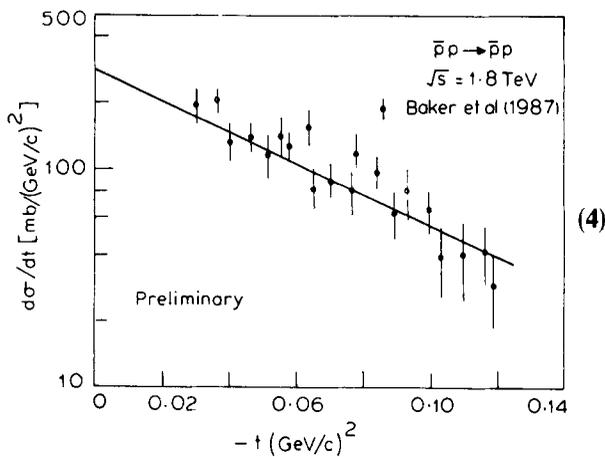
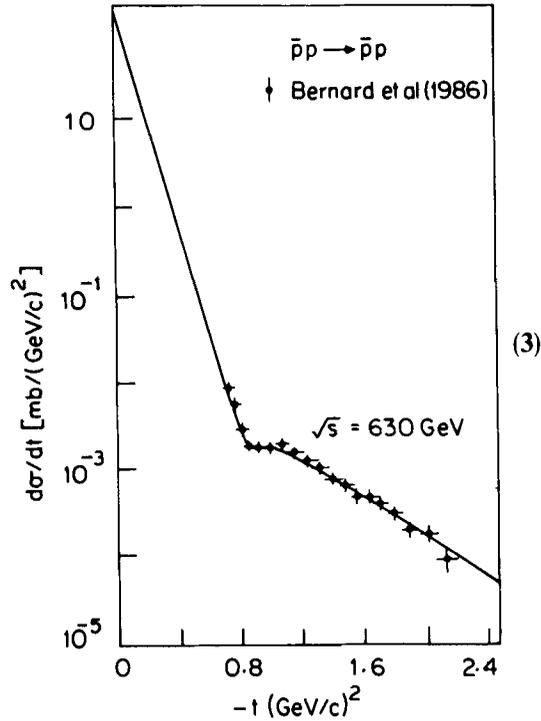
Bozzo *et al* (1985) reported results on $\bar{p}p$ elastic scattering at $\sqrt{s} = 546$ GeV in the momentum transfer range $0.45 < -t < 1.55$ (GeV/c)². The measurements were made at the CERN S $\bar{p}p$ S collider. These data extend to higher momentum transfer their earlier measurements (Battiston *et al* 1983; Bozzo *et al* 1984a) which covered the range up to 0.5 (GeV/c)². The absolute normalization of the t -distribution was obtained by smoothly joining the data to the earlier measurements (Battiston *et al* 1983; Bozzo *et al* 1984a). The results are shown in figure 1 where the errors shown are only statistical. An additional scale error of 10% is present due to the joining of the two sets of data and to the normalization error of Bozzo *et al* (1984a). No dip is observed in the data which show an exponential decrease with momentum transfer and a break at $-t \approx 0.9$ (GeV/c)² followed by a shoulder. Above 0.9 (GeV/c)², the differential cross-section is of the order of $1 \mu\text{b}/(\text{GeV}/c)^2$ and decreases slowly with $-t$.

Bernard *et al* (1986) presented new results at $\sqrt{s} = 630$ GeV in the t -range $0.7 < -t < 2.2$ (GeV/c)². The results of the experiment at $\sqrt{s} = 630$ GeV are shown in figure 3. The errors shown include statistics and uncertainty on the acceptance, which is sizeable at the lowest values of t . The data are affected by an additional scale error of 15% due to uncertainties on the normalization. The results confirm the previous observation (Bozzo *et al* 1985) on the existence of a break in the t -distribution at $-t \approx 0.9$ (GeV/c)² which is followed by a shoulder.

Baker *et al* (1987) presented preliminary data for $\bar{p}p$ elastic scattering obtained at Tevatron Collider at $\sqrt{s} = 1.8$ TeV. They have measured the differential cross-section up to $-t \approx 0.12$ (GeV/c)² and have obtained the experimental value of the slope parameter B as 18.1 ± 1.87 (GeV/c)⁻². Results of their measurements are shown in figure 4.

3. Calculations and discussion

The total number of helicity amplitudes for the reaction $\bar{p}p \rightarrow \bar{p}p$ is sixteen. If the principles of conservation of parity and invariance of time reversal and charge



Figures 3 and 4. Differential cross-section for $\bar{p}p$ elastic scattering at $\sqrt{s} = 630$ GeV and 1.8 TeV. The curves represent predictions of the model described in the text.

conjugation are made use of, then the number of independent helicity amplitude reduces to five. The five independent s -channel helicity amplitudes are usually denoted by $\Phi_1, \Phi_2, \Phi_3, \Phi_4$ and Φ_5 . It is useful to introduce the following linear combinations which correspond asymptotically to the exchange of definite quantum numbers in the t -

channel:

$$\begin{aligned}
 N_0 &= 1/2(\Phi_1 + \Phi_3), \\
 \left. \begin{aligned} N_1 &= \Phi_5 \\ N_2 &= 1/2(\Phi_4 - \Phi_2) \end{aligned} \right\} \text{natural parity exchange,} \\
 \left. \begin{aligned} U_0 &= 1/2(\Phi_1 - \Phi_3) \\ U_2 &= 1/2(\Phi_4 + \Phi_2) \end{aligned} \right\} \text{unnatural parity exchange.}
 \end{aligned}$$

The subscripts of N and U give the amount of t -channel helicity flip. The pomeron P and the Regge trajectories w , f , and A_2 (natural parity exchanges) can contribute to N_0 , N_1 and N_2 whereas U_0 corresponds to A_1 -like exchange and U_2 to π and B exchanges. Since at high energy, the low-lying trajectories do not make any significant contribution, the amplitudes U_0 and U_2 can be ignored. The double-flip amplitude N_2 is neglected to avoid any proliferation of parameters. Thus we have to fit the data for the reaction $\bar{p}p \rightarrow \bar{p}p$ by using two amplitudes N_0 and N_1 only.

At high energy, the pomeron P along with $P \times P$ cut dominates the scattering process. The non-flip amplitude N_0 will contain the Pomeron as well as $P \times P$ cut contribution. Pumpline and Kane (1975) have given a plausible theoretical argument for the existence of diffractive production of helicity flip amplitude. Following these authors we shall assume that the amplitude N_1 is dominated by the pomeron. Thus the two amplitudes may be written as

$$\begin{aligned}
 N_0(s, t) &= [\gamma_{++}(t) \{ -\cos(\pi\alpha_p(t)/2) + i \sin(\pi\alpha_p(t)/2) \}] s^\alpha P^{(t)} \\
 &\quad + [\gamma_c(t) \{ -\cos(\pi\alpha_c(t)/2) + i \sin(\pi\alpha_c(t)/2) \}] s^{\alpha_c(t)} / \ln s, \\
 N_1(s, t) &= \sqrt{-t} [\gamma_{+-}(t) \{ -\cos(\pi\alpha_p(t)/2) + i \sin(\pi\alpha_p(t)/2) \}] s^\alpha P^{(t)},
 \end{aligned}$$

where the subscript c corresponds to the $P \times P$ cut. The units of N_0 and N_1 are $\sqrt{\text{mb}}(\text{GeV})$. The scaling factor s_0 has been taken equal to 1 GeV^2 . The sine terms in the denominators of signature factors have been absorbed in residue functions. The differential and total cross-sections are obtained from the formula:

$$\begin{aligned}
 d\sigma/dt &= 1/s^2 [|N_0(s, t)|^2 + |N_1(s, t)|^2] \text{mb}/(\text{GeV}/c)^2 \\
 \sigma_T &= 4\sqrt{\pi} \sqrt{0.3895/s} [\text{Im} |N_0(s, t=0)|] \text{mb}.
 \end{aligned}$$

Following Saleem and Aleem (1981) and Aleem and Saleem (1983), we find that very good agreement with the experimental data at collider energies is obtained by the following phenomenological choice of parameters.

$$\begin{aligned}
 \gamma_{++} &= 2.35 \exp(4.2t - t^2); \quad \gamma_{+-} = 0.065 \exp(0.15t) \\
 \gamma_c &= 18.4 \exp(9.10t); \quad \alpha_p = 1.069 + 0.1t \\
 \alpha_c &= 1.138 + 0.05t.
 \end{aligned}$$

The rise in total cross-section σ_T with energy cannot be explained by a pomeron with unit intercept. To fit the experimental data at collider energies, we have to choose the pomeron intercept as $\alpha(0) = 1.069$. Of course, the unit intercept of the pomeron is the maximum value permitted by the Froissart bound so as to ensure the maximum

strength of the strong interaction under crossing. Collins *et al* (1974a, b; 1983), Martin (1974) and Chu *et al* (1975) have discussed in detail the repercussions of $\alpha(0) > 1$ on the Froissart bound and have shown that $\alpha_p(0) = 1.07$ violates the Froissart bound only at ultrahigh energies which might not be available in the foreseeable future. Actually $\sigma_T(s) = 27 s^{0.07}$ mb so that $\sigma_T(s) < 60 \log^2 s$ mb until $s \approx 10^{17.5}$ GeV².

Figure 1 shows the differential cross-section $d\sigma/dt$ plotted versus $-t$ at $\sqrt{s} = 546$ GeV. The experimental points have been taken from Bozzo *et al* (1984a, 1985). The curve represents the predictions of the pole plus cut model. The agreement between experimental data and theoretical results is very good. The theoretical predictions at this energy have been extended to $-t = 2.4$ (GeV/c)². The measurements of $d\sigma/dt$ in this region will further endorse the validity of this model.

Figure 2 shows the slope parameter $B(t)$ plotted versus $-t$ at $\sqrt{s} = 546$ GeV. The predictions of our model agree with the measured values of $B(t)$ within experimental errors.

Figures 3 and 4 show the differential cross-section $d\sigma/dt$ plotted versus $-t$ at $\sqrt{s} = 630$ GeV and 1.8 TeV. The experimental points have been taken from Bernard *et al* (1986) and Baker *et al* (1987) respectively. The predictions of the model agree with experiment. The measurements of $d\sigma/dt$ at 630 GeV for small $-t$ have not yet been made. Our model predicts the forward differential cross-section equal to 201.1 mb/(GeV/c)².

The σ_{el} calculated in the range -1.5 to 0 (GeV/c)² comes out as 13.0 mb and 13.4 mb at $\sqrt{s} = 546$ and 630 GeV respectively. The calculated values of σ_{el}/σ_T are 0.215 and 0.217 at these two energies. At $\sqrt{s} = 546$ GeV, the experimental results for σ_{el} and σ_{el}/σ_T are 13.3 ± 0.4 mb and 0.215 ± 0.005 respectively. This shows that theoretical results both for σ_{el} and σ_{el}/σ_T are in excellent agreement with the experimental data at $\sqrt{s} = 546$ GeV. The measurements of the corresponding values at $\sqrt{s} = 630$ GeV and 1.8 TeV have yet to be made.

The ratio ρ of the real and imaginary parts of the forward scattering amplitude at collider energies has been evaluated as 0.171, 0.172 and 0.174 at $\sqrt{s} = 546, 630$ GeV and 1.8 TeV, respectively. The experimental measurements of ρ at $\sqrt{s} = 546, 630$ GeV and 1.8 TeV will therefore further test the validity of this model.

The predictions of the model for differential cross-sections at $\sqrt{s} = 5$ and 40 TeV are shown in figure 5 and exhibit very interesting features. For instance, the shoulder which occurs at collider energies $\sqrt{s} = 546$ and 630 GeV disappears when an energy of 40 TeV is approached, and we do not observe any structure except a break in the vicinity of $-t = 0.8$ (GeV/c)².

In figure 6, theoretical predictions for the slope parameter B at $\sqrt{s} = 630$ GeV and 1.8, 5 and 40 TeV have been exhibited. The final data at 1.8 TeV will soon be available at the Fermilab Tevatron collider. The ultrahigh energies of 5 and 40 TeV will be attained at the Serpukhov UNK and the superconducting super collider (SSC), respectively.

4. Comparison with other models

It would be interesting to compare the results of our pole plus cut model with the predictions of impact picture representation of Bourrely *et al* (1984) for the next generation of collider machines with the highest energy $\sqrt{s} = 40$ TeV. Prediction of both the models for the total cross-section, ratio of elastic and total cross-sections, ratio

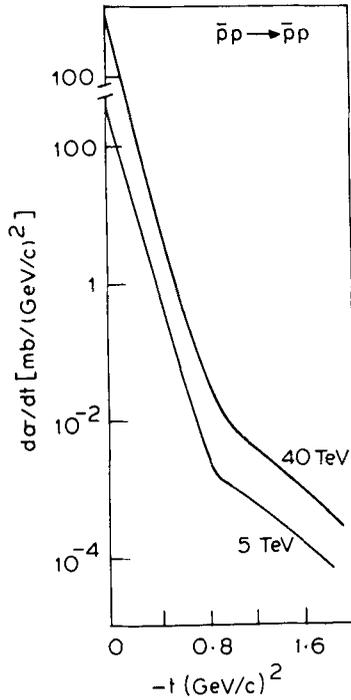


Figure 5. Predictions of the model described in the text for the differential cross-sections at 5 and 40 TeV.

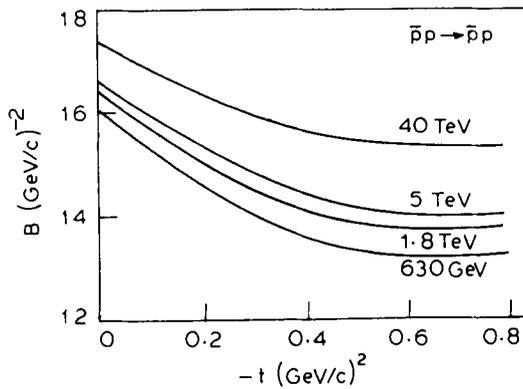


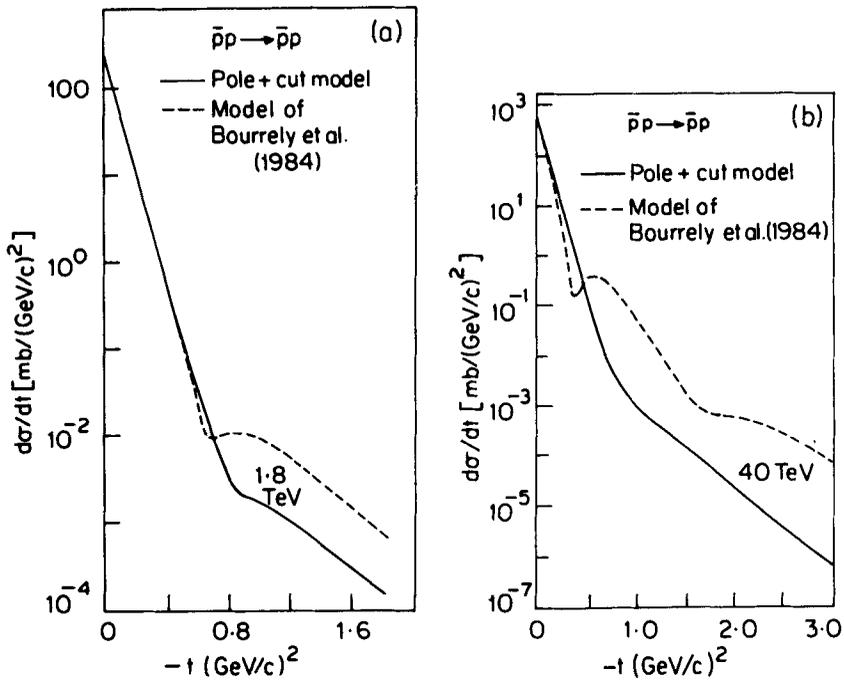
Figure 6. Predictions of the model described in the text for the slope parameter B at 630 GeV and 1.8, 5 and 40 TeV.

of the real and imaginary parts of the forward scattering amplitudes, position of the first minimum and the slope between $0 < -t < 0.15 (\text{GeV}/c)^2$ are given in table 1.

It can be seen from the table that at $\sqrt{s} = 0.55 \text{ TeV}$, the predictions of both the models for total cross-section σ_T , the ratio σ_{e1}/σ_T , the slope for $0 < -t < 0.15 (\text{GeV}/c)^2$ and the point/region where the shallow dip/shoulder occurs agree with experiment within errors. However in going to $\sqrt{s} = 40 \text{ TeV}$, the σ_T and the slope B in our model

Table 1. Comparison of the pole plus cut model and the model of Bourrely *et al* (1984) with the experimental data for various characteristics of $\bar{p}p$ elastic scattering.

\sqrt{s}	0.55 TeV			1.8 TeV		40 TeV	
	Pole + cut model	Model of Bourrely <i>et al</i>	Experimental value	Pole + cut model	Model of Bourrely <i>et al</i>	Pole + cut model	Model of Bourrely <i>et al</i>
$\sigma_T(\text{mb})$	60.6	62	61.9 ± 1.5	72.2	74.8	116.33	121.2
σ_{el}/σ_T	0.215	0.21	0.215 ± 0.005	0.25	0.23	0.37	0.3
$\rho(s, t=0)$	0.171	0.13	—	0.174	0.128	0.174	0.114
Position of first dip/break/shoulder in slope (GeV/c) ²	≈ 0.85	≈ 0.85	0.85	0.8	0.70	0.8	0.37
Slope (GeV/c) ⁻² for $0 < -t < 0.15$ (GeV/c) ²	15.5	15.6	15.3 ± 0.3	15.9	16.3	17.40	20.7


Figure 7. Comparison of the predictions of the model described in the text and the model of Bourrely *et al* (1984) for the differential cross-section at 1.8 and 40 TeV.

increases more slowly while σ_{e1}/σ_T rises more rapidly as compared with the predictions of the model proposed by Bourrely *et al* (1984). For $\rho(s, t = 0)$, our model shows a slight increase while the model of Bourrely *et al* (1984) shows a decrease as the energy increases from $\sqrt{s} = 0.55$ TeV to 40 TeV.

We also observe from figure 7 that in the pole plus cut model the shoulder in $d\sigma/dt$ which occurs at collider and Tevatron energies $\sqrt{s} = 546, 630$ GeV and 1.8 TeV disappears when SSC energy is approached. It turns into a break in the vicinity of $-t = 0.8$ (GeV/c)². This result is substantially different from that of Bourrely *et al* (1984) who predict that a shallow dip will not only persist but would also continue to move towards $t = 0$ with rise in energy. They predict a first dip at $-t = 0.7$ and 0.37 (GeV/c)² for $\sqrt{s} = 1.8$ and 40 TeV and a second dip at $-t = 1.67$ (GeV/c)² for $\sqrt{s} = 40$ TeV.

The value of the break/first dip as determined by experiment at Fermilab Tevatron and at SSC would be sufficient to discard one (or both) of these models.

References

- Aleem F and Saleem M 1983 *Phys. Rev.* **D27** 2068
 Arnison A *et al* 1983 *Phys. Lett.* **B128** 336
 Baker W *et al* 1987 Workshop "From colliders to superconductors", University of Wisconsin-Madison
 Bernard D *et al* 1986 *Phys. Lett.* **B171** 142
 Brodsky S and Lepage G P 1981 *Phys. Rev.* **D24** 1808
 Brodsky S and Ji C 1985 SLAC-PUB-3747
 Battiston R *et al* 1982a *Phys. Lett.* **B115** 333
 Battiston R *et al* 1982b *Phys. Lett.* **B128** 126
 Battiston R *et al* 1983 *Phys. Lett.* **B127** 472
 Bourrely C *et al* 1984 *Nucl. Phys.* **B247** 15
 Bozzo M *et al* 1984a *Phys. Lett.* **B147** 385
 Bozzo M *et al* 1984b *Phys. Lett.* **B147** 392
 Bozzo M *et al* 1985 *Phys. Lett.* **B155** 197
 Chu S Y *et al* 1975 *Phys. Rev.* **D13** 2985
 Collins P D B *et al* 1974a *Nucl. Phys.* **B80** 145
 Collins P D B *et al* 1974b *Nucl. Phys.* **B83** 241
 Collins P D B *et al* 1983 *Nucl. Phys.* **B43** 171
 Isgur N and Smith C L 1984 *Phys. Rev. Lett.* **52** 1080
 Martin A 1974 *Nucl. Phys.* **B77** 226
 Pumpline J and Kane G L 1975 *Phys. Rev.* **D11** 1183
 Saleem M and Aleem F 1981 *Phys. Rev.* **D24** 2403
 Saleem M and Aleem F 1983 *Phys. Rev.* **D27** 2235 (E)