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Double layers in a field-aligned current driven plasma

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Abstract. The solutions of small amplitude ion acoustic double layers in current-carrying plasma in the presence of the magnetic field have been presented. The electron beam along the magnetic field has only been considered and the drift velocity assumed to be less than the electron thermal velocity. The velocity, the width and the potential of such DLs are calculated.

Keywords. Ion acoustic double layers; electron beam; electron drift velocity.

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1. Introduction

Double layers (DLs) in a current-driven plasma in the presence of magnetic field have not been explored much, although DLs in unmagnetized plasma have been studied both theoretically and experimentally (Block 1972; Sato 1982; Schamel 1983; Sekar and Saxena 1984; Bujarbarua and Goswami 1985; Sutradhar and Bujarbarua 1987) extensively. Recently both weak ion acoustic DL and weak electron acoustic DL have been studied (Goswami and Bujarbarua 1985, 1987). Swift (1976) proposed a theory of laminar electrostatic shock, propagating obliquely to the magnetic field, supported by current-driven electrostatic ion cyclotron instability. Mozer *et al* (1977) observed large electric field in the auroral zone due to the ion cyclotron drift, which was of the order of electron thermal speed. Barnes *et al* (1985) showed in their simulation that weak DLs are formed in current-driven ion acoustic turbulence when the plasma is strongly magnetized and when the drift velocity is less than the electron thermal speed.

Recently Sutradhar Das and Bujarbarua (1988) studied the formation of weak ion acoustic DLs in an unmagnetized plasma, with electron drift velocity less than the electron thermal velocity. In the present study we have solved the small amplitude DLs in a plasma consisting of an electron beam in the presence of a magnetic field. However we have considered the magnetic field only along the direction of the beam as also the electron drift velocity less than the electron thermal velocity.

2. Basic equations

We consider a homogeneous and infinite plasma consisting of an electron beam in the presence of the magnetic field along the direction of the electron beam, and treat the ions, the beam electrons and the background plasma electrons as cold, warm and hot respectively. We describe the ions and the beam electrons by the fluid equation and assume a single Boltzmann model for the plasma electrons. The constant magnetic field B_0 is assumed to be in the z-direction and the ion acoustic wave is assumed to be propagating in the x-z plane. Hence we neglect the variation of all quantities in the y-direction. The fluid equations for ions in such a system are given by

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i u_x) + \frac{\partial}{\partial z}(n_i u_z) = 0, \tag{1}$$

$$\frac{\partial u_x}{\partial t} + \left(u_x\frac{\partial}{\partial x} + u_z\frac{\partial}{\partial z}\right)u_x + \frac{\partial \phi}{\partial x} = u_y, \tag{2}$$

$$\frac{\partial u_y}{\partial t} + \left(u_x\frac{\partial}{\partial x} + u_z\frac{\partial}{\partial z}\right)u_y = -u_x, \tag{3}$$

$$\frac{\partial u_z}{\partial t} + \left(u_x\frac{\partial}{\partial x} + u_z\frac{\partial}{\partial z}\right)u_z + \frac{\partial \phi}{\partial z} = 0. \tag{4}$$

We assume that the electron beam is moving in the z-direction, that is in the direction of the magnetic field. The fluid equations for the electron beam are given by

$$\frac{\partial n_b}{\partial t} + \frac{\partial}{\partial z}(n_b v_z) = 0, \tag{5}$$

$$\mu n_b\frac{\partial v_b}{\partial t} + \mu n_b v_b\frac{\partial u_b}{\partial z} - n_b\frac{\partial \phi}{\partial z} + \theta\frac{\partial n_b}{\partial z} = 0. \tag{6}$$

In equations (1)–(6), the ion velocity u and the beam velocity v_b are normalized by the ion acoustic speed $C_s(=(T_e/m_i)^{1/2})$. The ion density n_i and the beam density n_b are normalized by the equilibrium plasma electron density n_{e0}. The electric potential ϕ, the space co-ordinate x and time t are normalized by T_e/e, the ion Larmour radius $\rho(=C_s/\Omega_i)$ and Ω_i^{-1} respectively. Here $\Omega_i(=eB_0/m_{ic})$ is the ion gyrofrequency $\mu = m_e/m_i$ and $\theta = T_b/T_e$, where $m_e(m_i)$ is the mass of the electron (ion) and $T_b(T_e)$ is the temperature of the beam electron (plasma electron).

In solving equations (1)–(6), we have used Galiliean transformation $\xi = k_x x + k_z z - Mt$, where k_x and k_z are the direction cosine along x and z directions respectively and they are related by the equation $k_x^2 + k_z^2 = 1$. Integrating all the transformed equations and using the boundary condition $\phi = 0$, $v_i = 0$, $v_b = v_0$, $n_i = (1+\alpha)$, $n_b = \alpha$ at $\xi \to \alpha$, where $\alpha = n_{b0}/n_{e0}$, and after some proper substitution we get the following equations

$$n_b = \alpha\left[1 + \gamma_M\frac{\phi}{\theta} + \gamma_M\frac{\phi^2}{\theta^2} + \gamma_M\frac{\phi^3}{\theta^3}\right], \tag{7}$$

$$\frac{1}{2}\frac{\partial}{\partial \phi}\left(A\frac{\partial \phi}{\partial \xi}\right)^2 = A\left[\left(\frac{n_i}{1+\alpha} - 1\right) - \frac{k_z^2}{M^2(1+\alpha)^2}n_i I(\phi)\right] = -\frac{\partial v(\phi)}{\partial \phi}, \tag{8}$$

where

$$\gamma_M = -\frac{2\theta}{\mu k_z(v_0 - M/k_z)^2 - 2\theta}, \tag{9}$$

$$I(\phi) = \int_0^{\phi} n_i \partial\phi, \tag{10}$$

and

$$A = \left[1 - \frac{M^2}{n_i^3}(1+\alpha)^2 \frac{\partial n_i}{\partial \phi}\right]. \tag{11}$$

In the above equations $V(\phi)$ is the classical potential and since we have considered the small amplitude limit i.e. $\phi \ll 1$, we have retained all the terms upto ϕ^3 and neglected the higher terms. With this consideration we express the plasma electron density, which follows the Boltzmann relation, as

$$n_p(\phi) = 1 + \phi + \tfrac{1}{2}\phi^2 + \tfrac{1}{6}\phi^3. \tag{12}$$

Therefore, the total electron density $n_e = n_p + n_b$ is given by

$$n_e = (1+\alpha) + A_1\phi + A_2\phi^2 + A_3\phi^3, \tag{13}$$

where

$$A_1 = 1 + (\alpha\gamma_M/\theta), \tag{14a}$$

$$A_2 = \tfrac{1}{2} + (\alpha\gamma_M/\theta^2), \tag{14b}$$

$$A_3 = \tfrac{1}{6} + (\alpha\gamma_M/\theta^3). \tag{14c}$$

To the lowest order, we can write from (11) and (13), $A_1 \simeq 1 - M^2$.

In this system we assume quasineutrality, which is valid when the electron Debye length is much smaller than the ion gyro-radius. Therefore we substitute the total electron density n_e from (13) for the ion density n_i in (8) which gives

$$-\frac{\partial V}{\partial \phi} = \frac{A}{(1+\alpha)}[B_1\phi + B_2\phi^2 + B_3\phi^3], \tag{15}$$

where

$$B_1 = A_1 - (1+\alpha)(k_z^2/M^2), \tag{16a}$$

$$B_2 = A_2 - \tfrac{3}{2}A_1(k_z^2/M^2), \tag{16b}$$

$$B_3 = A_3 - (\tfrac{1}{2}A_1^2 + \tfrac{4}{3}A_2)(k_z^2/M^2). \tag{16c}$$

Integrating (20) and imposing boundary condition $V(0) = 0$ we get the classical potential as

$$-V(\phi) = \frac{A}{(1+\alpha)}(\tfrac{1}{2}B_1\phi^2 + \tfrac{1}{3}B_2\phi^3 + \tfrac{1}{4}B_3\phi^4). \tag{17}$$

Expressing the coefficients B_1 and B_2 in terms of B_3, using the DL condition, viz. $V(\psi) = \partial V/\partial\phi|_{\phi=\psi} = 0$, where ψ is the amplitude of the DL, we can express $V(\phi)$ as

$$-V(\phi) = \frac{A}{4(1+\alpha)} B_3 \phi^2 (\phi - \psi)^2. \tag{18}$$

Putting (18) in the energy law $\frac{1}{2}\phi'^2 + V(\phi) = 0$, we get the DL solution given by

$$\phi = \tfrac{1}{2}\psi[1 - \tanh k\xi], \tag{19}$$

where

$$k = \left[\frac{B_3}{8A(1+\alpha)}\right]^{1/2} \psi. \tag{20}$$

Equation (19) represents DL provided B_3 is positive.

We shall now calculate the velocity M of the DLs. The general expressions for M are rather difficult to solve as we get a fourth order algebraic equation in M. However assuming $\gamma_M \simeq 1$, for which the necessary condition should be $\mu k_z(v_0 - M/k_z)^2 \ll 2\theta$, M can easily be calculated and is given by

$$M = k_z \left\{\frac{(1+\alpha)}{(1+\alpha/\theta)}\right\}^{1/2} \left[1 - \frac{1}{3(1+\alpha/\theta)}\left\{\frac{1}{2} + \frac{\alpha}{\theta^2} - \frac{3}{2}\frac{(1+\alpha/\theta)}{(1+\alpha)}\right\}\tfrac{1}{2}\psi\right]. \tag{21}$$

For $\alpha = 0$, the velocity M of DLs reduces to that obtained by Goswami and Bujarbarua (1986).

Putting the linear value of M from (21) in (16c) B_3 can be rewritten, for $\gamma_M \simeq 1$, as

$$B_3 = \frac{1}{6} + \frac{\alpha}{\theta^3} - \left\{\tfrac{1}{2}(1+\alpha/\theta)^2 + \tfrac{4}{3}\left(\frac{1}{2} + \frac{\alpha}{\theta^2}\right)\right\}\frac{(1+\alpha/\theta)}{(1+\alpha)}. \tag{22}$$

Also using the linear value of M in the expression $A \simeq 1 - M^2$ we have from (25)

$$k = \left[\frac{B_3}{8(1+\alpha)\left\{1 - \dfrac{(1+\alpha)}{(1+\alpha/\theta)}k_z^2\right\}}\right]^{1/2} \psi. \tag{23}$$

The thickness of this DL is given by $\Delta = 2d$, where

$$d = 1/k = \left[\frac{8(1+\alpha)\left\{1 - \dfrac{(1+\alpha)}{(1+\alpha/\theta)}k_z^2\right\}}{B_3}\right]^{1/2} \Big/ \psi. \tag{24}$$

3. Discussion

We now discuss the formation of this type of DL for different values of α and θ. We have numerically calculated the values of B_3 and width d for three different cases of α and θ which are (1) $\alpha = \theta < 1$ (2) $\alpha < \theta < 1$ and (3) $\theta < \alpha < 1$ and found that DLs exist in all the three cases. But for cases (1) and (2), DL exists only for some specific values of α and θ whereas for (3) DL exists for any value of α and θ.

In the case (2) i.e. $\alpha < \theta < 1$, $\theta = 0{\cdot}05$ is the most preferable value since the calculated values of DLs thickness are in good agreement with the thickness found by Temerin *et al* (1982).

The width d depends on the value of k_z i.e. on the angle between the magnetic field and the direction of propagation of the DL. It is found that the width increases with the decrease of the value of k_z for fixed values of the amplitude. We have shown graphically the dependence of width d as on k_z, ψ, α and θ in figures 1–4.

Recently Goswami and Bujarbarua (1986) studied the propagation of ion acoustic DLs in a magnetized plasma and found that DLs can exist when the electrons are described by a two-Boltzmann distribution. In this paper we have considered a single

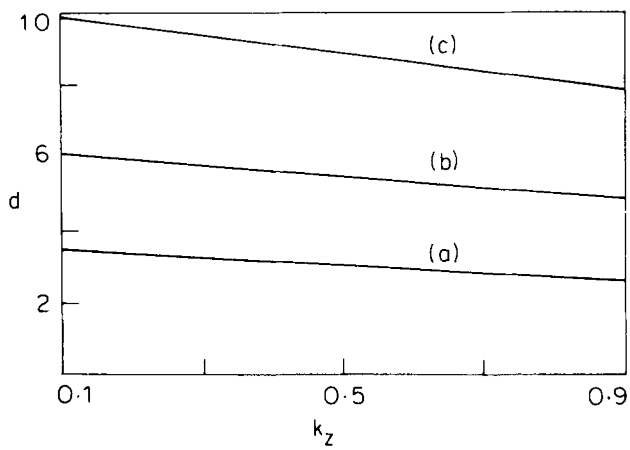

Figure 1. Variation of DL half width d with k_z for different values of beam density α for the case $\alpha = \theta$ and for the fixed value of the amplitude $\psi = 0{\cdot}1$. Curves a, b, c, correspond to $\alpha = 0{\cdot}1$, 0·15, 0·2 respectively.

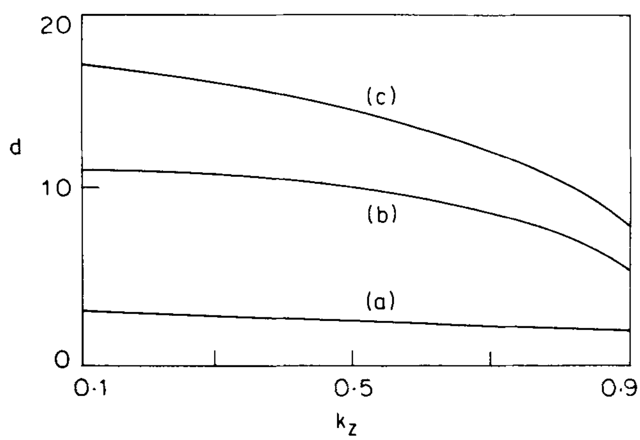

Figure 2. Variation of DL half width d with k_z for different values of beam density α for the case $\alpha < \theta < 1$ and for the fixed value $\theta = 0{\cdot}05$ and $\psi = 0{\cdot}1$. Curves a, b, c correspond to $\alpha = 0{\cdot}01$, 0·001 and 0·0005.

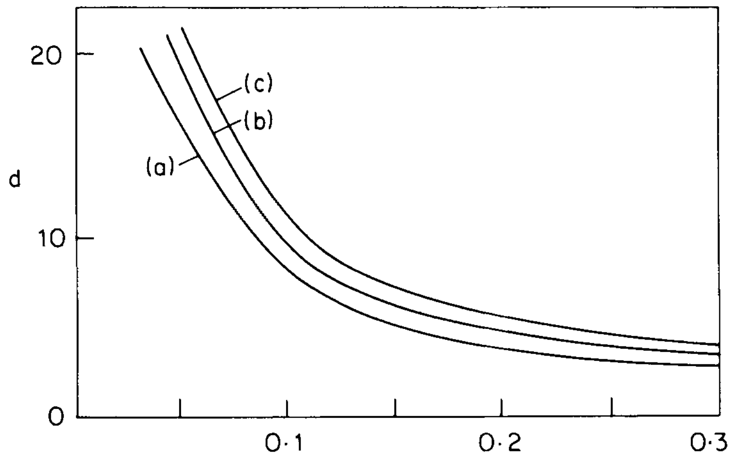

Figure 3. Variation of DL half width d with the amplitude ψ for different values of k_z for the case $\alpha < \theta < 1$ and for the fixed value $\alpha = 0{\cdot}001$ and $\theta = 0{\cdot}05$. Curves a, b, c correspond to $k_z = 0{\cdot}7, 0{\cdot}5$ and $0{\cdot}3$.

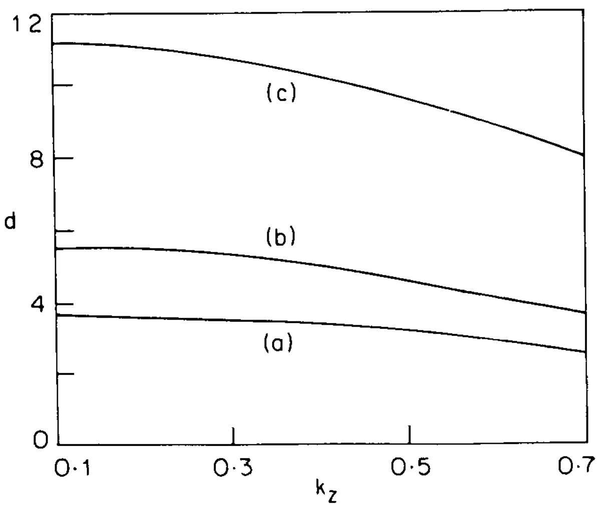

Figure 4. Variation of DL half width d with k_z for different values of the amplitude ψ for the case $\alpha < \theta < 1$ for the fixed value of $\alpha = 0{\cdot}001, \theta = 0{\cdot}05$. Curves a, b, c correspond to $\psi = 0{\cdot}3, 0{\cdot}2$ and $0{\cdot}1$.

Boltzmann distribution for the plasma electrons together with a beam of electrons moving parallel to the magnetic field. It can be seen from (21) that since $\theta = T_b/T_e < 1$, the presence of the electron beam reduces the velocity of the double layers.

In this paper we have assumed the quasineutrality condition and it is seen that the assumption of the quasineutrality amounts to the neglect of factor of the order of $(V_A/c)^2$ where V_A is the Alfven speed. Numerical estimates show that (V_A/c) is about 10^{-3} for laboratory plasma (Fujita *et al* 1984) and about 5×10^{-2} for auroral plasma (Goertz 1974). Therefore our assumption of quasineutrality is justified for the above physical situations. It may be noted that recently Bharuthram and Shukla (1985) investigated the dynamics and structure of multi-dimensional ion acoustic solitons and double layers in a magnetized plasma incorporating the departures from quasi-neutrality. However, their results show that departures from quasi-neutrality do not give any new effects.

It is to be noted that to obtain the DL solution we have assumed $\gamma_M \simeq 1$ which gives the condition $\mu k_z(v_0 - M/k_z)^2 \ll 2\theta$ and to satisfy this condition V_0 must be less than the electron thermal velocity.

Finally, although we have not studied the stability of the DLs discussed above, Schamel (1983) studied the stability of the slow ion acoustic DLs and found that such DLs are stable in one dimension but unstable for two or three dimensions. It may be conjectured that this result may hold good for the DLs discussed above.

The important result obtained in this paper is that DLs can be formed in a plasma when the electron drift v_0 along the magnetic field is less than or nearly equal to the electron thermal velocity.

We conclude that small amplitude ion acoustic DLs are formed in the field-aligned current-carrying plasma discussed in this paper and may be applicable in the laboratory plasma as well as in the auroral region of the space plasma.

References

Barnes C, Hudson M K and Lotko W 1985 *Phys. Fluids* **28** 1055
Bharuthram R and Shukla P K 1985 *Phys. Scr.* **34** 732
Block L P 1972 *Cosmic Electrodyn.* **3** 349
Bujarbarua S and Goswami K S 1985 *J. Geophys. Res.* **90** 7612
Fujita H, Matsuo K and Yagura S 1984 *Plasma Phys.* **26** 959
Goertz C K 1979 *Rev. Geophys. Space Phys.* **17** 418
Goswami K S and Bujarbarua S 1985 *Phys. Lett.* **A108** 149
Goswami K S and Bujarbarua S 1986 *Phys. Fluids* **29** 714
Goswami K S and Bujarbarua S 1987 *Pramana – J. Phys.* **28** 399
Mozer F S, Carlson C W, Hudson M K, Torbert R B, Parady B and Yalleau J 1977 *Phys. Rev. Lett.* **38** 292
Sato N 1982 Symp. on plasma double layers, Riso National Laboratory, June 16–18, p. 116, Riso, Denmark
Schamel H 1983 *Z. Naturforsch.* **38** 1170
Sekar A N and Saxena Y C 1984 *Pramana – J. Phys.* **23** 351
Sutradhar S and Bujarbarua S 1987 *J. Phys. Soc. Jpn.* **56** 139
Sutradhar Das S and Bujarbarua S 1988 *J. Phys. Soc. Jpn.* **57** 499
Swift D W 1976 *J. Geophys. Res.* **81** 3935
Temerin M, Cerney K, Lotko W and Mozer F S 1982 *Phys. Rev. Lett.* **43** 1318