Pramāṇa - J. Phys., Vol. 30, No. 6, June 1988, pp. 521–527. © Printed in India.

Post-collision interaction effects on the triple-differential cross-section for the ionization of helium by fast positrons

SADHANA SHARMA and M K SRIVASTAVA
Department of Physics, University of Roorkee, Roorkee 247 667, India

MS received 22 September 1987; revised 4 March 1988

Abstract. Triple differential cross-sections for the ionization of helium by fast positrons are calculated in a 'correlated' first Born approximation supplemented by the inclusion of post-collision interaction effects. The results are analysed with respect to electron-helium experimental data of Jung and coworkers in coplanar asymmetric geometry.

Keywords. Triple differential cross-section; coplanar asymmetric geometry; correlated first Born approximation; post-collision interaction effects.

PACS Nos 34·80; 34·90

1. Introduction

The triple differential cross-sections (TDCS) for the electron impact ionization of atoms are known to be quite sensitive to the choice of the scattering model used. This is particularly so in the case of asymmetric Ehrhardt type kinematical arrangement. The theory is generally not in satisfying agreement with experiment even at high energies. For the fast incident electrons (energy E_0) and a fixed asymmetric partitioning of energy the angular distribution of the slow electron (energy E_b), at a fixed small scattering angle θ_a for the fast electron (energy E_a) shows a two-peaked structure: a peak (binary peak) near the momentum transfer direction and another subsidiary one (recoil peak) near the opposite direction.

The developments in the theory beyond the first Born approximation have proceeded along the following lines: (i) distorted wave Born approximation (McDowell *et al* 1973; Baluja and Taylor 1976; Madison *et al* 1977; Bransden *et al* 1978, 1979; Smith *et al* 1979; Tweed 1980). (ii) Coulomb-projected Born approximation (Geltman 1971, 1974; Geltman and Hidalgo 1971, 1974; Hidalgo and Geltman 1972; Schulz 1973; Schubert *et al* 1979; Lal *et al* 1979; Pathak and Srivastava 1980, 1981; Ghosh *et al* 1984a, b). (iii) Second Born (B2) approximation which has been shown to essentially reproduce the main characteristic features (angular positions of the binary and recoil peak maxima and the ratio of binary to recoil peak intensities) of the TDCS angular distribution (Byron *et al* 1980, 1982; Ehrhardt *et al* 1982). (iv) Eikonal-Born series approach (Byron *et al* 1983, 1984, 1985) to consistently include all contributions up to order k^{-2} in the direct scattering amplitude. (v) Modified Glauber (MG) approximation (Byron and Joachain 1975; Gien 1976) which has recently been used to include still higher order ($n > 3$) terms of the direct scattering amplitude. The MG approximation is found to further improve the B2 results in the cases where the scattering angle θ_a

is not too small (Baliyan and Srivastava 1985, 1986a, b). (vi) Improvement in the description of the initial and/or final states of the target (Franz and Klar 1986; Sharma and Srivastava 1988, to be referred as I; Furtado and O'Mahony 1987; Srivastava and Sharma 1987). This leads to a considerable improvement in the binary to recoil peak intensity ratio even in a first order calculation. (vii) Incorporation of the effects of the post-collision interaction (PCI) between the three charged particles in the final state (Klar and Franz 1986). They alter the angles and energies at which the scattered and ejected electrons are detected from their values immediately after ionization. Their inclusion is found to shift the binary and recoil peaks to larger angles of ejection and some improvement in recoil peak size in the case of electron impact.

In this paper we study the PCI effects in the case of positron impact ionization of helium in the coplanar asymmetric kinematics. Very few theoretical studies have been carried out on positron impact ionization of helium (Joachain 1984; Ghosh *et al* 1985a, b; Basu *et al* 1985; Mandal *et al* 1986; Saxena and Srivastava 1987). There are no experimental data. A comparison of positron-induced and electron-induced ionization results is, however, expected to provide some additional insight into this process. The positron case differs from the corresponding electron case in the interference between the projectile-target nucleus and the projectile-target electron interactions and the PCI effects are expected to be very different in the two cases. The scattering amplitude is calculated in the 'correlated' first Born (CB1) approximation by following the procedure given in I. This by itself leads to identical results for incident electrons and positrons:

The method of calculation outlined in §2 and §3 contains our results and discussion.

2. Method

The direct scattering amplitude in the case of CB1 approximation is given by

$$f_{\mathrm{CB1}} = (2\pi)^{-1}\Big\langle \exp(i\mathbf{k}_a\cdot\mathbf{r}_0)\phi^{(-)}_{\mathbf{k}_b}(\mathbf{r}_1,\mathbf{r}_2) \Big| \frac{1}{r_{01}} + \frac{1}{r_{02}} \Big| \exp(i\mathbf{k}_0\cdot\mathbf{r}_0)\phi_i(\mathbf{r}_1,\mathbf{r}_2)\Big\rangle, \qquad (1)$$

where $\mathbf{k}_0$, $\mathbf{k}_a$ and $\mathbf{k}_b$ are the momenta of the incident, scattered and ejected particles respectively, $\mathbf{r}_0$ is the position vector of the incident positron, $\mathbf{r}_1$ and $\mathbf{r}_2$ are the position vectors of the target electrons, ϕ_i is the ground state wavefunction of helium atom which is taken to be the analytical fit to the Hartree-Fock wavefunction given by Byron and Joachain (1966):

$$\phi_i(\mathbf{r}_1,\mathbf{r}_2) = u(\mathbf{r}_1)u(\mathbf{r}_2)$$

with

$$u(\mathbf{r}) = = \sum_{i=1}^{2} \gamma_i \exp(-\alpha_i r),$$

$$\alpha_1 = 1{\cdot}41, \quad \alpha_2 = 2{\cdot}61,$$

$$\gamma_1 = 0{\cdot}73485, \quad \gamma_2 = 0{\cdot}58715.$$

The final state wavefunction of the helium sub-system is taken to be the symmetrized product of the He^+ ground-state wavefunction

$$v(\mathbf{r}) = (8/\pi)^{1/2} \exp(-2r)$$

for the bound electron with the continuum wavefunction $\Psi_{\mathbf{k}_b}^{(-)}$ (orthogonalized to the ground state orbital u) for the ejected electron with momentum $\mathbf{k}_b$:

$$\phi_{\mathbf{k}_b}^{(-)}(\mathbf{r}_1, \mathbf{r}_2) = \frac{1}{\sqrt{2}} [\Psi_{\mathbf{k}_b}^{(-)}(\mathbf{r}_1) v(\mathbf{r}) + v(\mathbf{r}_1) \Psi_{\mathbf{k}_b}^{(-)}(\mathbf{r})],$$
$$\Psi_{\mathbf{k}_b}^{(-)}(\mathbf{r}) = \psi_{\mathbf{k}_b}^{(-)}(\mathbf{r}) - \langle u | \psi_{\mathbf{k}_b}^{(-)} \rangle u(\mathbf{r}).$$

The continuum electron wavefunction $\psi_{\mathbf{k}_b}^{(-)}$ is obtained by solving the Schrödinger equation for $\ell = 0, 1, 2$ in the field of the static charge distribution of the residual helium ion and is taken to be the usual Coulomb wave for $\ell \geqslant 3$. It is orthogonalized to the target ground state orbital. The details of evaluating equation (1) are given in I.

The positronium formation channel, in the highly asymmetric geometry in which we are interested here, is not expected to contribute significantly and, therefore, has not been considered.

PCI effects have been included by following the method of Klar and Franz (1986). The interaction responsible for the energy transfer to the target atom is a short range interaction. After the collision, the long range Coulomb interaction leads to exchange of energy and angular momentum between the two escaping particles. This Coulomb interaction produces a deflection of their trajectories. This deflection and the change in energies have been estimated classically (Popov and Benyoun 1981; Popov and Erokhin 1983; Klar *et al* 1986; Klar and Franz 1986). The boundary values r_{0a} and r_{0b} from where on the PCI effects are estimated for the fast and the slow electron respectively have been chosen to be equal to 3·5 Å and 0·2254 Å. The value $r_{0b} =$ 0·2254 Å corresponds to the distance where the probability of finding the bound electron is maximum. The distance r_{0a} should be fairly large to correspond to a peripheral collision and has been chosen to lead to best fit to the electron-helium experimental data of Jung *et al* (1985). See Klar and Franz (1986) for details of calculation.

The triple differential cross-section is given by

$$\frac{d^3\sigma}{d\Omega_a d\Omega_b dE_b} = (2\pi)^{-3} \frac{k_a k_b}{k_0} |f_{\mathrm{CB1}}|^2. \qquad (2)$$

3. Results and discussion

Figures 1–4 show our CB1-PCI results at $E_0 = 600\,\mathrm{eV}$ incident positron energy for $\theta_a = 4°$, $E_b = 2{\cdot}5\,\mathrm{eV}$; $\theta_a = 8°$, $E_b = 2{\cdot}5\,\mathrm{eV}$; $\theta_a = 4°$, $E_b = 10\,\mathrm{eV}$; $\theta_a = 8°$, $E_b = 10\,\mathrm{eV}$. They also contain the corresponding CB1 and CB1-PCI results for the electron impact case. All these results are compared with recent absolute measurements of Jung *et al* (1985). The CB1 results are naturally the same for both the electron and positron impact. The PCI effects break the axial symmetry around the momentum transfer direction. The incorporation of PCI effects shifts the binary peak to larger angles θ_b relative to the

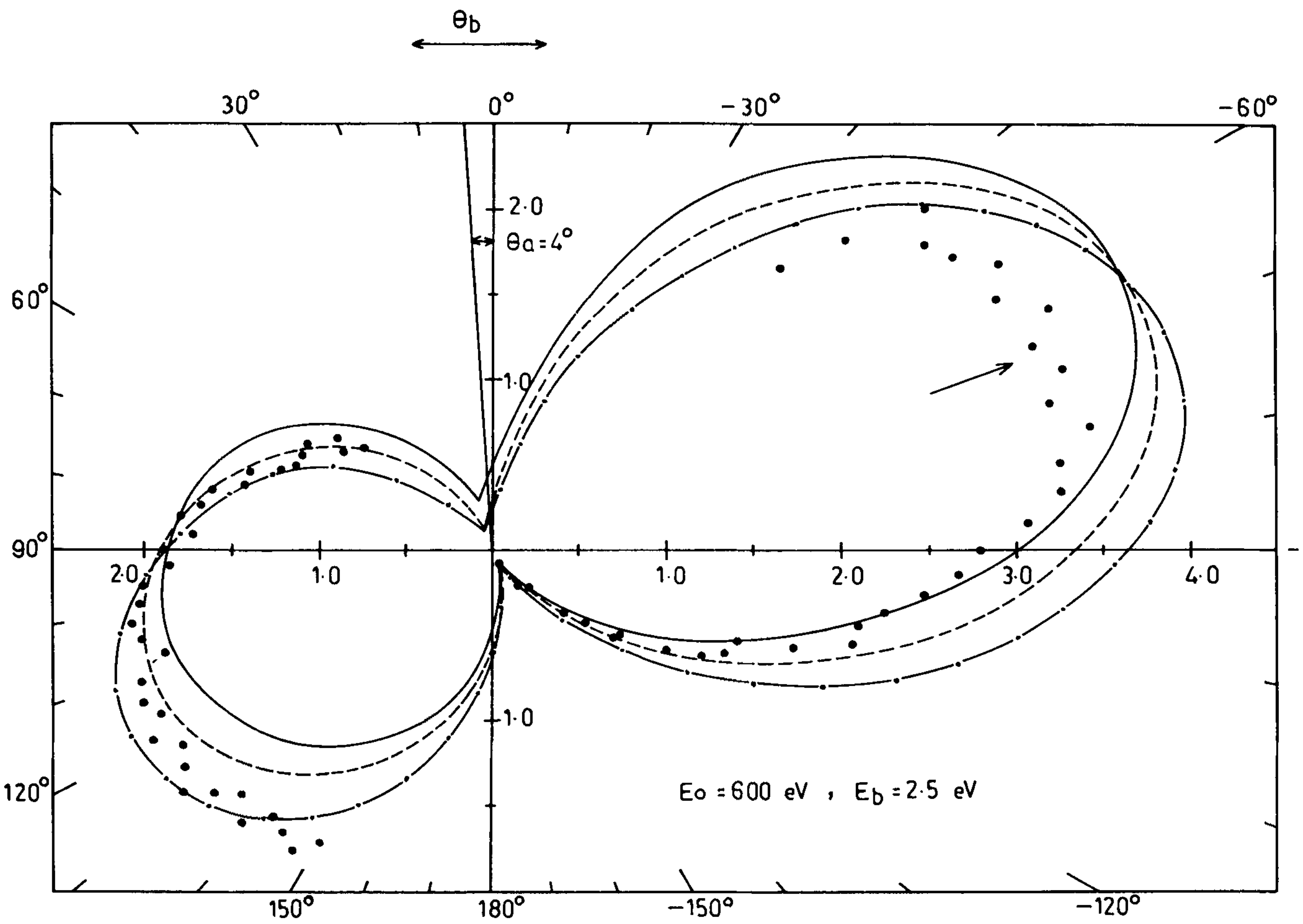

Figure 1. Triple differential cross-section in units of $10^{-22}\,m^2\,sr^{-2}\,eV^{-1}$ for the ionization of helium by positron impact at $E_0 = 600\,eV$, $E_b = 2{\cdot}5\,eV$, $\theta_a = 4°$. Theoretical results; 'correlated' first Born approximation for electron impact (CB1)----; 'correlated' first Born approximation for electron impact with PCI (CB1-PC1) -.-.-; present results for positrons ———. Experimental data are the absolute measurements of Jung *et al* (1985). The arrow indicates the direction of momentum transfer.

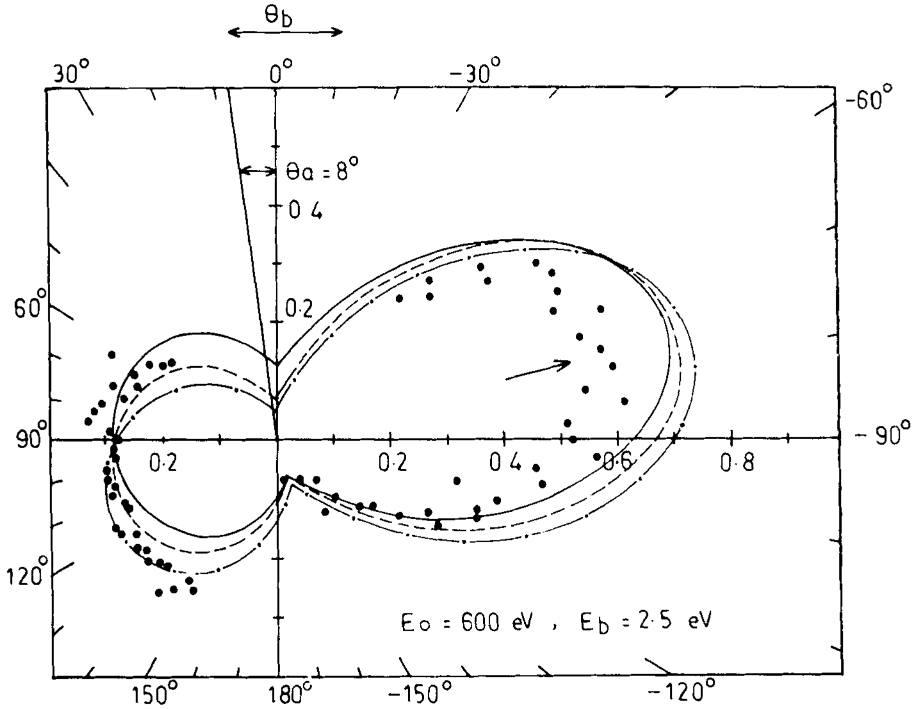

Figure 2. Same as figure 1 but $E_0 = 600\,\mathrm{eV}$, $E_b = 2{\cdot}5\,\mathrm{eV}$ and $\theta_a = 8°$.

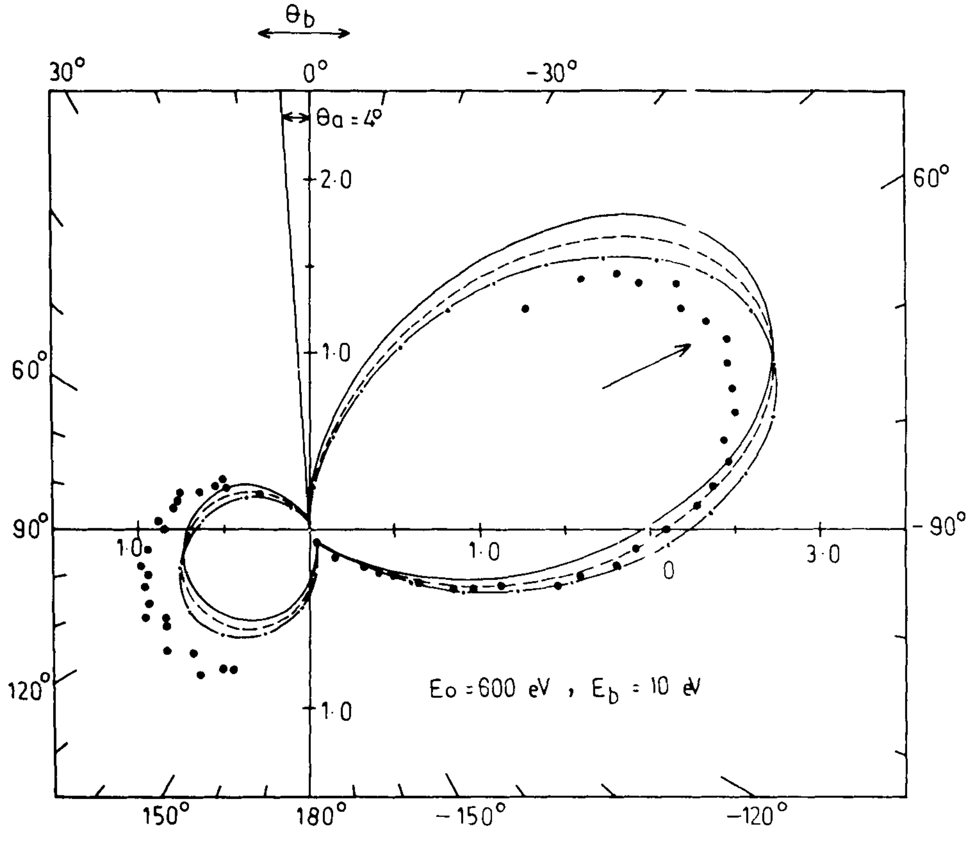

Figure 3. Same as figure 1 but $E_0 = 600\,\mathrm{eV}$, $E_b = 10\,\mathrm{eV}$ and $\theta_a = 4°$.

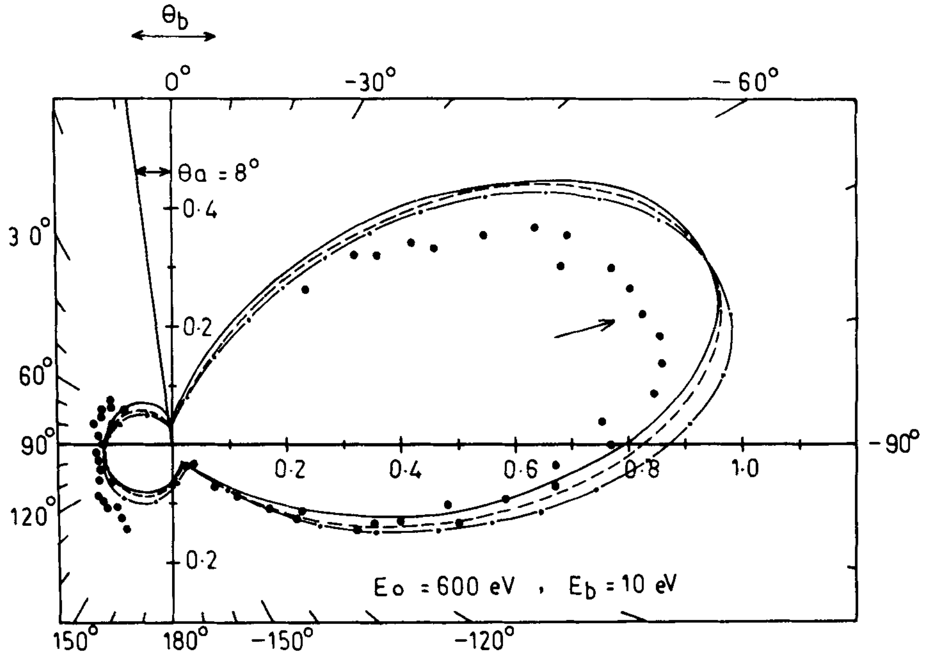

Figure 4. Same as figure 1 but $E_0 = 600\,\text{eV}$, $E_b = 10\,\text{eV}$ and $\theta_a = 8°$.

momentum transfer direction in the e^- impact case and to smaller angles in the e^+ case. Similar shift is observed in the recoil peak region. The PCI effects lead to a reduction in the recoil peak intensity in the case of e^+ impact in contrast to the e^- case where the recoil peak intensity increases. However, the binary peak size remains almost unchanged. There are two angles, θ_{b0} in the binary region and θ'_{b0} in the recoil region, at which the energy transfer to the slow electron is zero. The result is that, at these angles, the TDCS with and without PCI are almost identical. The angle θ_{b0} is close to the binary peak maximum and therefore the binary peak size remains almost unchanged. The angle θ'_{b0} is however much away from the recoil peak maximum. For a given E_b, both θ_{b0} and θ'_{b0} decrease as the scattering angle θ_a increases.

The main result of this paper is that the incorporation of PCI effects leads to changes which are in the right direction, that is, in the positron impact case the two peaks shift to smaller angles of ejection as one would expect. Another result is that a larger binary to recoil peak maxima ratio is expected in the positron case.

Acknowledgements

This work was supported by the Indian Space Research Organisation. One of us (SS) would like to thank the University Grants Commission for the award of a research fellowship.

References

Baliyan K S and Srivastava M K 1985 *Phys. Rev.* **A32** 3098
Baliyan K S and Srivastava M K 1986a *Phys. Rev.* **A33** 2155
Baliyan K S and Srivastava M K 1986b *J. Phys.* **B19** 3603
Baluja K L and Taylor H S 1976 *J. Phys.* **B9** 829
Basu M, Mazumdar P S and Ghosh A S 1985 *J. Phys.* **B18** 369
Bransden B H, Smith J J and Winters K H 1978 *J. Phys.* **B11** 3095
Bransden B H, Smith J J and Winters K H 1979 *J. Phys.* **B12** 1267
Byron F W Jr and Joachain C J 1966 *Phys. Rev.* **146** 1
Byron F W Jr and Joachain C J 1975 *J. Phys.* **B8** L284
Byron F W Jr, Joachain C J and Piraux B 1980 *J. Phys.* **B13** L673
Byron F W Jr, Joachain C J and Piraux B 1982 *J. Phys.* **B15** L293
Byron F W Jr, Joachain C J and Piraux B 1983 *Phys. Lett.* **A99** 427
Byron F W Jr, Joachain C J and Piraux B 1984 *Phys. Lett.* **A106** 299
Byron F W Jr, Joachain C J and Piraux B 1985 *J. Phys.* **B18** 3203
Ehrhardt H, Fischer M, Jung K, Byron F W Jr, Joachain C J and Piraux B 1982 *Phys. Rev. Lett.* **48** 1807
Franz A and Klar H 1986 *Z. Phys.* **D1** 33
Furtado F Mota and O'Mahony P F 1987 *J. Phys.* **B20** L405
Geltman S 1971 *J. Phys.* **B4** 1288
Geltman S 1974 *J. Phys.* **B7** 1994
Geltman S and Hidalgo M B 1971 *J. Phys.* **B4** 1299
Geltman S and Hidalgo M B 1974 *J. Phys.* **B7** 831
Ghosh A S, Mazumdar P S and Basu M 1984a *J. Phys.* **B17** 968
Ghosh A S, Mazumdar P S and Basu M 1984b *Z. Phys.* **A319** 13
Ghosh A S, Mazumdar P S and Basu M 1985a *J. Phys.* **B18** 1881
Ghosh A S, Mazumdar P S and Basu M 1985b *Can. J. Phys.* **63** 62
Gien T T 1976 *J. Phys.* **B9** 3203
Hidalgo M B and Geltman S 1972 *J. Phys.* **B5** 617
Joachain C J 1984 *Positron scattering in gases*, (eds) J W Humberston and M R C McDowell (New York: Plenum)
Jung K, Muller-Fiedler R, Schlemmer P, Ehrhardt H and Klar H 1985 *J. Phys.* **B18** 2955
Klar H and Franz A 1986 *Phys. Rev.* **A33** 2103
Klar H, Franz A and Tenhagen H 1986 *Z. Phys.* **D1** 373
Lal M, Tripathi A N and Srivastava M K 1979 *J. Phys.* **B12** 945
Madison D H, Calhoun R V and Shelton W N 1977 *Phys. Rev.* **A16** 552
Mandal P, Roy K and Sil N C 1986 *Phys. Rev.* **A33** 756
McDowell M R C, Morgan L A and Myerscough V P 1973 *J. Phys.* **B6** 1435
Pathak A and Srivastava M K 1980 *J. Phys.* **B13** 3257
Pathak A and Srivastava M K 1981 *J. Phys.* **B14** 4865
Popov Yu V and Benyoun J J 1981 *J. Phys.* **B14** 3413
Popov Yu V and Erokhin V F 1983 *Phys. Lett* **A97** 280
Schubert E, Schuck A, Jung K and Geltman S 1979 *J. Phys.* **B12** 967
Schulz M 1973 *J. Phys.* **B6** 2580
Saxena Sushma and Srivastava M K 1987 *Can. J. Phys.* **65** 458
Sharma Sadhana and Srivastava M K 1987 *Phys. Rev.* **A35** 1939
Smith J J, Winters K H and Bransden B H 1979 *J. Phys.* **B12** 1723
Srivastava M K and Sharma Sadhana 1988 (in press)
Tweed R J 1980 *J. Phys.* **B13** 4467