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Threshold pump strain for parametric amplification in *n*-indium antimonide

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Abstract. Phenomenological approach has been used to determine the threshold pump strain by assuming that at this strain the pump generates an electric field for which the drift velocity of the carriers equals the sound velocity. Numerical values obtained for *n*-InSb have been compared with similar studies which are based on Boltzmann transport approach.

Keywords. Threshold pump strain; parametric amplification; indium antimonide.

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1. Introduction

When more than one wave propagates through a piezoelectric crystal, they affect one another because each wave gives rise to its own bunching of carriers as a result of which the carrier distribution of one interacts with the field distribution of the other. The coupling due to this interaction is known as parametric coupling. In this case the nonlinear interaction between the carriers and the electric field results in the generation of new electrical and acoustical signal (Kroger 1964; Tell 1964; Elbaum and Truell 1964; Mauro and Wang 1967). In addition, the parametric interaction between a large and a small amplitude wave results in the amplification of the latter (Komar and Timan 1970; Dyakonov and Lisavskii 1970). The large amplitude wave, usually referred to as the pump, can produce amplification only above a certain fixed strain known as the threshold pump strain. Johri and Spector (1977) employed the Boltzmann transport approach to derive an expression for the threshold pump strain. However, studies based on the phenomenological approach are not available in literature. In the phenomenological approach one first derives an expression for the second order field acting on a given wave due to acoustoelectric interaction with the other wave. The wave equations are than set up for the coupled waves and solved for subharmonic and second harmonic generation. We have employed this approach to derive an expression for the threshold pump strain by assuming that at this strain the pump generates an electric field for which the drift velocity of the carriers equals the sound velocity. Under such a situation the carriers will remain in phase with the wave and will impart their energy to the signal which will, therefore, be amplified.

2. Theory

We consider an acoustic wave propagating in the x-direction of an n-type piezoelectric semiconducting medium and define a strain S, a stress T, and a displacement u such that

$$S = \partial u/\partial x \tag{1}$$

and

$$\partial T/\partial x = \rho \partial^2 u/\partial t^2, \tag{2}$$

where ρ is the mass density. We further assume that the medium is characterized by a piezoelectric constant e such that S produces an electric field E in the x-direction. The material equations corresponding to this problem are then given by

$$T = cS - eE \tag{3}$$

and

$$D = eS + \varepsilon E, \tag{4}$$

where ε is the dielectric permittivity at constant strain and c is the elastic constant at constant electric field. The expression for the current density is

$$J = q(n + fn_s)\mu E + (\mu/\beta)f(\partial n_s/\partial x), \tag{5}$$

where the first term is due to the drift and the second is due to the diffusion. Here q is the electronic charge, $\beta = 1/k_0 T$, μ is the electron mobility, n is the mean number density for the electrons and $(n + fn_s)$ is the instantaneous local density of electrons in the conduction band. The fraction f accounts for division of the space charge between conduction band and the bound states in the energy gap.

The Poisson's equation and the equation of continuity for this case can be respectively represented by

$$\partial D/\partial x = -qn_s \tag{6}$$

and

$$\partial J/\partial x = q\partial n_s/\partial t. \tag{7}$$

Combining (4), (5), (6) and (7), we get a relation between D and E in the form

$$\begin{aligned}\partial^2 D/\partial x\partial t = & -qn\mu\partial E/\partial x + f\mu\partial D/\partial x\cdot\partial E/\partial x \\ & + f\mu E\partial^2 D/\partial x^2 + (\mu f/\beta q)\cdot\partial^3 D/\partial x^3.\end{aligned} \tag{8}$$

Assuming the plane wave time and space dependence we can substitute

$$D = D_0 \exp i(kx - \omega t)$$

and

$$E = E_0 \exp i(kx - \omega t)$$

in (8), which yields

$$D = \frac{-i(nq\mu/\omega)E}{1 + 2(k/\omega)f\mu E + i\omega(k/\omega)^2\cdot(\mu f/q\beta)}. \tag{9}$$

For the case of small conductivity modulation the drift term fn_s in (5) is much less than n. As a result, the corresponding term $(2(k/\omega)f\mu E)$ in the denominator of (9) may be neglected. So

$$D = \frac{-i(\sigma/\omega)E}{1 + i\omega(k/\omega)^2(\mu f/q\beta)}, \tag{10}$$

where $\sigma = nq\mu$ is the average conductivity. For convenience, we define a conductivity frequency as

$$\omega_c = \sigma/\varepsilon$$

and a diffusion frequency as

$$\omega_D = (q\beta/f\mu)(\omega/k)^2 = (q\beta/f\mu)v_s^2.$$

The reciprocal of ω_c is the dielectric relaxation time and ω_D is the frequency above which the wavelength is sufficiently short for diffusion to smooth out carrier density fluctuations which have the periodicity of the acoustic wave.

On making these substitutions, we get

$$E = -\frac{e}{\varepsilon} S \frac{1 + i(\omega/\omega_D)}{1 + i(\omega/\omega_D + \omega_c/\omega)}. \tag{11}$$

We now assume that the signal will be amplified by the pump provided the pump strain generates an electric field for which the drift velocity is either equal to or greater than the velocity of the signal. The threshold pump strain S_{th} will, therefore, be that strain for which

$$v_s = v_d = \mu E = \mu\left(\frac{e}{\varepsilon}\right)\frac{1 + i(\omega/\omega_D)}{1 + i(\omega/\omega_D + \omega_c/\omega)} S_{th}$$

or

$$S_{th} = \left(\frac{v_s \varepsilon}{\mu e}\right)\frac{1 + i(\omega/\omega_D + \omega_c/\omega)}{1 + i(\omega/\omega_D)}.$$

Considering only the real terms, we finally get

$$S_{th} = \frac{v_s \varepsilon}{\mu e}\left(1 + \frac{\omega_c \omega_D}{\omega_D^2 + \omega^2}\right). \tag{12}$$

Which gives finite result in the limit $\omega \to 0$ and in the limit $\omega \to \infty$. The corresponding expression obtained from the Boltzmann transport approach (Johri and Spector 1977) is given by

$$S_{th} = \frac{v_s}{2\mu}\left(\frac{\varepsilon}{4\pi e}\right)\frac{[1 + (\omega_c/2\omega + 2\omega/\omega_D)^2]^{1/2}}{[1 + (\omega/2\omega_D)^2]^{1/2}}, \tag{13}$$

which diverges in the limit $\omega \to 0$.

3. Results and discussion

The results obtained from (12) and (13) are illustrated in figure 1 where the frequency dependence of the threshold pump strain for n-InSb has been shown. The physical parameters for this material have been taken to be $v_s = 4{\cdot}0 \times 10^5 \text{ cm sec}^{-1}$,

$$n = 2{\cdot}5 \times 10^{14} \text{ cm}^{-3}, \quad \varepsilon = 18,$$

$$\rho = 5{\cdot}8\,\mathrm{g\,cm^{-3}}, \quad e = 1{\cdot}8 \times 10^4\,\mathrm{esu\,cm^{-2}},$$

$$\mu = 2{\cdot}4 \times 10^7\,\mathrm{cm^2/esu/sec} \quad \text{and} \quad T = 77\,\mathrm{K}.$$

These are the parameters which have been used by Johri and Spector (1977). Curve 1 corresponds to the results obtained from (12) i.e. from the phenomenological approach whereas curve 2 corresponds to the results obtained from (13) i.e. from the Boltzmann transport approach. At $\omega = 1{\cdot}0 \times 10^8$, the former gives $S_{\mathrm{th}} = 0{\cdot}8 \times 10^{-4}$, whereas the latter gives $S_{\mathrm{th}} = 5{\cdot}2 \times 10^{-4}$. So it can be remarked that the order of the threshold pump strain obtained from the phenomenological approach is the same as that obtained from the Boltzmann transport approach. However, their magnitude differs from each other. Because of the lack of experimental data these results have not been compared with experiments. It may however be remarked that the phenomenological approach is valid only for the case for which the electron mean free path l is much smaller than the phonon wavelength $(2\pi/k)$ i.e. for the case for which $kl \ll 1$. In this case the electron will undergo many collisions in moving a wavelength. Consequently, their average velocity will be the drift velocity induced by the fields. As a result, only the drift field can give the electrons a velocity equal to that of the sound wave so that they can remain in phase with the wave and resonantly loose energy. In the opposite case of $kl \gg 1$, the average velocity of the electrons will be the resultant of the drift velocity and the Fermi velocity. Since the latter is much larger than the sound velocity, the drift field is not required to bring the carriers into resonance with the wave. The electrons in this case can remain in resonance with the wave even in the absence of the drift field (Eckstein 1963).

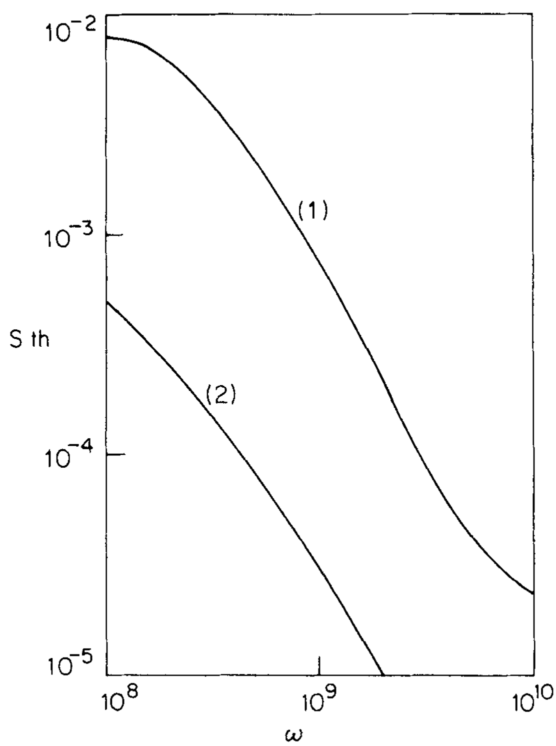

Figure 1. Threshold pump strain shown as a function of frequency in n-InSb when $kl \ll 1$.

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