Pramāṇa - J. Phys., Vol. 30, No. 4, April 1988, pp. 299–304. © Printed in India.

The superconformal anomaly in the 1 + 1 dimensional Wess-Zumino model

S G KAMATH

Department of Mathematics, Indian Institute of Technology, Madras 600036, India

MS received 7 September 1987; revised 13 January 1988

Abstract. The superconformal trace anomaly is worked out to one-loop order in perturbation theory for the 1 + 1 dimensional Wess-Zumino model.

Keywords. Wess-Zumino model; one + one dimensions; superconformal anomaly.

PACS No. 11·30

1. Introduction

There is now a considerable literature on the anomaly associated with superconformal invariance in supersymmetric theories in 3 + 1 dimensions (Lukierski 1977; Curtright 1977; Abbott *et al* 1977; Hagiwara *et al* 1980; Espriu 1985). The classical conservation law associated with this invariance viz $\gamma^\mu J_\mu = 0$ is broken by quantum corrections and the resulting anomaly has been computed via perturbation theory (Abbott *et al* 1977; Hagiwara *et al* 1980; Espriu 1985) as well as using the normal product method (Clark *et al* 1978).

This paper is a sequel to our recent study (Kamath 1987, hereafter referred to as I) on the trace anomaly associated with broken scale invariance in the 1 + 1 dimensional Wess-Zumino model. The model, with dimensional coupling constants and non-zero mass terms for the fields, exhibits explicit breaking of superconformal invariance classically; thus we have $\gamma^\mu J_\mu = i\sqrt{2}(-\lambda + \mu H + (3g/2)H^2)\psi$ in this case. However, just as in I, this relation becomes anomalous on inclusion of quantum corrections. To one-loop order, as we shall show below, we obtain

$$\gamma^\mu J_{\mu\,\mathrm{anom}} = \gamma^\mu J_\mu + \frac{3ig}{\sqrt{8\pi}}\psi.$$

Here ψ, H are the fields that occur in the lagrangian density for the model, with μ, g and λ being the dimensional coupling constants. For completeness, we record here the corresponding trace anomaly for the Belinfante energy-momentum tensor worked out in I; it reads as

$$\theta^{B\mu}_{\mu\,\mathrm{anom}} = \theta^{B\mu}_{\mu} - \frac{3g}{4\pi}F.$$

The computation of the superconformal anomaly here imitates the approach adopted for the trace anomaly in I. Thus, with the method of Coleman and Jackiw (1971) we first set up the superconformal trace identity in §2. The anomaly is then obtained diagrammatically to one-loop order using the method of dimensional regularization in §3. As will be seen in §2 there are a large number of diagrams that do not contribute to the anomaly because of cancellations in pairs either due to supersymmetry or because the HH loop is removed by a corresponding contribution from the F counterterm. These results were also noted in I.

2. The superconformal trace identity

The lagrangian density for the Wess-Zumino model is given by (Browne 1975)

$$2L = -(\partial_\mu A)^2 - \bar{\psi} i\partial\!\!\!/ \psi - F^2 + \rho F + 3g(A^2 F - \bar{\psi}\psi A). \tag{1}$$

The classical potential $2V(A,F) = -F^2 + \rho F + 3gA^2 F$ has a critical point at $F = 0$, $3gA^2 = -\rho$ at which $V = 0$. The other critical point viz $F = \frac{1}{2}\rho$, $A = 0$ at which $4V = \rho^2$, corresponds to the case where supersymmetry is broken spontaneously and is ignored here. Replacing A in (1) by $A_0 + H$, with $3gA_0^2 = -\rho$, we rewrite with $\mu = 3gA_0$,

$$2L = -(\partial_\mu H)^2 - \bar{\psi}(i\partial\!\!\!/ + \mu)\psi - F^2 + 2\mu HF + 3g(H^2 F - \bar{\psi}\psi H) - \lambda F, \tag{2}$$

where we have now included the infinite counterterm λF in L. The latter as shown by Browne (1975) is necessary to ensure that $\langle F(x)\rangle_0 = 0$, and is given to one-loop order by $2\lambda = -3giD_H(0)$. From (2) the relevant propagators are easily worked as follows*:

$$\begin{aligned} &iD_H(p) = i(-p^2 + \mu^2)^{-1}, \quad iS_\psi(p) = -i(p\!\!\!/ + \mu)^{-1}, \\ &iD_{HF}(p) = \mu i D_H(p), \quad iD_{FF}(p) = p^2 i D_H(p). \end{aligned} \tag{3}$$

It is now easy to show that (2) is invariant under the supersymmetry transformations $\sqrt{2}\delta H = \bar{\varepsilon}\psi$, $\sqrt{2}\delta\psi = (\varepsilon F - i\partial\!\!\!/ H\varepsilon)$, and $\sqrt{2}\delta F = -i\bar{\varepsilon}\partial\!\!\!/\psi$. The conserved current is easily worked out as

$$\sqrt{2}J^\mu = i\left(-\lambda + \mu H + \frac{3g}{2}H^2\right)\gamma^\mu\psi - \partial\!\!\!/ H\gamma^\mu\psi. \tag{4}$$

Following the method of Coleman and Jackiw (1971) the supersymmetry Ward identity can be worked out as

$$\partial^\mu\Gamma_\mu(y{:}x_1,x_2) = -i\sum_{r=1}^{2}\delta(y-x_r)\delta^{(r)}G(x_1,x_2), \tag{5}$$

with

$$\begin{aligned} &\Gamma_\mu = \langle 0|T^*(J_\mu(y)H(x_1)\bar{\psi}(x_2))|0\rangle_c, \\ &\delta^{(1)}G = \langle 0|T^*(\delta H(x_1)\bar{\psi}(x_2))|0\rangle_c, \\ &\delta^{(2)}G = \langle 0|T^*(H(x_1)\delta\bar{\psi}(x_2))|0\rangle_c. \end{aligned} \tag{6}$$

*Our conventions are: $g^{00} = -g^{11} = 1$, $\gamma_0^\dagger = \gamma_0$, $\gamma_1^\dagger = -\gamma_1$, $\varepsilon^{01} = -\varepsilon^{10} = 1$, $\gamma_5\gamma^\mu = i\varepsilon^{\mu\nu}\gamma_\nu$. Also $\int_p$ and $\int_x$ denote $\int(2\pi)^{-2}\mathrm{d}^2p$ and $\int\mathrm{d}^2x$ respectively.

Under superconformal transformations, for which $\sqrt{2}\delta H = -i\bar{\varepsilon}x\cdot\gamma\psi$, $\sqrt{2}\delta\psi = \not{\partial} Hx\cdot\gamma\varepsilon + ix\cdot\gamma F\varepsilon$ and $\sqrt{2}\delta F = -\bar{\varepsilon}x\cdot\gamma\not{\partial}\psi$, the superconformal current is easily shown to be $\tilde{J}^\mu = -ix\cdot\gamma J^\mu$. It obeys the divergence equation

$$\Delta \equiv \partial^\mu \tilde{J}_\mu = \sqrt{2}\left(-\lambda + \mu H + \frac{3g}{2}H^2\right)\psi. \tag{7}$$

Following Coleman and Jackiw (1971) the analog of (5) for the superconformal transformations can now be shown as

$$\partial^\mu \tilde{\Gamma}_\mu(y{:}x_1, x_2) = \Delta(y{:}x_1, x_2) - i\sum_{r=1}^{2}\delta(y - x_r)\delta^{(r)}G(x_1, x_2) \tag{8}$$

with the various Green's functions resembling (6), but with appropriate replacements to cover the superconformal transformation and $\Delta(y{:}x_1, x_2) = \langle 0|T^*(\Delta(y)H(x_1)\bar{\psi}(x_2))|0\rangle_c$.

With the relation $\tilde{J}^\mu = -ix\cdot\gamma J^\mu$, it is simple to recast (8) as

$$\gamma^\mu\Gamma_\mu(y{:}x_1, x_2) = i\Delta(y{:}x_1, x_2), \tag{9}$$

which is just the superconformal trace identity. In § 3 we verify that (9) is false to one-loop order and thus obtain the superconformal anomaly, using the method of dimensional regularization.

3. The superconformal anomaly

Some of the connected one-loop diagrams that are relevant for a verification of the trace identity (9) are shown in figure 1. To obtain the superconformal anomaly, however, it is enough to consider only the logarithmically divergent diagrams. It is easy to substantiate this using figure 1a itself, which incidentally, is the only source of the anomaly at the one-loop level. Using (4) we first write the contribution of figure 1a to

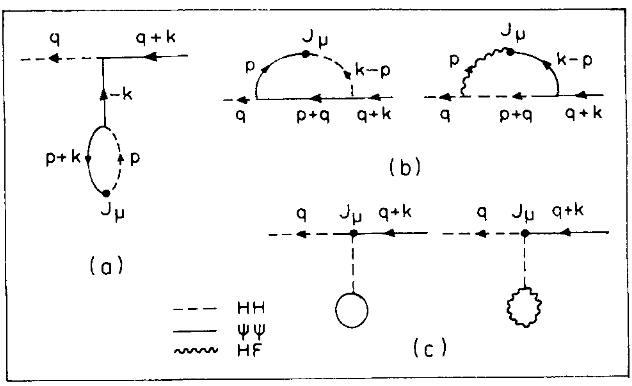

Figure 1. a. The one-loop contribution to the superconformal anomaly; **b.** One-loop diagrams that preserve the superconformal trace identity; **c.** An example of a pair of diagrams that cancel each other by supersymmetry.

the following matrix element to $O(g^2)$ as

$$\int_{xy} \exp(iq\cdot x + ik\cdot y)\langle 0|T^*(J_\mu(y)H(x)\bar{\psi}(0))|0\rangle_c$$
$$= iD_H(q)(I_\mu^{(1)} + I_\mu^{(2)})iS_\psi(q+k) \tag{10}$$

with $I_\mu^{(1)}$ arising from the first term in (4), and given by the finite momentum integral

$$\sqrt{2}I_\mu^{(1)} = (3ig/2)^2 4i\mu \int_p iD_H(p)\gamma_\mu iS_\psi(p+k)iS_\psi(-k). \tag{11}$$

$I_\mu^{(2)}$, on the other hand, is due to the second term in (4) and is given by

$$\sqrt{2}I_\mu^{(2)} = -(3ig/2)^2 4 \int_p i\not{p} iD_H(p)\gamma_\mu iS_\psi(p+k)iS_\psi(-k). \tag{12}$$

Clearly $I_\mu^{(2)}$ is logarithmically divergent and therefore $\gamma^\mu I_\mu^{(2)}$ cannot be equated to zero—as one would have done naively, using the relation $\gamma^\mu\gamma^\nu\gamma_\mu = 0$ in 2 dimensions—prior to the momentum integration. Put differently, one cannot exchange γ^μ with the momentum integration in (12). This is of course possible in (11) and would lead to

$$\sqrt{2}\gamma^\mu I_\mu^{(1)} = 8i\mu(3ig/2)^2 \int_p iD_H(p)iS_\psi(p+k)iS_\psi(-k). \tag{13}$$

It is easy to check that this is just the result that one would obtain when the $\sqrt{2}\mu H\psi$ term in (7) is inserted in place of J^μ in the loop in figure 1a. Put simply, the $\mu H\gamma^\mu\psi$ term in (4) obeys the superconformal trace identity. As we shall argue later on, the remaining entries in the first term in (4) also respect the superconformal trace identity. It is therefore easy to infer that the second term in (4) will lead to the anomaly, if any, which we shall now obtain. We shall subsequently comment on the uniqueness of figure 1a, at the one-loop level, as a source of the superconformal anomaly.

Using the method of dimensional regularization, the momentum integral in (12) can be reworked as

$$4\pi I_\mu = \int_0^1 x\not{k}\rho\gamma_\mu(\mu - \not{k}(1-x))\,dx + \tfrac{1}{2}\gamma^\nu\gamma_\mu\gamma_\nu\Gamma(1-n/2)\int_0^1 \rho^{n/2-1}\,dx \tag{14}$$

with $\rho^{-1} = k^2x(1-x) - \mu^2$. Clearly, the first integral in (14) yields zero on calculating $\gamma^\mu I_\mu$; ignoring the $\log\rho$ term in the second integral over x, which will also yield zero, it being finite, we rewrite

$$8\pi I_\mu = \gamma^\nu\gamma_\mu\gamma_\nu\Gamma(1-n/2) = 2(1-n/2)\gamma_\mu\Gamma(1-n/2) = 2\gamma_\mu \tag{15}$$

in 2 dimensions. In obtaining this result we have used the relation $\gamma^\nu\gamma_\mu\gamma_\nu = 2\gamma_\mu(1-n/2)$ in n dimensions. Clearly therefore (15) leads via (12) to the superconformal anomaly denoted by

$$\pi A = \pi\gamma^\mu I_\mu^{(2)} = (3g/2)^2\sqrt{2}iS_\psi(-k). \tag{16}$$

This is just the $O(g^2)$ contribution of the matrix element

$$\int_{xy} \exp(iq\cdot x + ik\cdot y)\left\langle 0\left| T\left(\frac{3ig}{\pi\sqrt{8}}\psi(y)H(x)\bar{\psi}(0)\right)\right|0\right\rangle_c \tag{17}$$

thus suggesting that as an operator the anomaly is just $\sqrt{8}\pi A = 3ig\psi$. The arguments offered above for figure 1a, particularly in respect of the non-anomalous contributions given by (11), can be repeated for figure 1b for example. The integration over the loop momentum leads to a finite result and so the relations $\gamma^\mu\gamma_\nu\gamma_\mu = 0$, $\gamma^\mu\gamma_\mu = 2$ in $1+1$ dimensions are readily applicable. Thus, those diagrams do not contribute to the anomaly. The same argument applies to diagrams which resemble figure 1b, but which have two instead of three internal lines in the loop; they arise from the $O(g)$ term in (4). Besides the aforementioned diagrams there are a few diagrams (see figure 1c, for an example) which cancel each other because of supersymmetry, whereby HF loops cancel the fermion loops. To reassure the reader, we give below the matrix elements in momentum space of figure 1c.
Fermion loop:

$$2\alpha iD_H(q)iD_H(0)\gamma_\mu iS_\psi(q+k)TriS_\psi(0).$$

Bose loop:

$$2\alpha iD_H(q)2iD_H(0)\gamma_\mu iS_\psi(q+k)iD_{\mathrm{HF}}(0). \tag{18}$$

Here $\sqrt{2}\alpha = (3ig/2)^2, iD_H(0)$ and $iD_{\mathrm{HF}}(0)$ are boson propagators with zero momenta, while $\mathrm{Tr}\, iS_\psi(0)$ and $iD_{\mathrm{HF}}(0)$ refer to the self-contracted fermion and boson propagators in position space. From (3), we obtain

$$\mathrm{Tr}\, iS_\psi(0) = \int_p \mathrm{Tr}\, iS_\psi(p) = -2\int_p iD_{\mathrm{HF}}(p); \quad iD_{\mathrm{HF}}(0) = \int_p iD_{\mathrm{HF}}(p). \tag{19}$$

Using (19), we see that the sum of the boson and fermion loop contributions displayed in (18) is zero. Finally the effect of the $\lambda\gamma^\mu\psi$ term in (4) which arises from the F counterterm in the lagrangian (2) is cancelled by corresponding diagrams involving HH loops.

As an example, we give below a pair of matrix elements which resemble those given in (18):
HH loop:

$$2\alpha iD_H(q)iD_{\mathrm{HF}}(0)\gamma_\mu iS_\psi(q+k)iD_H(0)$$

F counterterm:

$$2i\beta iD_H(q)iD_{\mathrm{HF}}(0)\gamma_\mu iS_\psi(q+k) \tag{20}$$

with $2\sqrt{2}\beta = 3ig\lambda$. Using the relation $2\lambda = -3giD_H(0)$ in (20) we see that the HH loop contribution cancels with the F counterterm.

From these arguments, one concludes that the anomaly arises solely from figure 1a, to one-loop order, and that

$$\gamma^{\mu} J_{\mu\,\mathrm{anom}} = \gamma^{\mu} J_{\mu} + \frac{3ig}{\pi\sqrt{8}}\psi. \tag{21}$$

The extension of these results for the superconformal anomaly to arbitrary loop order is presently being studied and will be reported elsewhere.

References

Abbott L F, Grisaru M T and Schnitzer H J 1977 *Phys. Rev.* **D16** 2995
Browne S 1975 *Phys. Lett.* **B59** 253
Clark T E, Piguet O and Sibold K 1978 *Nucl. Phys.* **B143** 443
Coleman S and Jackiw R 1971 *Ann. Phys. (NY)* **67** 552
Curtright T 1977 *Phys. Lett.* **B71** 185
Espriu D 1985 *Z. Phys.* **C27** 603
Hagiwara T, Pi S Y and Tsao H S 1980 *Ann. Phys. (NY)* **130** 282
Kamath S G 1987 submitted for publication
Lukierski J 1977 *Phys. Lett.* **B70** 183