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Baryon masses in the SU(4) Skyrme model

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Abstract. We consider the SU(4) Skyrme model with explicit chiral and flavour symmetry-breaking terms. Using the masses of the 15-plet pseudoscalar mesons as the input, we calculate the masses of the 20-plet baryons. The baryon masses predicted by this model agree with results based on quark model to about 15%. We find that the generalized Gell-Mann Okubo mass relation is very well satisfied.

Keywords. Skyrme model; SU(4) 20-plet baryons; SU(4) 15-plet pseudoscalar mesons; generalized Gell-Mann-Okubo mass relation.

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1. Introduction

The Skyrme model is a nonlinear σ model in which the baryons appear as topological solitons (Skyrme 1961; Balachandran *et al* 1982, 1983; Witten 1983). It has been a reasonably successful model of strong interactions at low energies. This can be seen by the fact that preliminary results relating to several static properties of the nucleons like their magnetic moments, charge radii, pion-nucleon coupling constant etc predicted using the SU(2) Skyrme model agree with experimental data to within 20–30% (Adkins *et al* 1983; Adkins and Nappi 1984). Investigations on dynamical properties like inter-nucleon potential and pion-nucleon scattering parameters are also encouraging (see for example, Jackson *et al* 1985; Mattis and Karliner 1985). In the SU(3) version of the model the D/F ratio of the meson-baryon coupling constants, mass symmetry-breaking pattern and magnetic moments of the baryons have been calculated with reasonable agreement with data (Guadagnini 1984; Sriram *et al* 1984; Adkins and Nappi 1985). It is therefore interesting to see how successfully the Skyrme model can be applied to studies relating to higher flavour symmetries like SU(4).

In an attempt to do this, we have considered here the SU(4) Skyrme model. The fact that it is a badly broken symmetry is taken care of by introducing a simple, though fairly general explicit symmetry breaking term in the Lagrangian. A calculation of the 20-plet baryon masses in this model would, therefore, give us a rough estimate of how well the Skyrme model in this form can be applied to studies relating to high flavour symmetries like SU(4). We have also studied the mass relationship between the baryons. We have reported the salient features of the calculation in the next section and have concluded with a brief discussion of our results.

2. Baryon masses

We start with the Lagrangian density

$$L_0 = \frac{f_\pi^2}{16}\mathrm{Tr}\,[(\partial_\mu U^+)(\partial^\mu U)] + \frac{1}{32e^2}\mathrm{Tr}\,[\partial_\mu U U^+, \partial_\nu U U^+]^2, \tag{1}$$

where $U \in \mathrm{SU}(4)$ is an order parameter. L_0 is invariant under the action of $\mathrm{SU}(4)_L \otimes \mathrm{SU}(4)_R$. In the trivial topology sector,

$$U = \exp\left[\frac{2i}{f_\pi}\sum_{j=1}^{15}\lambda_j \varphi_j\right],$$

where φ_j are the 15-plet pseudoscalar meson fields. In the soliton sector the winding number is identified with the baryon number B. The soliton solution corresponding to the $B = 1$, 20-plet baryons is given by

$$U_0 = \begin{bmatrix} \exp(if(r)\tau\cdot\hat{x}) & & 0 & 0 \\ & & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{2}$$

where $f(r)$ is solved from the Euler Lagrange equations and satisfies the boundary conditions $f(r) = \pi$ at $r = 0$ and $f(r) = 0$ at $r = \infty$. This is a direct generalization of the SU(2) ansatz.

The quantization of this Lagrangian in the baryon sector is done through the use of collective coordinates U given by

$$U = g(t) U_0 g^{-1}(t). \tag{3}$$

Here $g(t)$ is an element of SU(4) in the fundamental representation. The baryon wave functions are given by $D^{(N)}_{\alpha\beta}(g)$, the SU(4) generalizations of the D functions. In $D^{(n)}_{\alpha\beta}(g)$, n denotes the SU(4) representation considered and α symbolically represents the set of quantum numbers (I, I_3, Y, C) where I is the isospin, I_3 its third component, Y the hypercharge and C the charm quantum number. β represents the spin quantum number of the baryon state considered. In particular for the spin $\frac{1}{2}$ 20-plet the wave functions are $D^{20}_{I,I_3,Y,C;1/2,m,1,0}$ with $m = -\frac{1}{2}$ for spin up and $m = +\frac{1}{2}$ for spin down states.

In the meson sector we introduce an explicit symmetry-breaking term involving the meson masses. This will induce a definite symmetry breaking in the soliton sector as well. This extra term in the meson sector is given by

$$\Delta L = \frac{f_\pi^2}{32}(m_k^2 + m_D^2)\,\mathrm{Tr}\,[(U + U^+) - 2I]$$

$$- \frac{f_\pi^2}{8\sqrt{3}}(m_k^2 - m_\pi^2)\,\mathrm{Tr}\,[\lambda_8(U + U^+)]$$

$$-\frac{\sqrt{6}}{96}f_\pi^2(3m_D^2 - m_k^2 - 2m_\pi^2)\,\mathrm{Tr}\,[\lambda_{15}(U + U^+)]$$

$$-\frac{f_\pi^2}{192}(6m_{\eta_c}^2 - 9m_D^2 - m_k^2 + 4m_\pi^2)\,\mathrm{Tr}\,[\lambda_{15}U]\,\mathrm{Tr}\,[\lambda_{15}U^+]. \qquad (4)$$

In equation (4) I is the identity matrix and the last term though unusual is necessary in order to obtain the correct mass of the $C\bar{C}$ state η_c. The symmetry-breaking Hamiltonian corresponding to ΔL is obviously given by

$$H = -\int \mathrm{d}^3x \Delta L.$$

We can now evaluate the SU(4) symmetry breaking contribution to the baryon masses by computing the matrix element of ΔH between the various baryon states $B_{\alpha\beta}$, i.e.

$$\langle B_{\alpha\beta}|\Delta H|B_{\alpha\beta}\rangle = \tfrac{1}{20}\int \mathrm{d}\mu(g) D^{*20}_{\alpha\beta}(g)(\Delta H) D^{20}_{\alpha\beta}(g).$$

The integration is over the SU(4) group manifold, $\mu(g)$ is the appropriate measure and the D functions satisfy the orthonormality conditions

$$\int \mathrm{d}\mu(g) D^{*n}_{\alpha'\beta'}(g) D^{m}_{\alpha\beta}(g) = n\delta_{nm}\delta_{\alpha\alpha'}\delta_{\beta\beta'}.$$

Using (3) and the relationship

$$g^{-1}\lambda_r g = \lambda_\beta D^{15}_{\beta r}(g^{-1})$$

ΔH can be expressed in terms of D functions. The problem now reduces to evaluating the appropriate Clebsch-Gordon coefficients after performing the angular integrations.

We then obtain the following expressions for the baryon masses:

$$\underline{8}\begin{cases} m_N = m_0 - \dfrac{5\mu}{48} + 0{\cdot}145\delta - \dfrac{3\chi}{16}, \\ m_\Sigma = m_0 - \dfrac{5\mu}{48} + 0{\cdot}145\delta + \dfrac{7\chi}{144}, \\ m_\Lambda = m_0 - \dfrac{5\mu}{48} + 0{\cdot}145\delta - \dfrac{7\chi}{144}, \\ m_\Xi = m_0 - \dfrac{5\mu}{48} + 0{\cdot}145\delta + \dfrac{5\chi}{36}, \end{cases}$$

$$\underline{6}\begin{cases} m_{\Sigma_c} = m_0 - \dfrac{\mu}{72} + 0{\cdot}254\delta - \dfrac{47\chi}{432}, \\ m_{\Xi_c} = m_0 - \dfrac{\mu}{72} + 0{\cdot}254\delta - \dfrac{47\chi}{864}, \\ m_{\Omega_c} = m_0 - \dfrac{\mu}{72} + 0{\cdot}254\delta + \dfrac{47\chi}{216}, \end{cases}$$

$$\underline{3}^* \begin{cases} m_{\Xi_{c'}} = m_0 + \dfrac{\mu}{12} + 0{\cdot}178\delta + \dfrac{11\chi}{288}, \\ m_{\Lambda_c} = m_0 + \dfrac{\mu}{12} + 0{\cdot}178\delta - \dfrac{11\chi}{144}, \end{cases}$$

$$\underline{3} \begin{cases} m_{\Xi_{cc}} = m_0 + \dfrac{2\mu}{9} + 0{\cdot}265\delta - \dfrac{13\chi}{432}, \\ m_{\Omega_{cc}} = m_0 + \dfrac{2\mu}{9} + 0{\cdot}265\delta + \dfrac{13\chi}{216}, \end{cases}$$

where m_0 is the common mass for the 20 plet,

$$\mu = \pi f_\pi^2 (m_D^2 - \tfrac{2}{3} m_\pi^2 - m_k^2) \int_0^\infty (1 - \cos f(r)) r^2 \, \mathrm{d}r,$$

$$\chi = \pi f_\pi^2 (m_k^2 - m_\pi^2) \int_0^\infty (1 - \cos f(r)) r^2 \, \mathrm{d}r,$$

and

$$\delta = \frac{\pi f_\pi^2}{36} (6 m_{\eta_c}^2 - 9 m_D^2 - m_k^2 + 4 m_\pi^2) \int_0^\infty \sin^2 f(r) r^2 \, \mathrm{d}r.$$

Now,

$$\pi \int_0^\infty [1 - \cos f(r)] r^2 \, \mathrm{d}r \approx \frac{49\,\mathrm{MeV}}{m_\pi^2 f_\pi^2}$$

(Adkins and Nappi 1984). Using the known form for $f(r)$ we can compute the integral $\pi \int_0^\infty \sin^2 f(r) r^2 \, \mathrm{d}r$ to be approximately $69\,\mathrm{MeV}/m_\pi^2 f_\pi^2$. This in turn implies that

$$\mu \approx 8718\,\mathrm{MeV}; \quad \chi \approx 582\,\mathrm{MeV} \quad \text{and} \quad \delta \approx 2356\,\mathrm{MeV}.$$

3. Discussion

In table 1 we give the numbers corresponding to the baryon masses based on quark model predictions (Singh 1982; Singh *et al* 1985), experimental results wherever available (Review of Particle Properties 1986) and the values obtained by us within the Skyrme model framework.

We observe from our calculations the following: (i) the simple symmetry-breaking pattern we have assumed predicts baryon masses which agree with quark model predictions to $\approx 15\%$; (ii) A more interesting feature is that despite observation (i), the generalized Gell-Mann Okubo sum rule

$$2(m_{\underline{3}} + m_{\underline{8}}) = 3 m_{\underline{3}}^* + m_{\underline{6}}$$

is very well satisfied. This happens although ΔL does not transform in any simple manner under SU(4). Thus, this group theoretical aspect relating SU(4) baryon masses follows independent of the fact that the SU(4) symmetry-breaking term has a complicated transformation under the action of the group. Comment (i) stresses the

Table 1. Masses of the 20-plet baryons of SU(4) (in MeV).

Representation	Particle	Present analysis	Experiment	Quark model prediction
$\underline{8}$	N	938·9 (input)	—	—
	Σ	1076·0	1193·0	—
	Λ	1019·0	1115·6	—
	Ξ	1129·0	1318·0	—
$\underline{6}$	Σ_c	2029·0	**2450·0	2430·0
	Ξ_c	2124·0	*2460·0	2550·0
	Ω_c	2219·0	*2740·0	2730·0
$\underline{3}^*$	Ξ_c'	2782·0	—	2490·0
	Λ_c	2716·0	****2281·0	2300·0
$\underline{3}$	Ξ_{cc}	4159·0	—	3680·0
	Ω_{cc}	4211·0	—	3870·0

Note:

1) The experimental values are taken from the Review of Particle Properties, April 1986, and the quark model predictions reported in Table 1 are as given by Singh (1982).

2) The symbols*,** and **** experimental numbers have the same meaning as in the Review of Particle Properties, April 1986.

important role of the flavour symmetry in using the Skyrmion to address questions which go beyond pure group theory.

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