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Effect of the three-body force on trinucleon bound systems considering S and S'-state admixture

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Abstract. We report the calculation of binding energy, charge form factor and point-like proton density of both ^{3}H and ^{3}He by the hyperspherical harmonics method with the inclusion of two-pion exchange three-nucleon force (Fujita-Miyazawa type). For the two-body force the N–N Afnan-Tang S-3 potential is taken. Coulomb and three-body forces are treated nonperturbatively. In this calculation the mixed symmetry S'-state of the trinucleon ground state is considered along with the space totally symmetric S-state.

Keywords. Nuclear structure; trinucleon systems; three-body force; hyperspherical harmonics method.

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1. Introduction

The extent to which three-body force (3BF) affects bound state properties of the trinucleon system has been the subject of intense interest recently. The fact that neither the binding energy (BE) nor the charge form factor (CFF) of the trinucleon can be satisfactorily understood in terms of two-nucleon force (2BF) alone and the subsequent suggestion that the inclusion of 3BF may solve the riddle have been responsible for this surge of interest in this problem.

During the early stage of studies of this problem Ballot and Fabre (Fabre 1979; Ballot and Fabre 1979) performed a calculation considering the inclusion of two-pion exchange (TPE) three body force by the hyperspherical harmonic expansion (HHE) method and inferred that the inclusion of 3BF may remove the discrepancies of calculations using standard two-nucleon interactions with experimental results; however that calculation included only a few partial waves in the potential harmonic expansion of the ground state wave function and the lowest multipolar contribution of the p-wave Fujita Miyazawa 3BF (Fujita and Miyazawa 1957). Sato and Tanaka (1974) and Sato *et al* (1974) performed a variational calculation for ^{3}H including TPE-3BF and obtained an enhancement of BE by 1·16–1·50 MeV for different 2BF and $R_{rms} = 1{\cdot}323$ fm. Yang (1974) also performed a variational calculation including TPE-3BF and reported an increase of 2·32 MeV in BE of ^{3}H and $R_{rms} = 1{\cdot}6$ fm. But none of them studied the effect of 3BF on the CFF and point-like proton density.

Hadjimichael (1978), on the other hand, used impulse approximation to investigate various effects, including Δ-resonance on $F_{ch}(q^2)$ (no calculation of BE) and reported that the contribution arising only from Δ-resonance is relatively small to reproduce the experimental data. Only by adding other effects (meson exchange currents (MEC),

recoil effect), $F_{ch}(q^2)$ becomes comparable with experimental data. Nogami *et al* (1981) studied the effect of TPE-3BF on the charge density of ^{3}H and ^{3}He by performing a simple model and variational calculation. The charge density $\rho(r)$ was calculated by them directly from a simple trial wavefunction and it was shown that a central depression indeed results but is not enough to account for the result obtained from experimental $F_{ch}(q^2)$ by McCarthy *et al* (1977). Torre *et al* (1981) calculated the BE of ^{3}H including the TPE-3BF with super-soft core 2BF using the equation developed by Faddeev (1960) and obtained an enhancement of BE by about 650 keV. Recently Ishikawa and Sasakawa (1986) reported the calculation of BE of ^{3}H including 3BF (using the Faddeev equation method). By varying a phenomenological parameter (cut-off mass) of 3BF and introducing some phenomenological repulsive potential along with the two-body and three-body force they obtained the correct triton BE of 8·48 MeV.

Das and coworkers (Das *et al* 1982b, c; Das and Coelho 1982; Das 1982; Coelho *et al* 1982) calculated the BE, CFF and point-like proton density for ^{3}H and ^{3}He using the HHE method, including the Fujita-Miyazawa (1957) form of TPE-3BF, while the ground state of the trinucleon was represented by the totally symmetric S-state. A purely phenomenological cut-off parameter x_0 was introduced to restrict highly singular 3BF at extremely short separations and investigated the effect of the 3BF on BE, $F_{ch}(q^2)$ and $\rho(r)$ of trinucleon systems as a function of x_0. They reported that for $x_0 = 0{\cdot}42$ fm the enhancement due to the inclusion of 3BF is small (10% and 50% of the discrepancies in the value of the first maximum of $|F_{ch}(q^2)|$ and BE respectively). A small central hole was reported in point-like charge density for ^{3}H for a rather unphysical value of x_0.

In this paper we report our calculation of trinucleon properties by the HHE method, considering the admixture of totally symmetric S-state and mixed symmetry S'-states. For 2BF we have taken a semi-realistic S-3 potential (Afnan and Tang 1968) and for 3BF the Fujita-Miyazawa (1957) form of TPE-3BE.

2. Theory

In the hyperspherical harmonics expansion method (Erens *et al* 1971; Ballot and Navarro 1975; Ballot and Fabre 1980) the completely antisymmetric wave function $\Psi(r, \Omega)$ of the trinucleon system is expressed as a sum of product of space wave function of specific symmetry (σ) and the spin–isospin wave function ($\Gamma^{(\sigma)}_{st}$) of the composite symmetry (so as to make Ψ totally antisymmetric).

$$\Psi(r, \Omega) = \sum \phi^{(\sigma)}(r, \Omega)\Gamma^{(\sigma)}_{st}, \tag{1}$$

where s and t represent total spin and isospin respectively. The most important symmetry components (σ) of the trinucleon ground state are space totally symmetric $S(L=0)$ state, mixed symmetry $S'(L=0)$ state and the $D(L=2)$ state. The space wavefunction $\phi^{(\sigma)}(r, \Omega)$ for a given symmetry component (σ) is then expanded in the potential basis (Ballot and Fabre 1980) of the hyperspherical harmonics (HH) functions, $\{\mathscr{P}^{(\sigma)}_{K\alpha}(\Omega)\}$

$$\phi^{(\sigma)}(r, \Omega) = r^{-5/2} \sum_{K\alpha} \mathscr{U}^{(\sigma)}_{K\alpha}(r)\mathscr{P}^{(\sigma)}_{K\alpha}(\Omega), \tag{2}$$

where r is the hyperradial variable which is the invariant global length in a six-dimensional space and Ω represents a set of five hyperangles (Faddeev 1960; Yakubowski 1967; Schmid and Ziegelmann 1974; Ballot and Fabre 1980). The complete orthonormal HH set $\{\mathscr{P}^{(\sigma)}_{K\alpha}(\Omega)\}$ are the angular part of homogeneous harmonic polynomial of degree K in six-dimensional space. The label $K\alpha$ stands for five quantum numbers related to the five degrees of freedom in Ω.

Substitution of (2) into the non-relativistic Schrödinger equation (in the relative co-ordinates) for three nucleons of mass m and projection on to a particular HH lead to a system of coupled differential equations (Ballot and Fabre 1980).

$$\left(-\frac{d^2}{dr^2}+\frac{\mathscr{L}_K(\mathscr{L}_K+1)}{r^2}+k^2\right)\mathscr{U}^{(\sigma)}_{K\alpha}(r)+\sum_{\sigma'K'\alpha'}\langle\sigma K\alpha|v|\sigma'K'\alpha'\rangle\mathscr{U}^{(\sigma')}_{K'\alpha'}(r)=0, \tag{3}$$

where

$$\mathscr{L}_K=K+3/2,$$

$$k^2=-(m/\hbar^2)E, \quad (E<0 \text{ for bound state}),$$

$$v=(m/\hbar^2)V,$$

V being the full interaction potential including the two- and three-body forces. The matrix element $\langle\sigma K\alpha|v|\sigma'K'\alpha'\rangle$ is calculated by expanding v in potential multipoles using the same potential basis. Two separate expansions, one for the 2BF and another for the 3BF, are to be considered. The matrix element for the 2BF is given by

$$\begin{aligned}\langle\sigma K\alpha|v^{(2)}|\sigma'K'\alpha'\rangle &= \left\langle\sigma K\alpha\left|\sum_{K''\alpha''}v^{(2)}_{K''\alpha''}(r)\mathscr{P}_{K''\alpha''}(\Omega)\right|\sigma'K'\alpha'\right\rangle \\ &\equiv 3\sum_{K''\alpha''}(-1)^{K''}\langle\sigma K\alpha|K''\alpha''|\sigma'K'\alpha'\rangle v^{(2)}_{K''\alpha''}(r),\end{aligned} \tag{4}$$

where $\{\mathscr{P}_{K''\alpha''}\}$ is the appropriate set consistent with the nature (central or tensor) of the interaction and $v^{(2)}_{K''\alpha''}(r)$ is the corresponding potential multipole (Ballot and Fabre 1980; Das *et al* 1982b). The geometrical structure coefficients $\langle\sigma K\alpha|K''\alpha''|\sigma'K'\alpha'\rangle$ are independent of interaction and hence can be calculated only once and stored for eventual use.

A similar procedure is used to obtain the matrix element for the 3BF. Among the various versions of the 3BF we have taken the classic form of TPE-3BF given by Fujita-Miyazawa (1957) which consists of two terms, generated by the s and p waves of the virtual pions. The effect of the former is drastically reduced by the σ-meson exchange and will not be considered here. Only a part of the p-wave contribution generates angular correlation, which we consider here. Thus for the 3BF $V^{(3)}$, effective for the hyper-radial equation (3) we take (since for $\sigma=S$ or S', $s=t=\frac{1}{2}$)

$$\begin{aligned}V^{(3)} &= \left\langle\Gamma^{(\sigma)}_{\frac{1}{2}\frac{1}{2}}\left|\sum_k V^{(3)}_P(k)\right|\Gamma^{(\sigma)}_{\frac{1}{2}\frac{1}{2}}\right\rangle \\ &= \sum_{k=1,2,3}C^{(\sigma)}C_P(3\cos^2\theta_k-1)U_{(2)}(x_i)U_{(2)}(x_j),\end{aligned} \tag{5}$$

where θ_k is the angle between the direction $\mathbf{x}_j$ and $\mathbf{x}_i$ and $\mathbf{x}_k \equiv \mathbf{r}_{ij} = \mathbf{r}_i - \mathbf{r}_j$ (i, j, k cyclic permutation); C_P is the 3BF coupling coefficient, whose estimated value ranges between 0·46 and 1·0 MeV

$$C^{(\sigma)} = 1 \quad \text{for} \left\langle \Gamma^{(S)}_{\frac{1}{2}\frac{1}{2}} \middle| \sum_k V_P^{(3)}(k) \middle| \Gamma^{(S)}_{\frac{1}{2}\frac{1}{2}} \right\rangle$$

$$= \tfrac{1}{6} \quad \text{for} \left\langle \Gamma^{(S'-)}_{\frac{1}{2}\frac{1}{2}} \middle| \sum_k V_P^{(3)}(k) \middle| \Gamma^{(S'-)}_{\frac{1}{2}\frac{1}{2}} \right\rangle$$

$$= \tfrac{13}{18} \quad \text{for} \left\langle \Gamma^{(S'+)}_{\frac{1}{2}\frac{1}{2}} \middle| \sum_k V_P^{(3)}(k) \middle| \Gamma^{(S'+)}_{\frac{1}{2}\frac{1}{2}} \right\rangle$$

where $\Gamma^{(S)}_{\frac{1}{2}\frac{1}{2}}$ is the spin function of the completely symmetric S state; $\Gamma^{(S'+)}_{\frac{1}{2}\frac{1}{2}}$ and $\Gamma^{(S'-)}_{\frac{1}{2}\frac{1}{2}}$ are the spin functions of the mixed symmetry S'-state, symmetric and antisymmetric under exchange of particles 1 and 2 respectively.

$$U_{(2)}(x) = \left(1 + \frac{3}{\mu x} + \frac{3}{(\mu x)^2}\right) \exp(-\mu x)/\mu x \tag{6}$$

$$\mu = 0{\cdot}7\,\text{fm}^{-1}.$$

The matrix element for the 3BF is given by

$$\langle \sigma K\alpha | v^{(3)} | \sigma' K'\alpha' \rangle = 3 \sum_{K''\alpha''} (-1)^{K''} \langle \sigma K\alpha | K''\alpha'' | \sigma' K'\alpha' \rangle v^{(3)}_{K''\alpha''}(r), \tag{7}$$

where $v^{(3)}_{K''\alpha''}(r)$ is the potential multiple for the 3BF (Das *et al* 1982a).

Equation (3) is an infinite set of coupled differential equations (CDE). For practical purposes, this set of equations is truncated to a finite number of partial waves after achieving convergence in BE to a predetermined accuracy. The truncated set can be solved exactly numerically. However to reduce computer time and memory requirements we approximately decouple the set of equation (3) by the uncoupled adiabatic approximation (UAA) method (Levinger and Fabre 1981; Das *et al* 1982a; Coelho *et al* 1982).

When S, S' admixture is considered the charge form factor is given by

$$F^{T_3}_{\text{ch}}(q) = \frac{1}{T_3 + \frac{3}{2}} [(3G_{\text{ES}}(q) + 2T_3 G_{\text{EV}}(q)]$$

$$\times \sum_{(\sigma)} [F^{(\sigma,\sigma')}(q/\sqrt{3}) - 4\sqrt{2}\, T_3 G_{\text{EV}}(q) F^{(S'+,S)}(q/\sqrt{3})], \tag{8}$$

where

$$T_3 = \tfrac{1}{2} \text{ for } {}^3\text{He and } -\tfrac{1}{2} \text{ for } {}^3\text{H}$$

and

$$F^{(\sigma,\sigma')}(q) = 8 \sum_{K,K',K''} \langle \sigma K\alpha | K''\alpha'' | \sigma' K'\alpha' \rangle$$

$$\times \int_0^\infty \mathscr{U}^{(\sigma)}_{K\alpha}(r) \mathscr{U}^{(\sigma')}_{K'\alpha'}(r) \frac{J_{2K''+2}(qr)}{(qr)^2} \mathrm{d}r. \tag{9}$$

$G_{ES}(q)$ and $G_{EV}(q)$ are the scalar and vector electric form factor of the nucleons and are given in Collard *et al* (1965) and Janssens *et al* (1966).

The point-like proton density is given by

$$\rho(r) = \frac{1}{2\pi^2 r} \int_0^\infty F_{ch}^{T_3}(q) \sin(qr) q \, dq. \tag{10}$$

3. Results

The Afnan-Tang S-3 potential (Afnan and Tang 1968) which is considered to be reasonably realistic, although quite simple in structure, has been chosen to represent the 2BF for comparison with more restrictive earlier calculations. This potential, being purely central (although spin-dependent), does not couple the S, S' states to the D state ($L = 2$) of the trinucleon. Calculations are performed with twelve partial waves each of the S-state and S'-state; ensuring the convergence in BE for the 2BF. It is seen from (5) and (7) that 3BF is extremely singular ($\sim r^{-6}$) for $r \to 0$ and is attractive for the equilateral triangle configuration of the trinucleon. This will make the Hamiltonian unbounded below and cause in infinite oscillation in the wavefunction as $r \to 0$. The reason for this unphysical behaviour is the over-simplified choice of pion nucleon form factors. In fact the nucleons are not point particles and the 3BF of Fujita-Miyazawa (FM) is an over-simplified form.

Attempts were made by the Tucson-Melbourne (Coon *et al* 1979; Coon and Glockle 1981) and the Brazil groups (Coelho *et al* 1983; Robilota *et al* 1985) to regularize the 3BF at short separation (or equivalently at high momentum transfers) by introducing parametrical pion-nucleon form factors and including an appropriate diagram to cancel the singular part of 3BF. However one should note that an element of uncertainty always remains due to the arbitrariness of the phenomenological parameter or the neglected higher order (and heavier meson exchange) diagrams. This ignorance in the form of 3BF is less critical than it appears at first thought since the ignorance is essentially for the extremely short separations, which is masked by the hard core two nucleon potential. Since we are using a *soft core* potential the singularity in the FM-3BF is, however, crucial. To regularize the 3BF we introduce a phenomenological cut-off parameter x_0 and replace $U_{(2)}(x)$ by

$$\sim \begin{cases} U_{(2)}(x_0) & \text{for } x \leqslant x_0 \\ U_{(2)}(x) & \text{for } x > x_0. \end{cases}$$

Although extremely simple in nature this modification also removes the singularity in the FM-3BF for $r \to 0$ for a suitable choice of x_0.

Results of our calculation are presented in table 1. Our calculation shows that the S-3 potential overbinds the trinucleon system. Both BE and the first maximum of $|F_{ch}(q^2)|(F_{max})$ have been plotted as a function of x_0 in figure 1 for ^{3}H and in figure 2 for ^{3}He, which show that both BE and F_{max} depend strongly on x_0. Both these quantities show infinite discontinuities at specific values of x_0 and a smooth dependence between discontinuities. Each continuous region corresponds to a definite number of nodes near origin (NNO) in the wavefunction of UAA decoupled hyperradial equation. Figures 3 and 4 show the calculated point-like charge density $\rho(r)$. None of the curves shows any central hole.

Table 1. Results of calculation for the bound states of trinucleon system.

Description	Number of partial waves		x_0 (fm)	BE (MeV)	Percentage		$\lvert F_{ch}(q^2)\rvert$			rms charge radius (fm)
	S	S'			P_S	$P_{S'}$	Value at $q = 1\,\text{fm}^{-1}$	Position of first zero (fm^{-2})	$F_{max} \times 10^{+3}$	
^{3}H(2BF)	12	0		6·504	100		0·590	16·01	1·49	1·81
	12	12		9·238	95·9	4·1	0·624	15·87	2·29	1·71
^{3}H(2BF + 3BF)	12	0	0·340	7·628	100		0·616	16·53	1·84	1·73
$C_p = 0·9$	12	12	0·385	10·562	95·6	4·4	0·645	18·16	2·96	1·64
^{3}He(2BF)	12	0		5·802	100		0·565	15·95	1·06	1·89
	12	12		8·501	95·9	4·1	0·598	15·80	1·67	1·79
^{3}He(2BF + 3BF)	12	0	0·340	6·892	100		0·592	16·45	1·32	1·81
$C_p = 0·9$	12	12	0·385	9·784	95·3	4·7	0·617	18·12	2·16	1·70

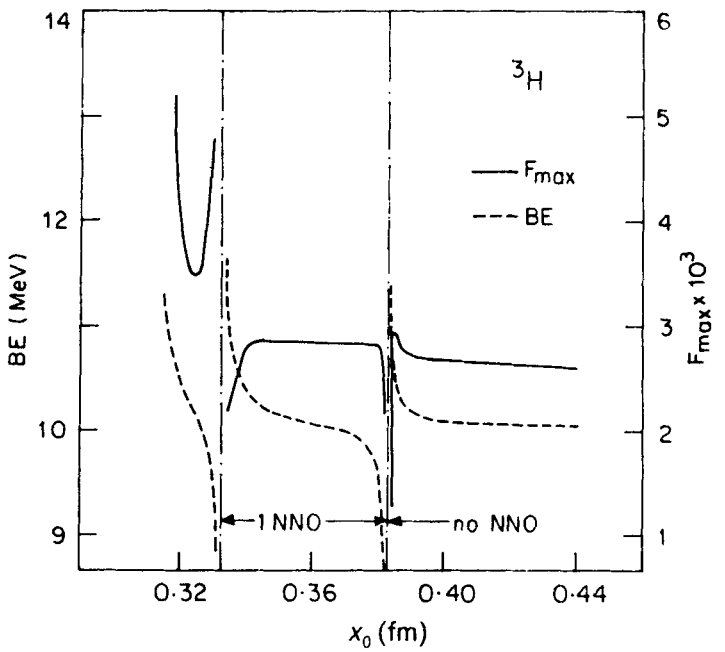

Figure 1. Calculated binding energies and F_{max} as a function of x_0 for triton.

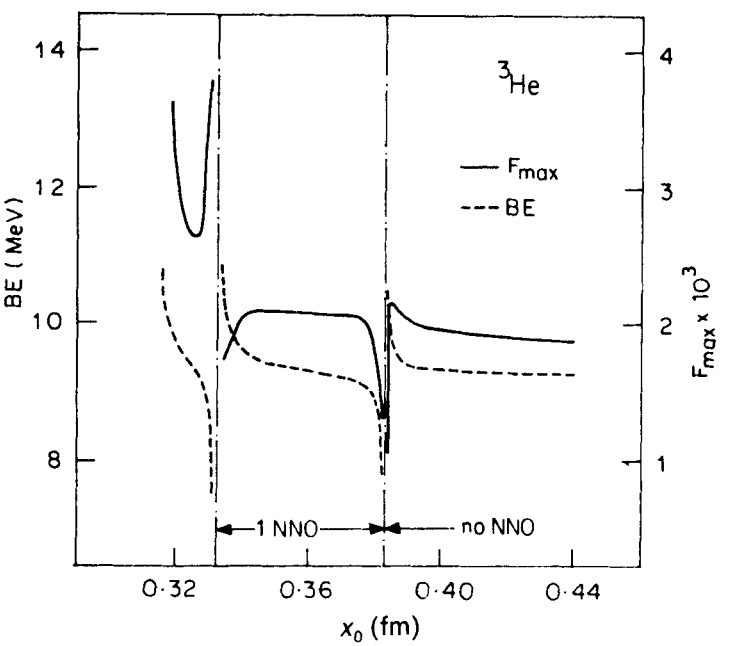

Figure 2. Calculated binding energies and F_{max} as a function of x_0 for ^{3}He.

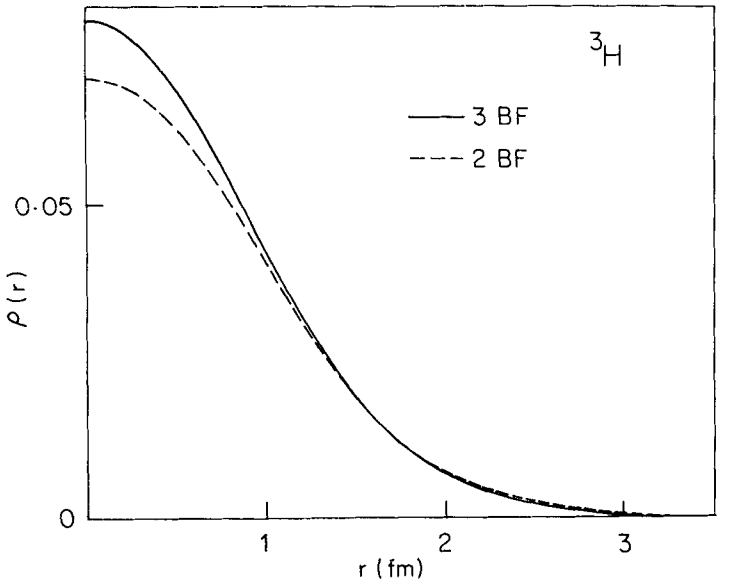

Figure 3. Calculated point-like charge density for ^{3}H (for 3BF $C_p = 0{\cdot}9$ MeV, $\mu = 0{\cdot}7$ fm^{-1} $x_0 = 0{\cdot}385$ fm).

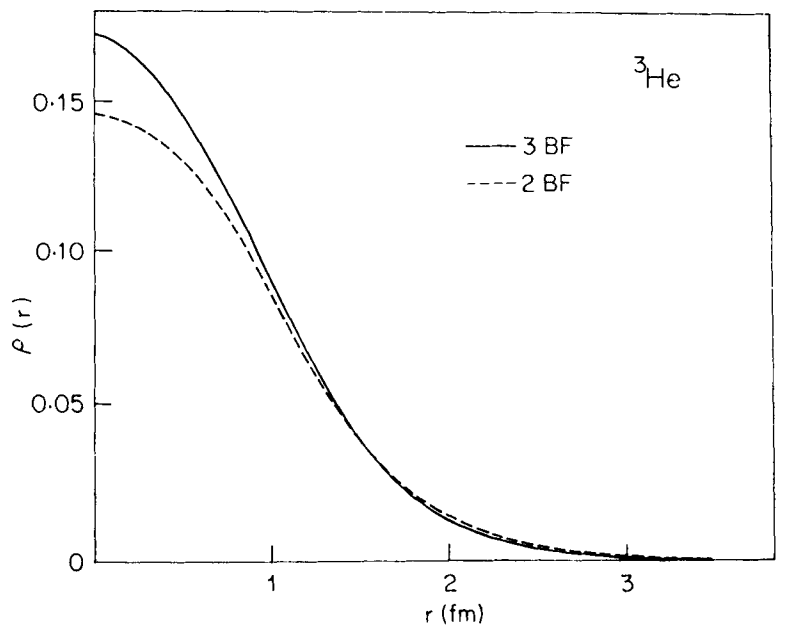

Figure 4. Calculated point-like charge density for ^{3}He (for 3BF $C_p = 0{\cdot}9$ MeV, $\mu = 0{\cdot}7$ fm^{-1}, $x_0 = 0{\cdot}385$ fm).

The nature of the curves of figures 1 and 2 is somewhat similar to those of Das *et al* (1982b), although in the present case, F_{max} for ^3H is nearly independent of x_0 in the one NNO region, away from the discontinuities. The nature of F_{max} in the two-NNO region is, however, markedly different from Das *et al* (1982b). Instead of a maximum, F_{max} has a minimum in this region. Further the inclusion of S' state shifts the position of a specific singularity towards a higher x_0 value compared to that for the S state alone (Das *et al* 1982b). Since NNO appears due to the strongly attractive 3BF, one concludes that the inclusion of the mixed symmetry S' state enhances the attractive contribution of 3BF necessitating a larger value of x_0 to compensate for this extra attraction. This also seems to be the reason for the markedly different behaviour of F_{max} in the two NNO region for the present case. The contribution arising from the S' state strongly influences F_{max} and its dependence on x_0 for small values of x_0 i.e. in the region where 3BF has a dominantly attractive contribution. This appears to compensate the variation of F_{max} on x_0 due to S state alone in such a way that F_{max} is nearly independent of x_0 in the one-NNO region and has an opposite dependence in the two-NNO region. However we note that wavefunctions presenting nodes do not correspond to physical realities in the trinuclear problem and one does not expect a node in the hyper-radial wavefunction for the ground state. Hence we choose a value of x_0, corresponding to the maximum of F_{max} in the zero-NNO region. From figures 1 and 2 it is found to be 0·385 fm. Incidentally these values are close to the hard core radius usually adopted in realistic two-nucleon potential (for example Reid hard core potential had hard core radius $r_c = 0{\cdot}4$ fm). This again lends some credibility to our contention that the strong singularity of the FM-3BF and the related ignorance of the exact form of 3BF at extremely short separations are essentially unimportant when a truly 'realistic' nucleon-nucleon force (which necessarily incorporates either a hard core or a sufficiently strong and repulsive soft core) is employed. However an essentially exact three-body calculation with the hard core potential is an extremely difficult problem and until it is satisfactorily tackled, one has to depend on 2BF with soft core of varying degree of 'softness', together with more or less phenomenological forms of 3BF.

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