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The one-loop Green's functions of dimensionally reduced gauge theories

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Abstract. We apply the dimensional regularization technique as well as that by dimensional reduction to the calculation of the regularized one-loop Green's functions in d_0-dimensional Yang-Mills theory with real massless scalars and spinors in arbitrary (real) representations of a gauge group G. As a particular example, the super-symmetrically regularized one-loop Green's functions of the $N = 4$ supersymmetric Yang-Mills model are derived.

Keywords. Regularization; gauge theories; supersymmetry.

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1. Introduction

The Kaluza-Klein idea (Kaluza 1921; Klein 1926) initiated the studying of gauge theories in higher dimensions of space-time. It has recently gained further importance due to developments of supersymmetric gauge theories including supergravities. Indeed, many interesting four-dimensional gauge models with extended supersymmetry are closely related to Lagrangian field theories in more than four space-time dimensions. The ten-dimensional supersymmetric Yang-Mills model (with $N = 4$ supersymmetry from the four-dimensional view point) and the ten-dimensional supergravity theories are of special importance due to their relevance to superstrings considered now as the most promising candidates for the unified theory of all fundamental interactions. These field theories arise from superstrings in the low-energy limit.

Introducing higher dimensions had been considered originally as a convenient mathematical tool for analyzing the structure of rather involved gauge theories with extended supersymmetry in four-dimensional space-time. However, recent advances in superstrings (Schwarz 1982, 1985) on the one hand, and the success in unifying Kaluza-Klein programme (Duff *et al* 1986) on the other, have reinforced the importance and possible physical significance of additional dimensions.

Therefore, it is of interest to elaborate the quantum structure of higher-dimensional gauge theories, as regards the effective action, the divergences, the ultraviolet asymptotics of Green's functions and so on. Despite considerable work in this field, only partial results are available for maximally extended supersymmetric gauge theories in view of the absence of a superfield description acceptable for quantum calculations in superspace. Hence, it is reasonable to further develop the conventional perturbative methods, which are familiar in the quantum non-Abelian gauge theory

(Slavnov and Faddeev 1978; Voloshin and Ter-Martirosyan 1984 and references therein), to apply them in the framework of d-dimensional (supersymmetric) Yang-Mills theory with an arbitrary d.

In the context of supersymmetric theories we encounter the problem of supersymmetric regularization, because conventional dimensional regularization seems to be in contradiction with supersymmetry. Nevertheless, Townsend and Nieuwenhuizen (1979), and Sezgin (1980) have shown in an explicit two-loop calculation that conventional dimensional regularization respects supersymmetry Ward identities in the Wess-Zumino model (Wess and Zumino 1974). However, if spin-1 fields are involved, the $N = 1$, $d_0 = 4$ supersymmetric Yang-Mills theory requires a modified version of dimensional regularization. Indeed, the number of bosonic and fermionic degrees of freedom depends on d in different ways, so this may be a source of potential discrepancies if usual dimensional regularization technique is used.

To maintain the supersymmetry Ward identities in quantum theory, Siegel (1979) (Siegel and Nieuwenhuizen 1980) has proposed a modified version of dimensional regularization based on dimensional reduction, which was proved to preserve the Ward identities of both supersymmetry and gauge invariance up to the second loop level in four space-time dimensions (Capper *et al* 1980). The difficulties in higher loop-orders were discussed by Avdeev *et al* (1981).

The dimensional reduction technique proposed by Siegel is based on analytically continuing only the number of coordinates and momenta, but not the number of components of the fields. This technique was subsequently termed the supersymmetric dimensional regularization by dimensional reduction (SRDR). In four dimensions, SRDR was applied to calculate the 3-loop β-functions in the N-extended supersymmetric Yang-Mills theories (Avdeev *et al* 1980; Avdeev and Tarasov 1982).

Our work is devoted to the calculation of the one-loop Green's functions in the d_0-dimensional (super) Yang-Mills theory, regularized by a continuation to d-dimensions*. We also consider the massless real scalar and spinor matter fields in arbitrary (real) representations of a compact and semi-simple Lie group G for a greater completeness of the results, primarily keeping in mind the applications to the supersymmetric models. We employ both conventional dimensional regularization (DR) and SRDR to facilitate the comparison of these approaches. This clearly shows that SRDR is a viable alternative to dimensional regularization in ordinary Yang-Mills theories. As an example of application of SRDR, we calculate the supersymmetrically regularized one-loop Green's functions in the $N = 4$ supersymmetric Yang-Mills model (SSYM), which emerges as the low-energy approximation to the open superstring. SRDR does not violate the Ward identities of supersymmetry, even when the supersymmetry is explicitly broken by the gauge fixing, whereas conventional dimensional regularization does.

2. DR and SRDR in multi-dimensional non-supersymmetric gauge theories and the one-loop two-point Green's functions

In this section we consider d_0-dimensional non-Abelian gauge theory with the total Lagrangian in accordance with Faddeev-Popov quantization prescription and

*We use throughout this paper "d" to designate the formally continuous number of space-time dimensions, whilst "d_0" takes only the positive integer values of d.

compare SRDR technique with that of conventional DR. The Lagrangian density is given by

$$L_B = -\tfrac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} + \frac{1}{2\alpha}(\partial_\mu A^{\mu a})^2 + \bar{c}^a\partial^\mu(\partial_\mu\delta^{ab} + f^{acb}A^c_\mu)c^b + \tfrac{1}{2}(\partial_\mu\Phi^A + A^a_\mu T^{aAB}\Phi^B)^2, \tag{1}$$

where we have introduced the following notations

$$\begin{aligned} F^a_{\mu\nu} &= \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + f^{abc}A^b_\mu A^c_\nu, \\ f^{adc}f^{bdc} &= \delta^{ab}G, \\ (T^aT^a)^{AB} &= C_1\delta^{AB}, \\ \mathrm{tr}\,(T^aT^b) &= C_2\delta^{ab}. \end{aligned} \tag{2}$$

f^{abc} are the totally antisymmetric structure constants of the underlying gauge group G, $\{T^a\}$ are the (antisymmetric) generators of a real representation of G, by which the real scalars Φ^A transform. The gauge fixing term and the corresponding ghost term in (1) have been chosen in α-gauge ($\alpha \equiv \gamma - 1$) to give a vector propagator of the form

$$D^{ab}_{\mu\nu}(p) = -i\delta^{ab}\frac{\eta_{\mu\nu}p^2 - \gamma p_\mu p_\nu}{p^4}. \tag{3}$$

The flat d_0-dimensional space-time is assumed to be Minkowski space with signature $\eta_{\mu\nu} = \mathrm{diag}(+, -, \ldots, -)$.

Introducing real (Majorana) fermions gives rise to the restrictions on d_0: the d_0-dimensional Minkowski space-time admits Majorana spinors if and only if $d_0 = 2; 4$ (mod 8). Under this condition we can add to (1) the fermion contribution as follows

$$L_F = \tfrac{1}{2}i\bar{\psi}^Q\gamma^\mu(\partial_\mu\psi^Q + A^a_\mu R^{aQT}\psi^T) + \tfrac{1}{2}i\bar{\psi}^Q[N^{QT}_A + i\gamma_{d_0+1}P^{QT}_A]\Phi^A\psi^T, \tag{4}$$

where both minimal and Yukawa-type couplings appear. Of course, the latter are supposed to be consistent with gauge invariance. $\{R^a\}$ are just the (antisymmetric) generators of a real representation of G:

$$\begin{aligned} (R^aR^a)^{ST} &= R_1\delta^{ST}, \\ \mathrm{tr}\,(R^aR^b) &= R_2\delta^{ab}. \end{aligned} \tag{5}$$

To set the notations, let us define also

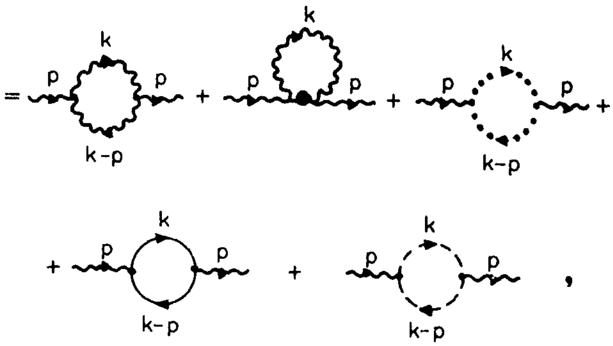
$$\begin{aligned} (N_AN_A)^{RT} &= N_1\delta^{RT}, \\ \mathrm{tr}\,(N_AN_B) &= N_2\delta_{AB}. \end{aligned} \tag{6}$$

The d_0-dimensional gamma matrices satisfy the usual equations

$$\begin{aligned} \{\gamma_\mu, \gamma_\nu\} &= 2\eta_{\mu\nu}, \\ \mathrm{tr}\,(\gamma_\mu\gamma_\nu) &= 2^{d_0/2}\eta_{\mu\nu}, \\ \gamma^2_{d_0+1} &= 1. \end{aligned} \tag{7}$$

The Feynman rules, which are needed in quantum perturbation theory, are easily read off from (1) and (4) in a standard way. As regards the one-loop Yang-Mills field two-point Green's function, one needs to calculate the contributions, which can be represented diagrammatically as follows

$$\langle A_\mu A_\nu \rangle_{\text{1-loop}} = D_{\mu\rho}(p)\Pi_{\rho\lambda}(p)D_{\lambda\nu}(p) \tag{8}$$



where each graph displays the contribution of Yang-Mills selfinteractions, ghosts, scalar and spinor minimal couplings, respectively. For example, the first graph in (8) can be reduced to the calculation of the following integral:

$$\begin{aligned}\Pi^{(A)ab}_{\mu\nu}(p) = \tfrac{1}{2}G\delta^{ab}\int\frac{d^dk}{(2\pi)^d}&\{\eta_{\mu\rho}(p+k)_\lambda + \eta_{\rho\lambda}(p-2k)_\mu + \eta_{\mu\lambda}(k-2p)_\rho\}\\ &\times\frac{1}{(p-k)^4k^4}(\eta_{\rho\alpha}k^2 - \gamma k_\rho k_\alpha)[\eta_{\lambda\beta}(p-k)^2 - \gamma(p-k)_\lambda(p-k)_\beta]\\ &\times\{\eta_{\nu\alpha}(k+p)_\beta + \eta_{\nu\beta}(k-2p)_\alpha + \eta_{\alpha\beta}(p-2k)_\nu\}.\end{aligned} \tag{9}$$

DR implies consideration of all the fields and the operators referred to the d-dimensional space-time. By making use of the Feynman reparametrization formulae

$$\frac{1}{a_1\cdots a_n} = \int_0^1 \mathrm{d}z_1\cdots\int_0^1 \mathrm{d}z_n\frac{(n-1)!\,\delta(1-z_1-\cdots-z_n)}{[a_1z_1+\cdots+a_nz_n]^n} \tag{10}$$

in rewriting the denominators, appropriately changing the integration variables and applying the integration formulae obtained via Wick rotation in the complex k^0-plane (all the operations are the standard ones)

$$\begin{aligned}\int\frac{(k^2)^m\,d^nk}{[k^2+p^2z(1-z)]^l} &= \frac{i\pi^{n/2}}{\Gamma\left(\frac{n}{2}\right)}(-1)^{m+l}[-p^2z(1-z)]^{m+\frac{n}{2}-l}\\ &\quad\times\frac{\Gamma\left(m+\frac{n}{2}\right)\Gamma\left(l-\frac{n}{2}-m\right)}{\Gamma(l)},\end{aligned} \tag{11}$$

the straightforward calculation of the polarization operator in (8) results in

$$\mathrm{DR}:\Pi^{ab}_{\mu\nu}(p)=\frac{i\delta^{ab}}{(4\pi)^{d/2}}(\eta_{\mu\nu}-p_\mu p_\nu)(-p^2)^{\frac{d}{2}-2}$$

$$\times\frac{\Gamma\left(\frac{d}{2}-1\right)\Gamma\left(\frac{d}{2}\right)\Gamma\left(2-\frac{d}{2}\right)}{\Gamma(d)}\left\{(3d-2)G-\tfrac{1}{2}\gamma(2d-7)\right.$$

$$\times(d-1)G-\tfrac{1}{4}\gamma^2(d-4)(d-1)G+C_2$$

$$\left.+\left(\frac{d}{2}-1\right)2^{(d_0/2)-1}R_2\right\}. \tag{12}$$

Any particular contribution of vector and ghost fields, scalars or spinors can be easily extracted from (12). $\Pi_{\mu\nu}(p)$ is transverse, as is required from the Ward identity $p^\nu\Pi_{\mu\nu}(p)=0$.

Alternatively, SRDR prescription consists of continuing in the number of space-time dimensions from d_0 to $d<d_0$, keeping the numbers of components of all other tensors fixed. Hence, we must take care in distinguishing between d_0-dimensional $\eta_{\mu\nu}$ resulting from $A_\mu A_\nu$-contractions and d-dimensional $\eta^{(d)}_{\mu\nu}$ resulting from $k_\mu k_\nu$ momentum integrals. We thus calculate pure Yang-Mills contribution (including contribution of ghosts) to the polarization operator

$$\mathrm{SRDR}:\Pi^{(A+c)ab}_{\mu\nu}(p)=\frac{i\delta^{ab}G}{(4\pi)^{d/2}}(-p^2)^{\frac{d}{2}-2}\frac{\Gamma\left(\frac{d}{2}-1\right)\Gamma\left(\frac{d}{2}\right)\Gamma\left(2-\frac{d}{2}\right)}{\Gamma(d)}$$

$$\times\left\{4(d-1)\eta_{\mu\nu}p^2-(d_0-2)\eta^{(d)}_{\mu\nu}p^2-(4d-d_0-2)p_\mu p_\nu\right.$$

$$-\frac{\gamma}{2}(d-1)[2(d-2)\eta_{\mu\nu}p^2-3\eta^{(d)}_{\mu\nu}p^2-(2d-7)p_\mu p_\nu]$$

$$\left.-\frac{\gamma^2}{4}(d-4)(d-1)[\eta^{(d)}_{\mu\nu}p^2-p_\mu p_\nu]\right\}. \tag{13}$$

The analogous SRDR-calculation of fermion contribution into the polarization operator results in

$$\mathrm{SRDR}:\Pi^{(\psi)ab}_{\mu\nu}(p)=\frac{i\delta^{ab}R_2 2^{n(d_0)}}{2(4\pi)^{d/2}}(-p^2)^{\frac{d}{2}-2}\Gamma\left(\frac{d}{2}-1\right)$$

$$\times\frac{\Gamma\left(\frac{d}{2}\right)}{\Gamma(d)}\Gamma\left(2-\frac{d}{2}\right)\{(d-1)\eta_{\mu\nu}-\eta^{(d)}_{\mu\nu}]p^2-(d-2)p_\mu p_\nu\}, \tag{14}$$

where we have introduced the notations

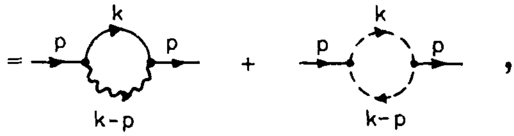
$$n_M(d_0)=\tfrac{1}{2}d_0-1,$$

$$n_{MW}(d_0) = \tfrac{1}{2}d_0 - 2 \tag{15}$$

for Majorana or Majorana-Weyl spinors, respectively. At the latter choice, the condition $d_0 = 2 \pmod 8$ is implied to be the case, as it is required for the existence of MW-spinors. Both the $\Pi^{(A+c)}_{\mu\nu}$ and $\Pi^{(\psi)}_{\mu\nu}$ are transverse in the d-dimensional vector field sector, as they should be.

The one-loop contribution to the scalar propagator in the theory (1) and (4) can be represented diagrammatically as follows

$$\langle \Phi\Phi \rangle_{\text{1-loop}} = D(p)M(p)D(p) \tag{16}$$



$$D(p)^{AB} = \frac{i\delta^{AB}}{p^2}.$$

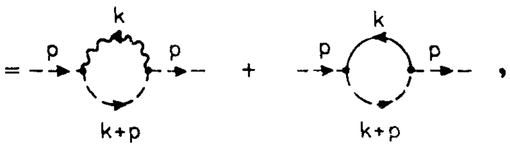
The straightforward DR-calculation of (16) results in

$$M^{AB}(p) = \frac{i\delta^{AB}}{(4\pi)^{d/2}}(-p^2)^{\frac{d}{2}-1}\Gamma\left(\frac{d}{2}-1\right)\frac{\Gamma\left(\frac{d}{2}\right)}{\Gamma(d)}$$
$$\times\Gamma\left(2-\frac{d}{2}\right)(d-1)\{2C_1(\gamma[3-d]-2)-2^{n(d_0)}N_2\}. \tag{17}$$

It should be noted here that by choosing the Feynman gauge ($\gamma = 0$) we recognize that the SRDR-derived $\Pi^{(A+c)}_{\mu\nu}(p)$ in the $(d_0 - d)$-dimensional sector in (14) is precisely the contribution of the $(d_0 - d)$ scalar/vector loop to the scalar mass operator $M(p)$ in the adjoint representation of a gauge group.

Finally, we present the result of the calculation of the fermion mass operator in the theory (1) and (4)

$$\langle \psi\bar{\psi} \rangle_{\text{1-loop}} = S(p)\Sigma(p)S(p) \tag{18}$$



$$S^{QR}_{\alpha\beta}(p) = \frac{i\delta^{QR}[p_\mu\gamma^\mu]_{\alpha\beta}}{p^2}.$$

In the one-loop approximation considered, the $\Sigma(p)$ arrived at is as follows

$$\Sigma^{QR}_{\alpha\beta}(p) = \frac{i\delta^{QR}}{(4\pi)^{d/2}}[p_\mu\gamma^\mu]_{\alpha\beta}(-p^2)^{\frac{d}{2}-2}$$

$$\times\Gamma\left(\frac{d}{2}-1\right)\frac{\Gamma\left(\frac{d}{2}\right)}{\Gamma(d)}\Gamma\left(2-\frac{d}{2}\right)(d-1)$$
$$\times\{(d-2)(\gamma-1)R_1-N_1\}. \tag{19}$$

3. The SRDR regularized one-loop Green's functions of $N = 4$ SSYM

The $N = 4$ super-Yang-Mills model (Brink *et al* 1977) has the simplest representation in the $d_0 = 10$ space-time in terms of the Yang-Mills field A^a_μ and the Majorana-Weyl spinor field ψ^a_α in the adjoint representation of a gauge group. The classical Lagrangian

$$L_S = -\tfrac{1}{4}(F^a_{\mu\nu})^2 + \frac{i}{2}\bar{\psi}^a\gamma^\mu(\partial_\mu\psi^a + A^c_\mu f^{acb}\psi^b) \tag{20}$$

is invariant (up to a total derivative) under the following supersymmetry transformation with a MW-spinor space-time independent parameter ε

$$\delta A^a_\mu = i\bar{\varepsilon}\gamma_\mu\psi^a,$$
$$\delta\psi^a = -\tfrac{1}{2}\gamma^\mu\gamma^\nu F^a_{\mu\nu}\varepsilon. \tag{21}$$

In the derivation of this result it is necessary to use the identity

$$f^{abc}(\bar{\psi}^a\gamma_\mu\psi^b)(\bar{\varepsilon}\gamma^\mu\psi^c) = 0, \tag{22}$$

which is only true in $d_0 = 2; 4; 6$ and 10 dimensions and is retained by dimensional reduction. This identity enables us to anticipate the failure of conventional DR in its naive form to preserve the supersymmetry Ward identities, since if we were to go down to $d < 10$ with d-dimensional gamma matrices, (22) would no longer hold. Instead, this identity is clearly valid with ten-dimensional 32×32 gamma matrices.

In quantum theory we consider the total effective Lagrangian with the gauge-fixing and ghost terms included according to general Faddeev-Popov prescription (in α-gauge). Now only the SRDR prescription works and so the formulae (13) and (14) should be applied in the particular case $d_0 = 10$ with the restrictions

$$\psi^a = \gamma_{11}\psi^a; \quad R_1 = R_2 = -G. \tag{23}$$

Adding contributions (13) and (14) together under these conditions results in

$$\Pi^{ab}_{\mu\nu}(p) = \frac{-i\delta^{ab}G}{(4\pi)^{d/2}}(-p^2)^{\frac{d}{2}-2}$$
$$\times\frac{\Gamma\left(\frac{d}{2}-1\right)\Gamma\left(\frac{d}{2}\right)\Gamma\left(2-\frac{d}{2}\right)}{\Gamma(d)}\left\{4(\eta^{(d)}_{\mu\nu}p^2 - p_\mu p_\nu) - \frac{\gamma}{2}(d-1)\right.$$
$$\times[2(d-2)\eta_{\mu\nu}p^2 - 3\eta^{(d)}_{\mu\nu}p^2 - (2d-7)p_\mu p_\nu] - \tfrac{1}{4}\gamma^2(d-4)(d-1)$$
$$\left.\times(\eta^{(d)}_{\mu\nu}p^2 - p_\mu p_\nu)\right\}. \tag{24}$$

We note, in passing, that some particular cancellations between (13) and (14) take place under the conditions (23).

The calculation of the SRDR-regularized fermion mass operator in the $N=4$ SSYM is quite analogous. The result turns out to be essentially the same as that in (19) with $N_1=0$ and $R_1=-G$.

$$\Sigma^{ab}_{\alpha\beta}(p)=\frac{i\delta^{ab}G}{(4\pi)^{d/2}}[p_\mu\gamma^\mu\tfrac{1}{2}(1+\gamma_{11})]_{\alpha\beta}(-p^2)^{\frac{d}{2}-2}$$
$$\times\frac{\Gamma\left(\frac{d}{2}\right)}{\Gamma(d)}\Gamma\left(\frac{d}{2}-1\right)\Gamma\left(2-\frac{d}{2}\right)(d-2)(d-1)(1-\gamma)$$
$$=\frac{-i\delta^{ab}G\alpha}{(4\pi)^{d/2}}[p_\mu\gamma^\mu\tfrac{1}{2}(1+\gamma_{11})]_{\alpha\beta}(-p^2)^{\frac{d}{2}-2}$$
$$\times\frac{\Gamma^2\left(\frac{d}{2}\right)}{2\Gamma(d-1)}\Gamma\left(2-\frac{d}{2}\right). \tag{25}$$

The supersymmetry Ward identity is derived by a set of standard tricks. First, one defines the generating functional $\Gamma(A^a_\mu,\psi^a)$ of the one-particle-irreducible (1PI) Green's functions by a functional Legendre transform of

$$Z(J^a_\mu,\bar{J}^a)=\int[\mathrm{d}A\,\mathrm{d}\psi\,\mathrm{d}\bar{c}\,\mathrm{d}c]\exp\left\{i\int\mathrm{d}^{10}xL_{S,\mathrm{tot}}\right\}, \tag{26}$$

where conventional sources J^a_μ and $\bar{J}^a$ coupled to A^a_μ and ψ^a, respectively, have been added to $L_{S,\mathrm{tot}}$. Second, one considers the variation of generating functional under an infinitesimal supersymmetry transformation (21) on the fields. Keeping in mind the invariance of the integration measure under the change of variables, varying Z and operating on the resulting equation in terms of the $\Gamma(A^a_\mu,\psi^a)$ with

$$\left(\frac{\delta}{\delta A^c_\nu(z)}\right)\left(\frac{\delta}{\delta\bar{\psi}^f(y)}\right), \tag{27}$$

the following identity is obtained in the Feynman gauge (cf Capper *et al* 1980)

$$\frac{\delta J^a_\mu(z')}{\delta A^c_\nu(z)}\frac{\delta\bar{J}^b(y')}{\delta\bar{\psi}^f(y)}\langle\delta^{10}(z'-x)\bar{\varepsilon}\gamma^\mu\psi^a(x)\psi^b(y')\rangle$$
$$+\frac{1}{2}\frac{\delta\bar{J}^a(y')}{\delta\bar{\psi}^f(y)}\frac{\delta J^{b\mu}(z')}{\delta A^c_\nu(z)}\langle\delta^{10}(y'-x)iA^b_\mu(z')\gamma^\rho\gamma^\sigma F^a_{\rho\sigma}(x)\varepsilon\rangle$$
$$+\frac{\delta^2\bar{J}^a(y')}{\delta A^c_\nu(z)\delta\bar{\psi}^f(y)}\langle\partial^\rho A^b_\rho(x)\bar{\varepsilon}\gamma^\sigma\partial_\sigma\psi^b(x)\bar{\psi}^a(y)\rangle$$
$$+\frac{\delta\bar{J}^a(y')}{\delta\bar{\psi}^f(y)}\frac{\delta J^{h\mu}(z')}{\delta A^c_\nu(z)}\langle\partial^\rho A^b_\rho(x)\bar{\varepsilon}\gamma^\sigma\partial_\sigma\psi^b(x)\psi^a(y')iA^h_\mu(z')\rangle$$

$$+\frac{\delta^2 \bar{J}^b(y')}{\delta A_\nu^c(z)\delta\bar{\psi}^f(y)}\langle f^{apq}\psi^b(y')\partial^\mu\bar{c}^a(x)\bar{\varepsilon}\partial_\mu\psi^p(x)c^q(x)\rangle$$

$$+\frac{\delta\bar{J}^b(y')}{\delta\bar{\psi}^f(y)}\frac{\delta J^{h\mu}(z')}{\delta A_\nu^c(z)}\langle f^{apq}\psi^b(y')iA_\mu^h(z')\partial_\mu\bar{c}^a(x)\bar{\varepsilon}\gamma^\mu\psi^p(x)c^q(x)\rangle. \tag{28}$$

The integration over the points y', z' and x is understood. The notation $\langle(\cdots)\rangle$ stands for

$$\int[\mathrm{d}A\,\mathrm{d}\psi\,\mathrm{d}\bar{c}\,\mathrm{d}c](\cdots)\exp\left\{i\int \mathrm{d}^{10}xL_{S,\mathrm{tot}}\right\}. \tag{29}$$

Now 1PI Green's functions are given by functional derivatives acting on sources, for example,

$$\frac{\delta J^{a\mu}(z')}{\delta A_\nu^c(z)}=\frac{\delta^2\Gamma}{\delta A_\mu^a(z')\delta A_\nu^c(z)}=G^{ac\mu\nu}(z',z). \tag{30}$$

In the tree approximation the validity of the Ward identity (28) is guaranteed by the identity (22), as it should. The verification of the identity (28) at the one-loop level is considerably more subtle. Here we emphasize the necessity to carefully distinguish between d_0-dimensional $\eta_{\mu\nu}$ and d-dimensional $\eta_{\mu\nu}^{(d)}$ Kronecker deltas, resulting from $A_\mu A_\nu$ contractions and from $k_\mu k_\nu$ momentum integrals, respectively, in actual quantum calculations. In particular, it results in the different ways in which d and d_0 enter Feynman rules. This essential technical device should be taken into account also if one wishes to extend SRDR prescription to higher orders of perturbation theory.

The verification of the Ward identity (28) by making use of the SRDR prescription goes exactly as in the well-known four-dimensional case (Capper *et al* 1980) and is not reported in detail here. The mismatch between $\eta_{\mu\nu}$ and $\eta_{\mu\nu}^{(d)}$, d and d_0, can be viewed as arising from ε-scalars ($\varepsilon=d_0-d$) if we decompose the ten-dimensional vector field A_μ into an d-dimensional vector A_i plus ε-scalars $A_\sigma(d\leqslant\sigma\leqslant d_0)$. To see how maintaining the difference between $\eta_{\mu\nu}$ and $\eta_{\mu\nu}^{(d)}$ allows the one-loop supersymmetry Ward identities to go through while setting $\eta_{\mu\nu}^{(d)}=\eta_{\mu\nu}$, i.e. going the case of conventional DR, causes breakdown in the proof, one should extract from (28) the contribution of the ε-scalars. This contribution results from the first and the second terms in (28) and it is just the difference between the SRDR- and the DR-calculated Yang-Mills and fermion two-point one-loop Green's functions, respectively. Examining this difference by making use of (13), (14), (19), (24) and (25) leads to the conclusion that the infinite parts of the diagrams satisfy the Ward identity independently on the dimensional regularization scheme chosen, but the finite parts only do if SRDR is employed (owing to the nonvanishing contribution of the ε-scalars in (28) from ε-scalar loop and ε-scalar/fermion loop).

The structure of the infra-red and ultra-violet divergences of the $N=4$ SSYM as well as the β-function are well-known, for example, from superstring considerations (Schwarz 1982) and computer calculations (Avdeev *et al* 1980) and are not discussed here.

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