

## Characterization of energy levels in the nucleus $^{170}\text{Lu}$

P C SOOD\*, R K SHELINE\*\* and R W HOFF\*\*\*

\*Department of Physics, Banaras Hindu University, Varanasi 221 005, India

\*\*Florida State University, Tallahassee, Florida 32306, USA

\*\*\*Nuclear Chemistry Division, Lawrence Livermore National Laboratory, Livermore, California 94550, USA

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**Abstract.** Configuration assignments are derived for the observed energy levels in the odd-odd deformed nucleus  $^{170}\text{Lu}_{99}$  based on the calculations of the two-particle band head energies for a zero range residual interaction, the beta-feeding characteristics, and the observed features for similar bands in the neighbouring nuclei. In particular, specific assignments are given for the  $J^\pi = 1^+$  levels at 198.4 keV, 349.0 keV and 785.5 keV. The ambiguities with respect to the assignments for the  $K^\pi = 3^-$  bands are discussed. A new isomer with  $J^\pi = 7^+$  and half-life of several seconds is predicted around  $(225 \pm 25)$  keV and experiments are suggested to identify it.

**Keywords.** Doubly-odd deformed nucleus  $^{170}\text{Lu}$ ; energy level Nilsson assignments;  $\log ft$  considerations; residual interaction energy calculations.

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### 1. Introduction

The experimental information on the energy levels of the doubly odd deformed nucleus  $^{170}\text{Lu}$  has come primarily from the electron capture of  $^{170}\text{Hf}$  studied during the late sixties (Treherne *et al* 1964, 1969; Bjornholm *et al* 1965; Harmatz and Handley 1966; Abou-Leila *et al* 1968; Chu and Reednick 1970). Even though this information is incomplete and the characterization of the identified levels is mostly tentative or speculative, no theoretical or experimental studies on this nucleus have been reported over the past seventeen years (Schmorak and Auble 1975; Lederer and Shirley 1978; Chunmei 1987). On the theoretical side, detailed studies predicting the band head energies of the full spectrum of the two quasi-particle states in the odd-odd nuclei based on a quantitative evaluation of the neutron-proton residual interaction energy are scarce. Matching of the observed states to the expected two-particle configurations is usually attempted on the basis of the summed neutron and proton single particle energies coupled with the empirical Gallagher-Moszkowski (1958) (GM) rule for the relative ordering of the  $K^+ = \Omega_p + \Omega_n$  and the  $K^- = |\Omega_p - \Omega_n|$  band heads. Whereas the GM splitting energies (Boisson *et al* 1976; Elmore and Alford 1976a) provide a measure of the spin-dependent part of the residual neutron-proton interaction  $V_{np}$ , the spin-independent (Wigner) part of this interaction remains undetermined in such studies. Consequently, relative placement of the various two particle bands in a given

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\*To whom all correspondence should be addressed.

nucleus cannot be attempted on a quantitative basis in this approach. Recently Sood and Singh (1982) developed a formulation to overcome this shortcoming of the model for odd-odd nuclei. By combining the atomic mass data with the observed GM splitting energies, they were able to determine both the Wigner and the spin-dependent parts of the interaction. They further derived analytical expressions for the matrix elements of the zero-range residual interaction and evaluated the two particle band head energies for the complete low energy spectrum of a given nucleus. The formulation has since been successfully applied to discuss the energy levels of the odd-odd actinides (Sood 1984a, b, c; Sood and Singh 1982, 1983, 1984) and also of several nuclei of the rare earth region (Sood 1983; Sood and Singh 1984; Sood and Sheline 1987; Sood *et al* 1986, 1987). In particular, the recent application of this formulation to the  $N = 99$  isotones  $^{166}\text{Ho}$  and  $^{168}\text{Tm}$  (Sood *et al* 1987; Sood and Sheline 1987) is relevant to the present study. A somewhat similar model, but without explicitly including the Wigner term in the interaction, has been developed by Hoff *et al* (1984, 1985).

We describe in §2 the present experimental situation, bringing into focus the open questions on the characterization of the known energy levels and on the nature of other predicted levels in the low energy spectrum of  $^{170}\text{Lu}$ . Our formulation for the calculation of the two-particle band head energies and the GM splitting energies is briefly outlined in §3. This is followed by a discussion of the results and the conclusions of the present study.

## 2. Experimental situation—open questions

All the available experimental information on the level structure of  $^{170}\text{Lu}$  has been obtained from the  $^{170}\text{Hf}$  decay studies except for the early identification (Bjornholm *et al* 1965) of the 0.67 second 93 keV  $4^-$  isomer through the  $(p, n)$  reaction study. The ground state  $K^\pi = 0^+$  band has been identified with the  $1^+, 2^+$ , and possibly  $3^+$  rotational levels placed at 98.5 keV, 44.5 keV, and 176.7 keV respectively. The only other definite assignments include a  $K^\pi = 1^-$  band head at 164.7 keV with a 212.6 keV  $2^-$  rotational level, another  $K^\pi = 1^-$  band head at 244.9 keV with a 283.9 keV  $2^-$  rotational level, and a  $K^\pi = 0^-$  band head at 407.5 keV with a 470.3 keV  $1^-$  rotational level. These studies have raised several significant questions, unanswered so far, which are discussed below and which provide the motivation for the present study.

The 785.5 keV  $1^+$  level receives 55% of the total population from the  $^{170}\text{Hf}$  decay with the  $\log ft$  value 5.3. It has 14 gamma transitions, many of them with definite multipolarity, leaving it. Still this level has an undefined character (Treherne *et al* 1969). In an attempt to find an available configuration for this state, Harmatz and Handley (1966) suggested a negative parity based on the unconfirmed multiplicities. However, the precise electron conversion data of Treherne *et al* (1969) (hereafter referred to as TVV) defined many multiplicities unambiguously; these, taken together with the beta-transition characteristics, confirm  $J^\pi = 1^+$  for this state. TVV (1969) could not suggest any configuration for this state and concluded that its structure 'cannot be explained by using Nilsson's orbitals and the GM coupling rules'. In our investigations we seek an acceptable configuration assignment for this 785.5 keV  $1^+$  level.

Another  $J^\pi = 1^+$  level with no characterization is placed at 198.4 keV on the basis of

a strong coincidence between the 99.9 keV and the 98.5 keV  $\gamma$ -rays; it receives 19% of the total population from the  $^{170}\text{Hf}$  decay with a  $\log ft$  value of 6.6. TVV (1969), not being able to characterize it, remarked that 'it is doubtful whether such a low-lying state has spin and parity  $1^+$ '. Also, a proposed  $J^\pi = 1^+$  level at 349.0 keV has been given (TVV 1969) a tentative assignment based on the expected couplings. While the  $J^\pi = 4^-$  93 keV 0.67 sec isomer is well established from independent studies (Bjornholm *et al* 1965; Harmatz and Handley 1966; Treherne *et al* 1969), the position of its  $K^\pi = 3^-$  spin-antiparallel GM counterpart is altogether uncertain. Harmatz and Handley (1966) had placed it at 148.1 keV, whereas TVV (1969) suggested a  $3^-$  level at 96.0 keV, terming their assignment as speculative. We examine the possible location of the expected low energy  $K^\pi = 3^-$  band head in this context.

The decay studies of the  $J^\pi = 0^+$  ground state of an even-even nucleus yield only limited information on the level structure of the daughter odd-odd nucleus in that such decays directly populate only the positive parity states with  $J = 1$  through allowed decays and the negative parity states with  $J \leq 2$  through first forbidden decays. We expect several low-lying high  $K$  bands and also high spin rotational members of the low  $K$  bands in the odd-odd nuclei which remain inaccessible in such decay studies. Of particular interest in the present case is the  $K^\pi = 7^+$  band of the ground state configuration which, because of its expected low excitation energy and its high  $J^\pi K$ , is likely to be a rather longlived isomer.

### 3. Calculations

The two-particle states in odd-odd nuclei arise from the superposition of the proton and the neutron single particle Nilsson orbitals  $\Omega^\pi[\text{Nn}_3\Lambda\Sigma]$  observed in the excitation spectra of the neighbouring odd-mass isotopic and isotonic nuclei. Each two-particle configuration  $(p_i n_j)$  gives rise to two bands  $K_{ij}^\pm = |\Omega_{p_i} \pm \Omega_{n_j}|$  whose ordering normally follows the spin-spin coupling according to the GM rule. The single particle orbitals of interest for the low energy spectrum of  $^{170}\text{Lu}$  are listed in table 1, which gives the observed excitation energies in each case from the known spectra of the  $(A-1)$  isotopic/isotonic nuclei. The table also lists the band numbers  $K$  for the two-particle configurations.

The two-particle (unmixed) band head energy is obtained from the expression

$$E_K(p_i n_j) = E(p_i) + E(n_j) + E_{\text{rot}} + E_{\text{int}}(\text{np}), \quad (1)$$

where the last two terms represent respectively the contributions from the zero-point rotational energy and the residual neutron-proton interaction. The residual interaction has an explicitly spin dependent  $(\vec{\sigma}_p \cdot \vec{\sigma}_n)$  term to describe the nearly universal validity of the empirical GM rule; this term causes a splitting  $E_{\text{GM}}$  between the lower spins-parallel (triplet  $K_T$ ) band and the upper spins-antiparallel (singlet  $K_S$ ) band. It also causes an odd-even shift (Newby 1962) of the levels in the  $K = 0$  bands. Thus  $E_{\text{int}}$  has three components (including one arising from the Wigner or spin-independent term)

$$\begin{aligned} E_{\text{int}} &= \langle V_{\text{np}} \rangle \\ &= E_W \pm 1/2 E_{\text{GM}} + E_N \delta_{K,0}. \end{aligned} \quad (2)$$

For the residual interaction we use the zero range interaction employed in our earlier

**Table 1.** The two-particle bands  $K^\pm = |\Omega_p \pm \Omega_n|$  expected in the low energy spectrum of  $^{170}\text{Lu}$ . The column headings denote the proton orbital ordering ( $p_i$ ), its observed excitation energy (in keV) in the  $(A - 1)$  isotope and its asymptotic quantum numbers  $\Omega^\pm [Nn_3 \Lambda \Sigma]$ . The rows present the same information for the neutron orbitals. The band numbers  $K$  are listed according to the GM rule, i.e.  $K_T (\Sigma = 1)$  followed by  $K_S (\Sigma = 0)$ ; the numbers below these entries are the values  $\times 10^3$  of the matrix elements  $A_0$  and  $A_\sigma$  for the respective  $(p_i, n_j)$  configuration evaluated for the deformation  $\delta = 0.28$ .

$n_j \downarrow$	$p_i \rightarrow$	$p_0$	0	$p_1$	29.0	$p_2$	97.4	$p_3$	186.7
		$7/2^+$	[404 $\downarrow$ ]	$1/2^-$	[541 $\downarrow$ ]	$1/2^+$	[411 $\downarrow$ ]	$5/2^+$	[402 $\uparrow$ ]
$n_0$	0	$0^+$	$7^+$	$3^-$	$4^-$	$3^+$	$4^+$	$6^+$	$1^+$
$7/2^+$	[633 $\uparrow$ ]	87.58		62.22		80.56		55.36	
$n_1$	24.2	$4^-$	$3^-$	$1^+$	$0^+$	$1^-$	$0^-$		
$1/2^-$	[521 $\downarrow$ ]	53.33		86.9		142.79			
$n_2$	191.2	$1^-$	$6^-$	$2^+$	$3^+$				
$5/2^-$	[512 $\uparrow$ ]	80.50		61.2					
$n_3$	569.8	$6^-$	$1^-$						
$5/2^-$	[523 $\downarrow$ ]	93.61							
$n_4$	590.7	$1^+$	$6^+$						
$5/2^+$	[642 $\uparrow$ ]	78.42							

studies. The details of the formulation have been presented earlier (Sood and Singh 1982). Here we present only the essential results. For the zero range interaction we obtain

$$E_{\text{int}}(K) = (1 - \alpha)WA_0(K) \pm \alpha WA_\sigma(K) + (-1)^J B \delta_{K,0}, \quad (3)$$

where  $W$  is the strength parameter of the spin-independent (Wigner) term and  $\alpha W$  is the fractional strength of the spin-dependent term in the residual interaction, and  $A_0$  and  $A_\sigma$  are the respective interaction matrix elements calculated for each configuration  $(p_i, n_j, K)$  using the two-particle product Nilsson wave functions. The odd-even shift coefficient  $B$  may be obtained theoretically (Boisson *et al* 1976; Elmore and Alford 1976a; Sood and Ray 1986) or empirically from the lowest two observed members of the  $K = 0$  band using the expression (Elmore and Alford 1976a)

$$B = \frac{1}{2}(-1)^J [E_0(J) - E_0(J+1)] + \frac{\hbar^2}{2\mathcal{I}} \{(-1)^J(J+1) + a_p a_n \delta_{\Omega_{1/2}}\}, \quad (4)$$

where  $a_p$  and  $a_n$  are the decoupling parameters for the  $\Omega_p = \Omega_n = 1/2$  case. The rotational energy term is approximately given by the expression (Sood and Singh 1982)

$$E_{\text{rot}} = \frac{\hbar^2}{2\mathcal{I}} [K - (\Omega_p + \Omega_n)] \quad (5)$$

which vanishes for the  $K^+ = (\Omega_p + \Omega_n)$  band. For evaluating the interaction parameters, we note that the ground state band head energy may be written in terms of the atomic mass difference

$$E_M = [M(Z, A) - M(Z, A-1) - M(Z-1, A-1) + M(Z-1, A-2)] \quad (6a)$$

$$= E_{\text{rot}} + (-1)^J B \delta_{K,0} + (1 - \alpha)WA_0(K_{gs}) + \alpha WA_\sigma(K_{gs}). \quad (6b)$$

The GM splitting energy for the ground state configuration is

$$E_{\text{GM}} = 2\alpha W |A_{\sigma}(K_{gs})|. \quad (7)$$

The interaction parameters  $\alpha$  and  $W$  can now be evaluated using (6) and (7) for any ground state configuration provided both the bands of the configuration are known. If only the lower band is known, thus yielding  $E_M$ , or if both bands are known only as excited bands, thus yielding  $E_{\text{GM}}$ , we adopt the average value  $\alpha = 0.25$  and use it with known  $E_M$  or  $E_{\text{GM}}$  to determine  $W$ . In case no experimental information on a configuration is available, we adopt  $\alpha = 0.25$  with the value  $\alpha W = 0.89$  MeV derived (Elmore and Alford 1976a) from a least squares fit of the known GM splitting energies of the whole region.

The matrix elements  $A_0$  and  $A_{\sigma}$  have been evaluated for each of the two-particle configurations as a function of the deformation parameter  $\delta$  using the analytical expressions given by Sood and Singh (1982). The values obtained for  $\delta = 0.28$  applicable to the nucleus under study are listed in table 1. The two-particle band head energy for each of these configurations can now be calculated using equations (1) to (5). The predicted excitation energy of the band relative to the ground state is then given by subtracting  $E_M$  obtained in (6a) from the calculated band head energy of the excited band from (1). Thus our predicted band head energies do not use any input from the nucleus under consideration except the ground state information.

We now proceed to discuss the results of our calculations for the intrinsic energy levels in  $^{170}\text{Lu}$  to deduce the configuration assignments for the known levels, and to predict in particular the low-lying high  $K$  bands not observed so far.

#### 4. Results and conclusions

The configuration assignments deduced by us for the known levels in  $^{170}\text{Lu}$  are listed in table 2. We discuss in the following the basis for our revised or new assignments and the predictions for as yet unidentified low-lying high  $K$  bands.

##### 4.1 The ground state $K^{\pi} = 0^+$ and $K^{\pi} = 7^+$ bands

The ground state  $K^{\pi} = 0^+$  band with  $44.5$  keV  $2^+$  rotational level (yielding  $\hbar^2/2\mathcal{J} = 7.4$  keV) and  $98.5$  keV  $1^+$  rotational level (yielding  $B = -42$  keV) corresponds to the  $(p_0n_0)$  configuration  $\{7/2^+[404\downarrow]_p - 7/2^+[633\uparrow]_n\}$ . From this information and the recent atomic mass table (Wapstra and Audi 1985), we obtain using equations (5) and (6)

$$E_{\text{rot}}(p_0n_0; 0^+) = -52 \text{ keV}; \quad E_M = -455 \text{ keV}. \quad (8)$$

Substituting these values and the  $(p_0n_0)$  calculated matrix elements from table 1 in (6b), we obtain

$$87(1 - \alpha)W + 58\alpha W = 361,$$

or

$$W(3 - \alpha) = 12.45. \quad (9)$$

**Table 2.** Configuration assignments deduced in the present study for the observed energy levels of the nucleus  $^{170}\text{Lu}$ .

Experimental excitation energy (keV)	$J^\pi K$	$\Omega^\pi[\text{Nn}_3\Lambda]_p \pm \Omega^\pi[\text{Nn}_3\Lambda]_n$
(a) configurations indicated earlier and confirmed in the present study		
0.0	$0^+ 0$	$7/2^+ [404]_p - 7/2^+ [633]_n$
44.5	$2^+ 0$	
98.5	$1^+ 0$	
176.7	$3^+ 0$	$7/2^+ [404]_p + 1/2^- [521]_n$
93.0	$4^- 4$	
164.7	$1^- 1$	$7/2^+ [404]_p - 5/2^- [512]_n$
212.6	$2^- 1$	
244.9	$1^- 1$	$1/2^+ [411]_p + 1/2^- [521]_n$
283.9	$2^- 1$	
407.5	$0^- 0$	$1/2^+ [411]_p - 1/2^- [521]_n$
470.3	$1^- 0$	
(b) configurations assigned from the present study		
96	$3^- 3$	$1/2^- [541]_p - 7/2^- [633]_n$
148	$3^- 3$	$7/2^+ [404]_p - 1/2^- [521]_n$
198.4	$1^+ 1$	$1/2^- [541]_p + 1/2^- [521]_n$
349.0	$1^+ 1$	$5/2^+ [402]_p - 7/2^+ [633]_n$
785.5	$1^+ 1$	$7/2^+ [404]_p - 5/2^+ [642]_n$

Since the other member of the  $(p_0n_0)$  GM band pair has not been identified so far, its position may be predicted by using the global fit value of  $\alpha = (0.25 \pm 0.05)$ . Combined with (9) this gives

$$W = (4.52 \pm 0.10) \text{ MeV}; \quad (10)$$

$$E_{\text{GM}}(p_0n_0) = (131 \pm 25) \text{ keV} \quad (11)$$

and

$$E(p_0n_0, 7^+) = (225 \pm 25) \text{ keV}. \quad (12)$$

It is of interest to comment on the predicted GM splitting energy for this configuration since it implies an isomeric character for the  $7^+$  state discussed later in this paper. O'Neil and Burke (1972) had investigated this configuration in  $^{174}\text{Lu}$  experimentally as well as theoretically. Their Coriolis mixing calculations found the optimum unperturbed energy difference between the  $7^+, 0^+$  band heads to be 112 keV. Experimentally they assigned the particle group at  $\sim 433$  keV excitation energy in  $^{174}\text{Lu}$  to the unresolved spin  $J = 3$  and  $J = 4$  members of the  $K^\pi = 0^+$  band and the  $J = 7$  member of the  $K^\pi = 7^+$  band. Dewberry *et al* (1982), in their study of the energy levels of the isotonic nucleus  $^{166}\text{Ho}$ , termed their assignment of the  $7^+, 0^+$  band heads as tentative since the experimental GM splitting of 34 keV conflicted with the calculated value of 159 keV. A recent photoneutron study (Tsai *et al* 1986) of the lowest levels in the isotonic nucleus  $^{168}\text{Tm}$  deduced  $J^\pi = 0^+$  for the 17 keV excited state and suggested it as the triplet member of the GM pair with the earlier identified (Preibisz *et al* 1973)  $7^+$  band head at 312 keV as its singlet counterpart. This tentative assignment has since been confirmed by us (Sood and Sheline 1987) in a detailed analysis, yielding the

experimental GM splitting energy of 148 keV in  $^{168}\text{Tm}$  for this configuration. This value is in good agreement with the calculated value of Dewberry *et al* (1982) for this configuration in  $^{166}\text{Ho}$  and our predicted splitting in (11) for  $^{170}\text{Lu}$ . In view of the situation outlined above, it is of interest to experimentally re-examine the location of these states in  $^{166}\text{Ho}$  (Sood *et al* 1987) and in  $^{174}\text{Lu}$  and to identify the  $K^\pi = 7^+$  band heads in  $^{170}\text{Lu}$  and in  $^{172}\text{Lu}$ . In the latter nucleus also, only the low-lying  $K^\pi = 0^+$  band has been identified (Toth *et al* 1979) with the  $0^+$  band-head at 66 keV.

#### 4.2 The 785.5 keV $1^+$ level

According to the Nuclear Data evaluators (Chunmei 1987), the 785.5 keV level receives 55% of the  $\beta$ -feeding from the ground state decay of  $^{170}\text{Hf}$  with the  $\log ft$  value of 5.3. It also has 14 gamma transitions leaving it, 8 of them with known multiplicities. These features establish its  $J^\pi = 1^+$  assignment. However, as noted earlier, TVV (1969) were unable to assign a Nilsson configuration to this level. On examining the possible bands expected in  $^{170}\text{Lu}$  from the coupling of the available neutron and proton Nilsson orbitals as given in table 1, we find the  $(p_0n_4)$  configuration  $\{7/2^+[404]_p - 5/2^+[642]_n\}$  as the only candidate for a  $1^+$  band head with  $(E_p + E_n) > 200$  keV. In the following we examine if this assignment for the 785.5 keV  $1^+$  level is in agreement with the theoretical and the experimental considerations.

We note that this configuration has not so far been experimentally identified in any nucleus. Accordingly, as discussed in the previous section, we adopt the average interaction parameters  $\alpha = 0.25$  and  $\alpha W = 0.89$  MeV obtained from 'global' fits over the whole region. The  $5/2^+[642]$  neutron orbital has been observed (Lederer and Shirley 1978) at 590.7 keV in the isotonic  $(A - 1)$  nucleus  $^{169}\text{Yb}$ . The calculated  $E_{\text{rot}}$  for this configuration is  $-38$  keV. Combining these data with the calculated values of the interaction matrix elements listed in table 1, we evaluate the band head energy.

$$E(p_0n_4; K^\pi = 1^+) = 767 \text{ keV} \quad (13)$$

in close agreement with the observed 785 keV  $1^+$  level. This assignment is further supported by a consideration of the  $\beta$ -decay systematics. The  $^{168}\text{Lu} 6^-\{7/2^+[404]_p + 5/2^-[523]_n\}$  ground state decays to the  $5^-\{5/2^+[642]_p + 5/2^-[523]_n\}$  1999 keV level in the nucleus  $^{168}\text{Yb}$  with  $\log ft = 5.2$ . In the present case our assignment corresponds to the analogous decay of the  $0^+\{7/2^+[404]_p\}$  ground state configuration in  $^{170}\text{Hf}$  to the  $1^+\{7/2^+[404]_p - 5/2^+[642]_n\}$  785 keV state in  $^{170}\text{Lu}$  with  $\log ft = 5.3$ . Thus in the decays of both  $^{168}\text{Lu}$  and  $^{170}\text{Hf}$ , the  $7/2^+[404]_p \rightarrow 5/2^+[642]_n$  transitions are involved, resulting in very similar  $\log ft$  values.

Accordingly based on the consideration of the calculated two-particle band head energy and of the beta-decay characteristics we conclude that the 785.5 keV  $1^+$  level in  $^{170}\text{Lu}$  has the  $1^+\{7/2^+[404]_p - 5/2^+[642]_n\}$  configuration.

#### 4.3 The 198.4 keV $1^+$ level

The level at 198.4 keV, which receives almost all of its population directly from the  $^{170}\text{Hf}$  decay (with  $\beta$ -intensity  $\sim 19\%$  and  $\log ft = 6.6$ ), has been established from the observed strong coincidence of the 99.9 keV and the 98.5 keV decay gamma transitions. Its feeding and decay characteristics suggest  $J^\pi = 1^+$ ; however, TVV (1969) termed this assignment doubtful for such a low-lying state.

Looking at the possible bands in table 1, we expect the lowest  $K^\pi = 1^+$  band from the  $(p_1n_1)$  configuration and propose its identification with the observed 198.4 keV  $1^+$  state based on the following considerations:

- In the neighbouring  $^{172}\text{Lu}$  isotope, the  $1^+$  band-head of this configuration is seen (Toth *et al* 1979) at 109.4 keV above the  $4^-$  ground state. This proposed assignment in  $^{170}\text{Lu}$  places the same  $1^+$  band head 105.5 keV above the analogous  $(p_0n_1)$   $4^-$  isomer.
- Our calculations using the  $^{172}\text{Lu}$  interaction parameters place the  $K^\pi = 1^+(p_1n_1)$  band head at 195 keV in excellent agreement with the observed  $1^+$  excitation energy.
- This assignment is further supported by the observed characteristics of the beta-decay feeding this state. The  $^{170}\text{Hf}0^+$  ground state is expected to have a  $1/2^- [541]^2$  coupled proton pair component in view of the very low excitation energy (29 keV) for this proton orbital in the  $(Z-1, A-1)$  nucleus. Thus the  $\beta$ -decay feeding the  $\{1/2^- [541]_p + 1/2^- [521]_n\}$  coupled state in the daughter nucleus involves the  $1/2^- [541]_p \rightarrow 1/2^- [521]_n$  transition. This transition has been observed in  $^{177}\text{W}$  decay ( $\log ft = 6.2$ ),  $^{179}\text{W}$  decay ( $\log ft = 6.5$ ),  $^{181}\text{Os}$  decay ( $\log ft = 6.8$ ), and also in  $^{172}\text{Ta}$  decays ( $\log ft = 6.4$  and  $6.6$ ). The observed  $\log ft = 6.6$  in the present case is consistent with the proposed assignment to the 198 keV state in  $^{170}\text{Lu}$ .

Accordingly we conclude that the 198.4 keV level is the  $K^\pi = 1^+$  band head with the  $\{1/2^- [541]_p + 1/2^- [521]_n\}$  configuration. However, its observed decay to the  $1^+0$  level of the  $(p_0n_0)$  ground band would have been hindered if the 198 keV had pure  $(p_1n_1)$  character. This decay requires the 198 keV level to have some admixture of the nearest available  $1^+$  band from  $(p_3n_0)$ , which we suggest as the dominant configuration for the 349 keV  $1^+$  level as discussed below. This assignment for the 198 keV  $1^+$  level further requires a  $K^\pi = 0^+$  band as the singlet member of the GM pair with the  $(p_1n_1)$  configuration. Our calculations place this band head about 20 keV above its  $K^\pi = 1^+$  counterpart. Experimental identification of this  $K^\pi = 0^+$  and/or its associated rotational levels in the 200–300 keV excitation energy range would be of considerable interest.

#### 4.4 The 349.0 keV $1^+$ level

A  $J^\pi = 1^+$  level has been suggested on the basis of its population through beta-feeding and the non-coincidence of the 349.0 keV with a low energy  $\gamma$ -ray. TVV (1969), on qualitative considerations, suggested that it may have the  $(p_1n_1)$  configuration which we have now established for the 198 keV  $1^+$  level. Looking at table 1 we find  $1^+(p_3n_0)$  as the only available choice for a  $J^\pi = 1^+$  level in this energy range. Our band head energy calculations are in agreement with this assignment. The proposed assignment is also consistent with the failure to observe the gamma population of the  $(p_3n_0)$  349 keV level by the  $(p_0n_4)$  785 keV level decay which would involve a simultaneous change of both the neutron and the proton orbitals.

We accordingly assign the configuration  $1^+\{5/2^+[402]_p - 7/2^+[633]_n\}$  to the observed 349 keV  $1^+$  level. The  $K^\pi = 6^+$  GM counterpart of this configuration is expected to lie  $(80 \pm 20)$  keV below this  $1^+$  level.

#### 4.5 The $K^\pi = 3^-$ and the $K^\pi = 4^-$ bands

In table 1 we find two  $K^\pi = 3^-$  bands appearing in the low energy spectrum of  $^{170}\text{Lu}$ , both having  $K^\pi = 4^-$  as their GM counterpart bands. For the  $(p_0n_1)$  configuration the  $K^\pi = 4^-$  is the spins-parallel member expected to lie lower than its  $K^\pi = 3^-$  counterpart. On the other hand, for the  $(p_1n_0)$  configuration the  $K^\pi = 3^-$  is the lower-lying spins-parallel band. Since the summed single-particle energies in both cases are similar, it is reasonable to expect the  $3^-(p_1n_0)$  as the lower one of the two  $K^\pi = 3^-$  bands.

Experimentally only the  $K^\pi = 4^-$  band head at 93.0 keV with the  $(p_0n_1)$  configuration  $4^-\{7/2^+[404]_p + 1/2^-[521]_n\}$  has been well-established. The  $K^\pi = 3^-$  singlet member of this configuration has been suggested at 148.1 keV by Harmatz and Handley (1966) and at 96.0 keV by TVV (1969). Both the  $K^\pi = 4^-$  and  $K^\pi = 3^-$  band heads of this configuration have been experimentally observed in  $^{172}\text{Lu}$  (Elmore and Alford 1976b) as the ground state GM band pair and in  $^{174}\text{Lu}$  (O'Neil and Burke 1972) as excited bands. In both cases the observed energy separation of the two band heads is 68 keV, corresponding to the experimental GM splitting energy (with correction for the zero point rotational energy) of 75 keV. The calculated GM splitting energy (Boisson *et al* 1976; Elmore and Alford 1976a) for this configuration has been given values in the range of 55 keV to 83 keV, depending on the nature of the residual interaction, with the lower values obtained for the zero range interaction adopted in our study. The suggested (Harmatz and Handley 1966) assignment of 148 keV  $3^-$  as the singlet member of the configuration  $\{7/2^+[404]_p \pm 1/2^-[521]_n\}$  forming a GM pair with its 93 keV  $4^-$  triplet member is thus supported by our band head energy calculations, earlier calculations of GM splitting energies, and the experimental information available for this configuration in  $^{172}\text{Lu}$  and  $^{174}\text{Lu}$ . The suggested 96 keV  $3^-$  state of TVV (1969) is not acceptable as the GM counterpart of the 93 keV  $4^-$  band head in  $^{170}\text{Lu}$  on any of the above counts. However the 148 keV  $3^-$  level has not been included in the level scheme deduced by TVV (1969) or in the Nuclear Data Sheets compilations. None of the studies so far, which mainly are the decay studies with the inherent limitation of directly populating only the  $I \leq 2$  states, can yield a direct confirmation of the  $3^-$  state. In view of the situation outlined above, reaction studies leading to the states in  $^{170}\text{Lu}$  are needed to deduce information on states with  $I > 2$ .

The  $K^\pi = 3^-$  and  $K^\pi = 4^-$  band pair with the  $(p_1n_0)$  configuration  $\{1/2^-[541]_p \pm 7/2^+[633]_n\}$  has been experimentally observed (Preibisz *et al* 1973) in the excitation spectrum of the isotonic nucleus  $^{168}\text{Tm}$ . Based on this information, we calculate the  $(p_1n_0) 3^-$  band head energy as  $(110 \pm 15)$  keV with its  $K^\pi = 4^-$  singlet counterpart lying about 75 keV above it. The proposed (Treherne *et al* 1969) 96 keV  $3^-$  level could thus have the  $(p_1n_0)$  configuration.

#### 4.6 Other low-lying bands

The experiments (Harmatz and Handley 1966; Treherne *et al* 1969) have identified the 164.7 keV  $1^-$  level as the  $K^\pi = 1^-$  band head with the  $(p_0n_2)$  configuration  $\{7/2^+[404]_p - 5/2^-[512]_n\}$  and the 244.9 keV  $1^-$  level as the  $K^\pi = 1^-$  band head with the  $(p_2n_1)$  configuration  $\{1/2^+[411]_p + 1/2^-[521]_n\}$ . The band head of the singlet member of the latter configuration has been identified with the 407.5 keV  $0^-$  level. These assignments are consistent with the observed systematics of these configurations in the

neighbouring nuclei and also with the band head energy calculations. The  $K^\pi = 6^-$  singlet member of the  $(p_0 n_2)$  configuration is predicted to lie about 160 keV above its triplet  $K^\pi = 1^-$  counterpart.

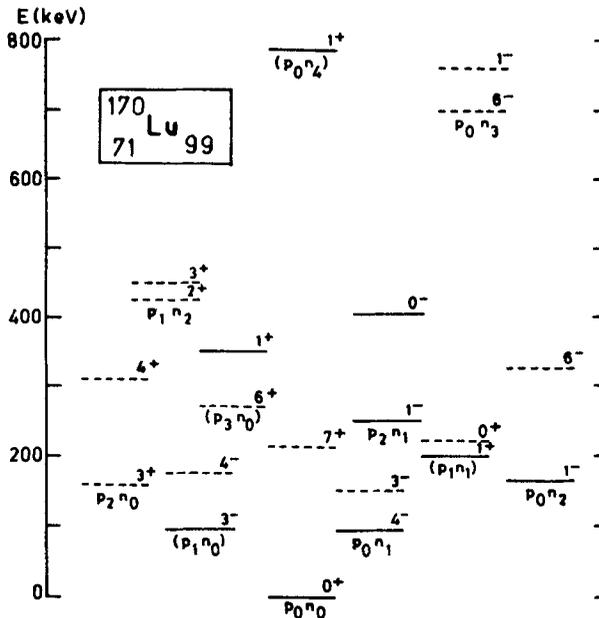
The other intrinsic states expected to lie up to about 400 keV excitation energy in  $^{170}\text{Lu}$  are the  $K^\pi = 3^+$  and  $K^\pi = 4^+$  bands with the  $(p_2 n_0)$  configuration  $\{1/2^+[411]_p \pm 7/2^+[633]_n\}$ . This configuration has been observed as the ground state in the isotonic nucleus  $^{168}\text{Tm}$ . Based on this information, we calculate the  $(p_2 n_0)$   $K^\pi = 3^+$  band head energy as  $(160 \pm 20)$  keV with the  $K^\pi = 4^+$  singlet member lying about 150 keV above it.

Another positive parity band pair involving the low-lying single particle orbitals arises from the  $(p_1 n_2)$  configuration with  $K^\pi_7 = 2^+$  and  $K^\pi_5 = 3^+$ . We predict these bands to lie just around 400 keV excitation with about 20 keV separation energy.

The conclusions of our study are summarized in figure 1 which shows the positions of all the two-particle band heads listed in table 1 except the  $K^\pi = 6^+(p_0 n_4)$  band head which is predicted to lie at around 900 keV. The experimentally known energy levels are shown by solid lines and the predicted levels by dashed lines. The configurations  $(p_i n_j)$  shown within brackets correspond to our assignments to the known levels.

#### 4.7 The 225 keV $7^+$ as an isomer

We have predicted the  $K^\pi = 7^+$  band head of the ground state configuration in  $^{170}\text{Lu}$  to lie at  $(225 \pm 25)$  keV. Considering its high spin, this state is expected to be a rather



**Figure 1.** The band head energies of the two-particle configurations for  $^{170}\text{Lu}$  in the notation of table 1 are plotted. The full lines correspond to the experimentally known levels with the configurations  $p_i n_j$  within brackets assigned from the present study. The dashed lines correspond to the predicted band head energies for the, as yet unobserved, specified configurations.

longlived isomer since, as seen in table 1, all the lower-lying bands have  $K \leq 4$ . Thus any transition from the  $J^\pi K = 7^+ 7$  will have the  $K$  hindrance factor of  $\Delta K \geq 3$ . On examining in detail the expected lower-lying complete energy spectrum, the  $7^+ 7$  can possibly decay to the following  $J^\pi K$  states with  $\Delta J \leq 3$ :

- (i) The 93 keV  $4^- 4$  and the predicted  $\sim 190$  keV  $5^- 4$  with the configuration  $\{7/2^+[404]_p + 1/2[521]_n\}$ .
- (ii) The predicted  $\sim 145$  keV  $4^+ 0$  level with the configuration  $\{7/2^+[404]_p - 7/2^+[633]_n\}$ .
- (iii) The predicted  $\sim 100$  keV  $4^- 3$  and  $\sim 160$  keV  $5^- 3$  levels with the configuration  $\{1/2^-[541]_p \pm 7/2^+[633]_n\}$ ; the perturbed  $4^- 4$  band head of this configuration is expected at  $\sim 240$  keV.
- (iv) The  $4^+ 3\{1/2^+[411]_p - 7/2^+[633]_n\}$  predicted at  $\sim 220$  keV.

Thus the most likely decay modes of the 225 keV  $7^+$  state are the  $(40 \pm 25)$  keV  $M2$  ( $\Delta K \geq 3$ ),  $(130 \pm 25)$  keV  $E3$  ( $\Delta K = 3$ ), and  $(76 \pm 25)$  keV  $M3$  ( $\Delta K = 7$ ) transitions. The expected lifetimes for these transitions may be estimated based on the following considerations. The observed 93 keV 0.67 second  $4^-$  isomer decays by a 48.5 keV  $M2$  ( $\Delta K = 4$ ) transition to the 44.5 keV  $2^+ 0$  involving the  $1/2^- [521]_n \rightarrow 7/2^+ [633]_n$  orbitals. The 24.2 keV  $E3$  ( $\Delta K = 3$ ) transition involving the same orbitals in the isotonic ( $A - 1$ ) nucleus  $^{169}\text{Yb}$  has a 46 second half-life. The decay of the 207.8 keV 2.28 second isomer in  $^{167}\text{Er}$  to the ground state by an  $E3$  transition also involves the same orbitals. Thus the decay mode indicated in (i) above is expected to have a half-life of the order of a second or more. The decay mode (ii) is an  $M3$  transition with  $\Delta K = 7$ , and is expected to have even a longer half-life. The decay mode (iii), involving the same orbital change, has been observed in the 29.0 keV  $E3$  decay of the 2.7 m isomer of the isotopic ( $A - 1$ ) nucleus  $^{169}\text{Lu}$ .

We conclude, on the basis of the known corresponding transitions involving the same orbitals in  $^{170}\text{Lu}$  and the neighbouring nuclei, that the predicted 225 keV  $7^+$  level in  $^{170}\text{Lu}$  will have a half-life of the order of a second or more.

We further suggest the  $^{168}\text{Yb}(\alpha, d)$  reaction study to look for this  $7^+$  isomer as well as the other high spin states in  $^{170}\text{Lu}$ . The nucleus  $^{168}\text{Yb}$  provides a stable target and the  $(\alpha, d)$  reactions have been proven to be particularly useful in studying the high spin states. For instance, Daehnick *et al* (1981) established that the most strongly populated states in their  $^{208}\text{Pb}(\alpha, d)$   $^{210}\text{Bi}$  reaction study tended to have  $J > 5$ .

In summary, we have discussed the location and the character of 22 intrinsic two-particle excitations in the low energy spectrum of the doubly odd nucleus  $^{170}\text{Lu}$ . New or revised configuration assignments have been proposed, as given in table 2, for five known levels. Thirteen new levels not identified so far have been predicted. Another rather longlived  $7^+$  isomer at  $(225 \pm 25)$  keV with half-life in seconds has also been predicted. A specific experimental study has been suggested to look for this isomer and other high spin states. Hopefully these investigations should provide valuable guidelines for further experimental studies of this nucleus.

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