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An age-dependent model of cavity radiation and its detection II. Anti-bunched and thermal characteristics

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Abstract. The model of cavity radiation introduced earlier is analysed further by considering special types of age-specific population growth interpretable in terms of evolution through phases. The model is shown to be versatile enough to admit anti-bunched photon statistics provided the process of spontaneous emission is appropriately modelled. A four-phase model is analysed and the resulting radiation is shown to correspond to the one obtained by the superposition of two independent thermal streams each with a Lorentzian spectrum.

Keywords. Age dependent model; non-Markov evolution; population point process; cavity radiation; photon statistics; anti-bunching; thermal stream; Lorentzian spectrum; superposition of thermal streams.

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1. Introduction

In a previous contribution (Srinivasan 1986a, herein after referred to as I), the cavity radiation was modelled as a population point process with marginal age-specific birth rate $\lambda(x)$ and constant death rate μ supported by an immigration process with a constant rate v. The detection process was modelled as an emigration process with a constant rate η. If the analysis of the process is confined to a particular (marginal) age-specific rate $\lambda(x)$ where

$$\lambda(x) = \lambda\alpha \exp(-\lambda x)(\lambda x)^{n-1}/(n-1)!, \tag{1}$$

where λ and α are positive parameters, it is possible to construct a phase dependent population process. When $n = 2$ the conditional life span of a photon conditional upon its survival from death (cavity absorption) and detection can be interpreted to be the sum of three phases; the first two phases are assumed to have durations T_1 and T_2 that are independent and exponentially distributed with the same parameter $\lambda(>0)$. In the first phase, the birth rate is assumed to be zero while in the second phase it is a constant equal to $\alpha(>0)$. The third phase is defined to be the residual one with indefinite life span during which the birth rate is zero again; the photon will eventually be absorbed or removed by detection since the life span is defined to be conditional. The reformulation of the age specific population evolution in terms of phases renders the analysis of the problem tractable. However it should be noted that while the marginal (age-specific) birth rates are the same as those of the original Kendall Process, the evolution in terms of the phases leads to a slightly more general process in as much as the (simultaneous) conditional birth rates at two different epochs are no longer independent. In paper I, the detection process was also analysed and it was shown that an appropriate choice of the parameter can render the resulting radiation thermal with Lorentzian characteristics at least up to second order. In view of this result we proceed to examine in this paper whether the population model of the assembly of photons can also lead to features corresponding to anti-bunched light or thermal light with more general spectral properties.

At the outset it is to be noted that the population evolution equation proposed in paper I for the Markov type of population evolution can be derived directly from the fully quantum mechanical evolution equation proposed by Scully and Lamb (1966, 1967). The connection between population point process and the density matrix formalism has been discussed in considerable detail by Srinivasan and Vasudevan (1987) who have shown that the quantum mechanical evolution equation of Scully and Lamb (1966) leads to the population evolution equation. In particular the Markov nature of the cavity evolution is shown to be a consequence of the special manner in which the coarse graining average is performed. Thus a different type of averaging can lead to non-Markov behaviour of the cavity population and non-Markov models of population evolution implied by age/phase dependent population parameters are justifiable from this angle.

The layout of the paper is as follows. In § 2 we modify the process of spontaneous emission by introducing a non-Markov element and it turns out that such an evolution can give rise to photon statistics that are anti-bunched. In § 3 we consider a general four-phase model with differential absorption rates. The resulting radiation is shown to possess characteristics identical with those of the radiation resulting from the superposition of two independent chaotic thermal beams each with a Lorentzian profile.

2. Antibunching and sub-Poissonian characteristics

We first show that the non-Markov evolution modelled and dealt with in I can also lead to anti-bunched and sub-Poissonian photon statistics. The part of the amplification process of the field corresponding to spontaneous emission has been modelled as a Poisson process which in turn implies Markov evolution. If this is also modelled as a non-Markov process, then it can lead to a reduction of bunching in general (Srinivasan 1986b). We shall accordingly model the immigration process (spontaneous emission) as a process of delayed Poisson stream the delay itself being an exponentially distributed random variable. In other words spontaneous emissions normally occur at an expected rate per unit of (coarse-grained) time; however after each emission, there is a dead time distributed exponentially with the parameter $\beta(>0)$. Thus after the expiry of the dead time, the process of spontaneous emission which was frozen during the dead period, continues. The process of absorption by cavity and stimulated emission is modelled as in paper I. Thus the analysis and results of paper I as far as they relate to the functions $g_i(z_1, z_2, z_3, t)$ $(i = 1, 2)$ are valid in the present model; however equations (3), (4) and (5) of I are no longer valid and have to be replaced by new ones.

At the outset we note that the process of spontaneous emission alternates between dead and live phases. During the dead phase, no spontaneous emission is possible, while during the live phase the time to the first emission is exponentially distributed with parameter ν. It is to be noted that each spontaneous emission triggers a new span of dead phase with duration exponentially distributed with parameter β. Thus it is convenient to introduce the two valued process $Z(t)$ taking values 0 and 1 corresponding to the dead and live phase of the process of spontaneous emission. We introduce the generating function

$$G_i(z_1, z_2, z_3, z, t) \quad (i = 0, 1) \text{ by}$$

$$G_i(z_1, z_2, z_3, z, t) = E[z_1^{X_1(t)} z_2^{X_2(t)} z_3^{X_3(t)} z^{X(t)} \mid W(0) = 0, \nu \neq 0, Z(0) = i] \tag{2}$$

where $X(t)$ represents the total number of photons. By analysing the backward differential relation, we obtain

$$\frac{\partial G_0(z_1, z_2, z_3, z, t)}{\partial t} = \beta G_0(z_1, z_2, z_3, z, t) + \beta G_1(z_1, z_2, z_3, z, t) \tag{3}$$

$$\frac{\partial G_1(z_1, z_2, z_3, z, t)}{\partial t} = -\nu G_1(z_1, z_2, z_3, z, t) + \nu G_0(z_1, z_2, z_3, z, t) g_1(z_1, z_2, z_3, z, t) \tag{4}$$

with the initial conditions

$$G_i(z_1, z_2, z_3, z, 0) = 1 \quad (i = 0, 1). \tag{5}$$

The generating function G_i contains implicitly the probability of the population size and its limit as $t \to \infty$ will yield the limiting distribution of the photon numbers. It is indeed difficult to obtain the functions G_i explicitly; however the factorial moments of the photon population can be obtained explicitly. It is to be noted that the limit as $t \to \infty$ of the various moments correspond to the moments of the number operator in the fully quantized version. We now turn our attention, as in paper I, to the determination of the first two moments of the population size.

To obtain the moments we introduce the following notation for $i = (0, 1)$:

$$a_i(t) = \frac{\partial G_i}{\partial z}(z_1, z_2, z_3, z, t) | z = z_1 = z_2 = z_3 = 1$$

$$b_i(t) = \frac{\partial G_i}{\partial z^2}(z_1, z_2, z_3, z, t) | z = z_1 = z_2 = z_3 = 1. \tag{6}$$

In addition to the moments and cross-correlations $A_k^i(t), A^i(t), B_K^{ij}(t), B^{ij}(t)$ defined by (10) and (11) of paper I, we introduce the following further notation:

$$A_k(t) = \sum_{i=1}^{3} A_k^i(t), \quad B_k(t) = \sum_{i,j=1}^{3} B_k^{ij}(t). \tag{7}$$

We next differentiate (2) and (3) with respect to z and evaluate the values at $z_1 = z_2 = z_3 = z = 1$ to obtain

$$\frac{\mathrm{d}a_0(t)}{\mathrm{d}t} = -\beta a_0(t) + \beta a_1(t), \tag{8}$$

$$\frac{da_1(t)}{dt} = -va_1(t) + v(a_0(t) + A_1(t)), \tag{9}$$

$$\frac{db_0(t)}{dt} = -\beta b_0(t) + \beta b_1(t), \tag{10}$$

$$\frac{db_1(t)}{dt} = -vb_1(t) + v[b_0(t) + B_1(t)] + 2va_0(t)A_1(t), \tag{11}$$

with the initial conditions

$$a_i(0) = b_i(0) = 0 \quad (i = 0, 1). \tag{12}$$

Using Laplace transform technique, we obtain

$$a_0^*(s) = \frac{\beta v A_1^*(s)}{s(s + \beta + v)}, \quad a_1^*(s) = \frac{(\beta + s)A_1^*(s)}{s(s + \beta + v)} \tag{13}$$

$$b_0^*(s) = \frac{\beta v[B_1^*(s) + L(s)]}{s(s + \beta + v)}, \quad b_1^*(s) = \frac{(\beta + s)v[B_1(s) + L(s)]}{s(s + \beta + v)} \tag{14}$$

where $L(s)$ is given by

$$L(s) = 2\int_0^\infty a_0(t)A(t)\exp(-st)\,dt. \tag{15}$$

Actually we need only $L(0)$ and it can be evaluated either directly by using the convolution theorem; we obtain after some computation,

$$L(0) = \frac{\beta v}{2(\lambda + \mu + \eta)(\beta + v + \lambda + \mu + \eta)^2 - \lambda\alpha} \times \{(2\lambda + \mu + \eta)^2(\lambda\alpha + 3(\lambda + \mu + \eta)^2 + 2(\beta + v)(\lambda + \mu + \eta)/[(\lambda + \mu + \eta)^2 - \lambda\alpha]^2 - 1\}. \tag{16}$$

We are now comfortably placed to obtain the first two moments of the equilibrium cavity photon population. As before neglecting the population in phase 3, we find the first moment is given by

$$\lim_{t\to\infty} a_0(t) = \lim_{t\to\infty} a_1(t) = \beta v A_1^*(0)/(\beta + v), \tag{17}$$

where $A_1^*(0)$ is evaluated using (7) and (18), (19) and (20) of paper I:

$$A_1^*(0) = (2\lambda + \mu + \eta)/[(\lambda + \mu + \eta)^2 - \lambda\alpha]. \tag{18}$$

The second factorial moment is evaluated in a similar way:

$$b_0(\infty) = b_1(\infty) = \beta v[B_1^*(0) + L(0)]/(\beta + v) \tag{19}$$

$$B_1^*(0) = \lambda\alpha(3\lambda + 2\mu + 2\eta + \alpha)/2[(\lambda + \mu + \eta)^2 - \lambda\alpha]^2. \tag{20}$$

As usual introducing the bunching factor by $\mathscr{B}$

$$\mathscr{B} = b_i(\infty)/[a_i(\infty)]^2, \tag{21}$$

we find

$$\mathscr{B} = \frac{(\beta+\nu)}{2\beta\nu(2\lambda+\mu+\eta)^2}\{\lambda\alpha(3\lambda+2\mu+2\eta-\alpha)+\beta\nu(2\lambda+\mu+\eta)^2 \times\left(3\lambda+3\mu+\eta+2\beta+2\nu+\frac{\lambda\alpha}{\lambda+\mu+\eta}\right) - \frac{[(\lambda+\mu+\eta)^2-\lambda\alpha]^2}{\lambda+\mu+\eta}\bigg/[(\beta+\nu+\lambda+\mu+\eta)^2-\lambda\alpha]. \tag{22}$$

If we choose $\mu+\eta=2\lambda$, $\alpha=\lambda/2, \nu=\lambda, \beta=\lambda/2$, we find $\mathscr{B}=0{\cdot}66$ showing that the photon statistics is anti-bunched and sub-Poissonian in its character. If we wish to obtain the product densities of the detection process, we need follow the method outlined in § 4 of paper I. Since the method of analysis is identical, we do not discuss it any further.

3. General phase model: evolution through four phases

We first consider the phase model corresponding to (1) when $n=3$ and generalize it by relaxing the identical nature of the distribution of the duration of the first three phases. We however retain the exponential nature of the distribution. Thus the first three phases are characterized respectively by the parameters $\lambda_1, \lambda_2, \lambda_3$, of the exponential distribution of the life spans. We further assume that the death (absorption) rate in the first three phases are constants respectively equal to μ_1, μ_2, μ_3. We assume for the sake of simplicity that the death rate in the residual phase is μ_3. The birth rate is zero in all phases except the third when it has the value $\alpha(>0)$. In the first instance we assume that immigrations (spontaneous emissions) is at a constant rate η. The detection is assumed to be at a constant rate per unit time. The primary motivation for the use of differential death rate stems from the fact that absorption is also part of evolution and hence non-Markov nature should get reflected in this process as well.

3.1 *Generating function of the population statistics*

We introduce the following notation:

$X_i(t)$: the number of photons in phase i at time $t (i=1,2,3,4)$

$X(t)$: the total number of photons at time t

$Y(t)$: the number of photons detected over the time interval $(0,t)$.

A comprehensive description of the statistics of the photon population can be provided by the generating functions $g_i (i=1,2,3,4)$ and g defined by

$$g_i(z_1,z_2,z_3,z,w,t) = E\left[\left(\prod_{j=1}^{3} z_j^{X_j(t)}\right) z^{X(t)} w^{Y(t)} \,|\, X_i(0)=X(0)=1, \nu=0\right] \tag{23}$$

$$g(z_1, z_2, z_3, z, w, t) = E\left[\left(\prod_{j=1}^{3} z_j^{x_j(t)}\right) z^{X(t)} w^{Y(t)} | X_i = 0 = X = 0, \nu \neq 0\right]. \quad (24)$$

The connecting relation between g and g_1 remains the same as in I:

$$g(z_1, z_2, z_3, z, w, t) = \exp - \nu \int_0^t [1 - g_1(z_1, z_2, z_3, z, w, \tau)]\, d\tau. \quad (25)$$

The function g_4 can be directly determined by using the non-proliferating nature of the photons in phase 4:

$$g_4(z_1, z_2, z_3, z, w, t) = z \exp[-(\mu + \eta)t] + \frac{\mu + w}{\mu + \eta}\{1 - \exp[-(\mu + \eta)t]\}. \quad (26)$$

As in I, we can keep out of our perview the photons in phase 4; alternatively we can assume that the death rate in phase 4 to be $\mu_3 + \lambda_3$ to deal with the population generated by the

$$\frac{\partial g_i}{\partial t} = -(\lambda_i + \mu_i + \eta) g_i + \lambda_i g_{i+1} + \mu_i + \eta z \quad (i = 1, 2). \quad (27)$$

For the population generated by the photons in phase 3 we use the backward differential relation combined with the branching nature of the photon population to obtain

$$\frac{\partial g_3}{\partial t} = -(\lambda_3 + \mu_3 + \eta + \alpha) g_3 + \alpha g_3 g_1 + \lambda_3 + \mu_3 + \eta z. \quad (28)$$

The initial conditions are given by

$$g_i(z_1, z_2, z_3, z, w, 0) = z_i z \quad (29)$$

3.2 *Factorial moments and cross correlations*

We next introduce the factorial moments and cross correlations by

$$a_i^j(t) = \frac{\partial g_i}{\partial z_j}, \quad a_i(t) = \frac{\partial g_i}{\partial z}$$

$$b_i^{jk}(t) = \frac{\partial^2 g_i}{\partial z_j z_k}, \quad a^j(t) = \frac{\partial g}{\partial z_j}, \quad b^{jk}(t) = \frac{\partial^2 g}{\partial z_j z_k} \quad (30)$$

where the derivatives are evaluated at the point $z_1 = z_2 = z_3 = w = z = 1$. At the outset we note

$$a_i(t) = \sum_j a_i^j(t) \quad (31)$$

we next differentiate both sides of (27) and (28) to obtain

$$\frac{\mathrm{d}a_1^j(t)}{\mathrm{d}t} = -\alpha_1 a_1^j(t) + \lambda_1 a_2^j(t) \tag{32}$$

$$\frac{\mathrm{d}a_2^j(t)}{\mathrm{d}t} = -\alpha_2 a_2^j(t) + \lambda_2 a_3^j(t) \tag{33}$$

$$\frac{\mathrm{d}a_3^j(t)}{\mathrm{d}t} = -\alpha_3 a_3^j(t) + \alpha a_1^j(t) \tag{34}$$

with the initial condition

$$a_i^j(0) = \delta_{ij} \tag{35}$$

where δ_{ij} is the Kornecker delta and the constants α_i are given by

$$\alpha_i = \lambda_i + \mu_i + \eta \quad (i = 1, 2, 3). \tag{36}$$

We solve the above set of equations by Laplace transform (LT) technique, using the symbol as a superscript to distinguish the Laplace transforms* of the corresponding function, we obtain after some computation

$$a_i^{j*}(s) = F_i^j(s)/D(s), \quad a_i^*(s) = F_i(s)/D(s) \tag{37}$$

where

$$\begin{aligned}
&D(s) = (\alpha_1 + s)(\alpha_2 + s)(\alpha_3 + s) - \lambda_1\lambda_2\alpha,\\
&F_1^1(s) = (\alpha_2 + s)(\alpha_3 + s), \quad F_1^2(s) = \lambda_1(\alpha_3 + s), \quad F_1^3(s) = \lambda_1\lambda_2,\\
&F_2^1(s) = \alpha\lambda_2, \quad F_2^2(s) = (\alpha_1 + s)(\alpha_3 + s), \quad F_2^3(s) = \lambda_2(\alpha_1 + s),\\
&F_3^1(s) = \alpha(\alpha_2 + s), \quad F_3^2(s) = \alpha\lambda_1, \quad F_3^3(s) = (\alpha_1 + s)(\alpha_2 + s),\\
&F_1(s) = (\alpha_3 + s)(\alpha_2 + s + \lambda_1), \quad F_2(s) = (\alpha_1 + s)(\alpha_3 + s + \lambda_2) + \alpha\lambda_2,\\
&F_3(s) = (\alpha_2 + s)(\alpha_1 + s + \alpha) + \alpha\lambda_1.
\end{aligned} \tag{38}$$

Let $\Gamma_1, \Gamma_2, \Gamma_3$ be the roots of the equation $D(s) = 0$, then the solution can be written as

$$\begin{aligned}
a_i^j(t) = {} & \frac{F_i^j(\Gamma_1)\exp(\Gamma_1 t)}{(\Gamma_1 - \Gamma_2)(\Gamma_1 - \Gamma_2)} + \frac{F_i^j(\Gamma_2)\exp(\Gamma_2 t)}{(\Gamma_2 - \Gamma_1)(\Gamma_2 - \Gamma_3)}\\
& + \frac{F_i^j(\Gamma_3)\exp(\Gamma_3 t)}{(\Gamma_2 - \Gamma_1)(\Gamma_3 - \Gamma_2)}(i, j = 1, 2, 3).
\end{aligned} \tag{39}$$

We can follow exactly the method of analysis presented in I and obtain explicit expressions for the LT of the b-functions. However we will not do so; rather we will consider a special case that will turn out to be interesting. We put the following condition on the zeros Γ_i of $D(s)$

$$2\Gamma_3 = \Gamma_1 + \Gamma_2. \tag{40}$$

The condition will prepare the ground for identifying the radiation as the one corresponding to the superposition of two Lorentzian profiles. The condition (40) will be satisfied if the parameter α_i, λ_i and α satisfy the condition

$$(\alpha_1+\alpha_2+\alpha_3)[-\tfrac{5}{18}(\alpha_1+\alpha_2+\alpha_3)^2+\tfrac{1}{2}(\alpha_1^2+\alpha_2^2+\alpha_3^2)] \\ = 3(\alpha\lambda_1\lambda_2-\alpha_1\alpha_2\alpha_3). \tag{41}$$

It can be verified that (41) is feasible in the sense that physically meaningful values α_i, λ_i and α satisfying (41) do exist. Using (40) we can rewrite (39) in the form

$$a_i^j(t) = 2\{F_i^j(\Gamma_1)\exp(\Gamma_1 t)+F_i^j(\Gamma_2)\exp(\Gamma_2 t) \\ -2F_i^j(\Gamma_3)\exp(\Gamma_3 t)\}/(\Gamma_1-\Gamma_2)^2 \quad i,j=1,2,3. \tag{42}$$

It then follows

$$a_i(t) = 2\{F_i(\Gamma_1)\exp(\Gamma_1 t)+F_i(\Gamma_2)\exp(\Gamma_2 t) \\ -2F_i(\Gamma_3)\exp(\Gamma_3 t)\}/(\Gamma_1-\Gamma_2)^2 \quad i=1,2,3. \tag{43}$$

We next perform appropriate differentiation to obtain for $i,j=1,2,3$

$$\frac{db_1^{ij(t)}}{dt} = -\alpha_1 b_1^{ij(t)}+\lambda_1 b_2^{ij(t)} \tag{44}$$

$$\frac{db_2^{ij(t)}}{dt} = -\alpha_2 b_2^{ij(t)}+\lambda_2 b_3^{ij(t)} \tag{45}$$

$$\frac{db_3^{ij}(t)}{dt} = -\mu_3 b_3^{ij}(t)+\alpha b_1^{ij}(t)+\alpha a_3^j(t)a_1^i(t)+a_3^i(t)a_1^j(t) \tag{46}$$

with the initial condition

$$b_k^{ij}(0)=0 \quad k=1,2,3. \tag{47}$$

Using Laplace transform technique we obtain

$$b_1^{ij*}(s) = \alpha\lambda_1\lambda_2\int_0^\infty [a_3^j(t)a_1^i(t)+a_3^i(t)a_1^j(t)]\exp(-st)\,dt/D(s). \tag{48}$$

For the special case when (19) holds, further simplification can be effected leading to

$$b_1^{ij}(0) = 2\alpha\lambda_1\lambda_2\Sigma_k\xi_k a_3^{j*}(-\Gamma_k)F_1^i(\Gamma_k)+a_3^{i*}(-\Gamma_k)F_1^j(\Gamma_k)/[(\Gamma_1-\Gamma_2)^2 D(0)] \tag{49}$$

where

$$\xi_k=1, \quad \text{for } k=1,2; \quad \xi_3=-2. \tag{50}$$

Using (4) we finally obtain

$$b^{ij}(\infty) = v^2 a_1^{i*}(0)a_i^{j*}(0)+vb_1^{ij*}(0) \tag{51}$$

so that (51) taken with (49) yields explicit formulae for the second factorial and cross moments of the stationary cavity population in different phases.

3.3 *Superposition of two fields*

The product densities of the detection process can be obtained in exactly the same manner as in *I*. In this case we find when the cavity and detector are conditioned to be in equilibrium the functions $f_1(\cdot)$ and $h_{sty}(\cdot)$ as defined by (37) and (38) are given by:

$$f_1(\cdot) = h_1(\infty) = \eta v a_1^*(0) = \eta v[\alpha_2\alpha_3 + \lambda_1\alpha_3 + \lambda_1\lambda_2]/D(0) \tag{52}$$

$$h_{sty}(t) = [h_1(\infty)]^2 + 2\eta^2 \sum_i E_i \xi_i \exp(\Gamma_i t)/(\Gamma_1 - \Gamma_2)^2 \tag{53}$$

where

$$E_i = h_1(\infty) v F_1(\Gamma_i)/(\eta\Gamma_i) + \sum b^{jk} F_k(\Gamma_i). \tag{54}$$

Next we observe that the radiation resulting from the four phase model has thermal characteristics if

$$2[h_1(\infty)]^2 = 2\eta^2 \sum_{ijk} \xi_i b^{jk}(\infty) F_k(\Gamma_i)/(\Gamma_1 - \Gamma_2)^2 \tag{55}$$

in view of (31) we have the identity

$$2\sum_i \xi_i F_k(\Gamma_i) = (\Gamma_1 - \Gamma_2)^2 \tag{56}$$

which reduces to (55) to

$$2(h_1(\infty))^2 = \eta^2 \sum_{jk} b^{jk}(\infty) \tag{57}$$

a relation which expresses the thermal condition directly in terms of moments.

We establish the feasibility of (55) and (41) by taking some specific values for the parameters; this can be done by trial and error method. Actually there are too many parameters at our disposal; we fix some of them by making the choice

$$\alpha_1 = 7\mu, \quad \alpha_2 = 3\mu, \quad \alpha_3 = 2\mu \tag{58}$$

where μ is some parameter having the dimension of 1/time; the condition (41) now reduces to

$$\alpha\lambda_1\lambda_2 = 6\mu^3 \tag{59}$$

where Γ_i themselves are given by

$$\Gamma_1 = -1{\cdot}354\mu, \quad \Gamma_2 = -6{\cdot}646\mu, \quad \Gamma_3 = -4\mu. \tag{60}$$

There is a choice of λ_1, λ_2 and α are given by

$$\lambda_1 = 4{\cdot}62\mu, \quad \lambda_2 = \mu, \quad \alpha = 1{\cdot}299\mu \tag{61}$$

satisfying (59) which when substituted in (57) determines:

$$v = 0{\cdot}644\mu. \tag{62}$$

For this choice of parameters, we have

$$f_1(\cdot) = h_1(\infty) = 0{\cdot}3553\eta \tag{63}$$

$$h_{sty}(t) = \eta^2(0\cdot3553)^2 + 0\cdot1165(\exp(-0\cdot677\mu|t|) + 0\cdot0402\exp(-3\cdot223\mu|t|)^2\eta^2. \tag{64}$$

If at this stage, we compare the formula with the one derived by using the quantum theory of detection (Glauber 1966; Srinivasan and Vasudevan 1967), we can identify the correlation function. For Gaussian light we have the identification.

$$h_{sty}(t) = \bar{I} + |G^{(1)}(rt_1, rt_2)|^2; \quad t_2 - t_1 = t \tag{65}$$

where $\bar{I}$ is the average intensity of the incident beam and $G^{(1)}$ is the coherence function which can be identified by

$$G^{(1)}(rt_1, rt_2) = (\bar{I}/1\cdot0402)(\exp(-0\cdot677\mu|t|) + 0\cdot0402\exp(-3\cdot223\mu|t|)). \tag{66}$$

Hence we conclude that the resulting radiation corresponds to the superposition of two independent Gaussian (chaotic) Lorentzian fields. Thus the superposition of chaotic fields has a nice interpretation in terms of the cavity evolution; a non-Markov evolution characterized by age dependent emission and absorption rates results in the radiation whose second order characteristics coincide with those of a thermal field obtained by the superposition of two independent thermal fields, with a Lorentzian spectrum.

The above identification assumes special importance for two reasons. First the derivation of the statistical characteristics of the radiation resulting from the superposition is a fairly difficult problem even in the case of chaotic fields (Barakat and Blake 1980); the method of analysis presented can be adapted to consider more intricate situations. Second there is an interpretation of the origin of non-Markov nature of the chaotic field in question. For instance the condition (55) or (57) characterising the Bose-Einstein fluctuation can be dispensed with; then we have a more general field with non-Gaussian characteristics. The analysis then leads to the determination of the second order characteristics of the resulting radiation field.

4. Summary and conclusion

Population point process models of cavity radiation have acquired importance mainly because of their viability and their ability to bring out many of the features relating to light amplification and its detection. In the past several models have been analysed and the earliest mode is that of Shimoda *et al* (1957) providing a description of the population characteristics of an assembly of photons subject to amplification and detection. Later Shepherd (1981) improvised the model to take into account the perpetual nature of the cavity-field detector interaction and obtained the statistical characteristics of the cavity population as well as the photo electron statistics. In these models, the population evolution parameters are constants and hence the model is essentially Markov in character, a property that leads to the Gaussian Lorentzian nature of the spectral profile of the resulting radiation. Hence a natural question arises as to what would be the characteristics of the spectral profile if Markov constraint is relaxed. This point has been examined in detail (Srinivasan and Vasudevan 1986).

It turns out that due to its very general nature the non-Markov evolution is very complex; depending on the choice of parameters, a non-Markov evolution can lead to a radiation with non-Gaussian character on the one hand and anti-bunched photon statistics on the other. In paper I a specific non-Markov model arising from age-dependent birth rate was analysed. In particular the resulting radiation was shown to be chaotic with a Lorentzian spectrum. We have now shown that a modification of the process of spontaneous emissions (to take into account the quantum jump undergone by the atomic system) leads to anti-bunched statistics in a very natural way. On the other hand if the spontaneous emissions are assumed to occur at a constant rate, the model brings out the characteristics leading to bunching. In view of this we have analysed the memory dependence by modelling the non-Markov evolution in terms of a 4-phase evolution. It turns out, as it should, that the model with the appropriate choice of the parameters describes radiation with spectral characteristics corresponding to the stream obtained by the superposition of two independent chaotic beams, each with a Lorentzian profile. Thus we conclude that the non-Markov modelling arising from age-dependent population parameters is viable enough to describe radiation of a non-classical nature on one hand and thermal field of a very general nature on the other.

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