Pramāṇa – J. Phys., Vol. 30, No. 1, January 1988, pp. 35–42. © Printed in India.

Gribov-Lipatov inequality and inclusive e^+e^- processes

D K CHOUDHURY and A K MISRA*

Department of Physics, *Department of Chemistry, University of Gauhati, Gauhati 781 014, India

MS received 4 May 1987; revised 15 September 1987

Abstract. We study the inclusive e^+e^- processes at the PETRA energy range within QCD and a fixed point theory using the phenomenological Gribov-Lipatov inequality suggested in an earlier analysis. Theoretical justification is provided within QCD and its possible implication in hadronization is discussed.

Keywords. Gribov-Lipatov inequality; quantum chromodynamics; hadronization.

PACS Nos 13·65; 13·60

1. Introduction

In this paper we address ourselves to the problem of inclusive e^+e^- processes $e^+e^- \rightarrow h + x$. Such processes have been studied recently with various fragmentation models (Field and Feynman 1978; Anderson *et al* 1983; Peterson *et al* 1983; Gottschalk 1983, 1984; Webber 1984). Attempts have also been made to study them in perturbative QCD (Kato *et al* 1983; Peterson *et al* 1983). Monte Carlo models (Ali *et al* 1983; Hoyer *et al* 1979) have been constructed to that ei d.

Choudhury and Vanryckghem (1978) studied these processes with the moment method (Tung 1975; Eilam and Glück 1976) assuming the validity of the Gribov-Lipatov relation (Gribov and Lipatov 1971, 1972a, b). Data from the SPEAR energy (Morehouse 1975, unpublished) were used.

Theoretical and phenomenological studies (Brandelik *et al* 1979; Kubota 1980; Floratos *et al* 1981; Konishi *et al* 1978, Dokshitzer 1977; Kawabe 1981) suggest the breakdown of the Gribov-Lipatov relation. A more recent phenomenological study (Choudhury and Misra 1987) of PETRA regime (Wu 1984) suggests its breakdown for proton and antiproton data with a definite trend in the form of an inequality.

The present work is an attempt to update our earlier analysis (Choudhury and Vankryckghem 1978) using the "inequality" version of the Gribov-Lipatov relation and the new experimental information (Wu 1984) at the PETRA regime. Theoretical justification for testing the inequality is provided within the QCD. A plausible implication of the relative degree of violation of the Gribov-Lipatov relation in pion and kaon data vs proton/antiproton is also discussed in the present paper.

2. Basic formalism

2.1 *Moment relations*

Let us define the moment integral in the usual way (Politzer 1974)

$$\int_0^1 \mathrm{d}x\, x^{n-2} F(x, Q^2) = M(n, Q^2). \tag{1}$$

The renormalizable field theories predict a factorizable Q^2 dependence

$$M(n, Q^2) Q \to \infty \sim C(n) \exp(-\lambda(n) k). \tag{2}$$

Here $C(n)$ is an unknown constant, $\lambda(n)$ the calculable anomalous dimension (Parisi 1972; Callan and Gross 1972; Politzer 1974) of the leading tensor operator in the Wilson expansion of the product of currents in the definition of the structure function and k is defined as

$$k = \begin{cases} \ln(Q^2/Q_0^2) \quad \text{(fixed point theory)} & \text{(FT)} \\ \ln[\ln(Q^2/\Lambda^2)/\ln(Q_0^2/\Lambda^2)] & \text{(QCD)}, \end{cases} \tag{3}$$

where Q_0^2 is some reference value and Λ is the QCD cut-off parameter. We also use the following forms of the anomalous dimension (Parisi 1972; Callan and Gross 1972; Politzer 1974)

$$\lambda(n) = \begin{cases} A\left(1 - \dfrac{6}{n(n+1)}\right) & \text{(FT)} \\ G\left(-3 - \dfrac{18}{n(n+1)} + 4 \displaystyle\sum_{m=1}^{n} \frac{1}{m}\right) & \text{(QCD)}. \end{cases} \tag{4}$$

Using the inverse Mellin transformation (Eilam and Glück 1976) and given the structure function $F(x, Q_0^2)$ at some reference value Q_0^2 for $x_0 < x < 1$, equations (1) and (2) can be used to evaluate the structure function $F(x, Q^2)$ for all Q^2 and $x_0 < x < 1$.

2.2 *Gribov-Lipatov relation*

The Gribov-Lipatov relation (Gribov and Lipatov 1971, 1972a, b) connects the structure function in the time-like and space-like region. It reads

$$\frac{z}{\beta} \frac{\mathrm{d}\sigma}{\mathrm{d}z}(e^+ e^- \to h + x)\bigg|_{z=\frac{1}{x}} = \frac{4\pi\alpha^2}{3S} F_2^{eh}(x, Q^2), \tag{5}$$

where β is the ratio of the three momentum to energy of the detected hadron and x and z are the usual variables for $eh \to ex$ and $e^+e^- \to h + x$ processes.

This relation has a simple intuitive basis in the parton model (Sullivan J D 1974, unpublished) and represents a reciprocity relation between the structure function $F_2^{eh}(x)$ and the fragmentation function $D_{qi}^h(z)$:

$$\frac{1}{x} F_2^{eh}(x) = \sum e_i D_{qi}^h(z)|_{z=\frac{1}{x}}. \tag{6}$$

In the leading logarithmic approximation, this relation holds even in perturbative

QCD (Dokshitzer 1978), while in the higher order, it breaks down (Floratos *et al* 1981). On the basis of the preconfinement hypothesis of Amati and Veneziano (1978), Kawabe (1981) also reported its violation. Defining

$$\bar{R}(x,Q^2) = \frac{F_2^{eh}(x,Q^2)}{\left.\dfrac{z}{\beta\sigma(e^+e^- \to \mu^+\mu^-)}\dfrac{d\sigma}{dz}(e^+e^- \to h+x)\right|_{x=\frac{1}{z}}}, \tag{7}$$

one then has

$$\bar{R}(x,Q^2) \neq 1 \tag{8}$$

when the Gribov-Lipatov relation breaks down.

The question then naturally arises whether $\bar{R}(x,Q^2) < 1$ or > 1. It was found by Brandelik *et al* (1979) using the empirical structure function of Riordan *et al* (1971) that the theoretical curve based on the Gribov-Lipatov relation falls short of the observed $\bar{P}$ yield implying $\bar{R} < 1$. This is confirmed by our recent analysis (Choudhury and Misra 1987) at the PETRA range (Wu 1984) up to $\sqrt{S} = 34\,\text{GeV}$. We therefore explore the theoretical basis of $\bar{R} < 1$ and attempt to distinguish the various factors that might contribute to the breakdown of the equality (5).

As noted earlier, both next to the leading QCD connections and the preconfinement mechanism yield $\bar{R} \neq 1$. In the preconfinement picture, the hadrons are produced from the colour singlet clusters like $qg\ldots g\bar{q}$ (g = gluon, q = quark) but $g \to q\bar{q}$ is forbidden. Such a selection rule however destroys the KLN theorem (Kinoshita 1962; Lee and Nauenberg 1964). As a consequence (Kawabe 1981), there is a kind of compensation between the infrared singularities of virtual corrections and the real emission terms. The net result is an enhancement of the fragmentation function occurring in

$$\frac{d\sigma}{dz}(e^+e^- \to h+x)$$

so that

$$D_{qi}^h(z)|_{\text{preconfinement}} > D_{qi}^h(z)|_{\text{without preconfinement}}. \tag{9}$$

Equation (9) was explicitly demonstrated by Kawabe (1981) using the recombination model of Chang and Hwa (1980). Next to leading QCD analysis (Floratos *et al* 1981) does not however clearly demonstrate such a feature.

Using (9) in (5) one obtains

$$\bar{R}(x,Q^2) < 1 \tag{10}$$

earlier inferred from phenomenological analyses (Brandelik *et al* 1979; Choudhury and Misra 1987). One is therefore tempted to attribute the preconfinement mechanism (rather than next to the leading correction) to be the basis of the inequality version of the Gribov-Lipatov relation $\bar{R}(x,Q^2) < 1$.

2.3 *Application of equation (10)*

Let us now use equation (10) for pion and kaon structure functions and study their consequences. Using (2) and (10) for pion and kaon inclusive distributions, we obtain

$$\frac{3s}{4\pi\alpha^2}\int dx\, x^{n-2}\frac{x}{\beta}\frac{d\sigma}{dx}(e^+e^- \to \pi, K + x) > e^{\pi,K}\exp(-\lambda(n)k). \qquad (11)$$

Hence, once the inclusive distribution

$$\frac{x}{\beta}\frac{d\sigma}{dx}(e^+e^- \to \pi, K + x)$$

is known for some reference value Q_0^2, it is possible to evaluate its lower bound for all Q^2 using (11). The Q^2 dependence should be governed by the same anomalous dimension $\lambda(n)$ of (4).

Let us record the values of numerical parameters occurring in our analysis. For four flavour model $G = 4/25$. On the other hand, for fixed point theories there is no theoretical value for A. We take $A = 0{\cdot}25$ and $0{\cdot}4$ from Tung (1975) and Choudhury and Vanryckeghem (1978) respectively. We also attempt to obtain the best value of A from PETRA data.

There is some arbitrariness in the QCD cut-off parameter as well. While the earlier analyses (Buras and Gaemers 1978; Fox 1977; Roy *et al* 1978) kept $\Lambda = 0{\cdot}3 - 0{\cdot}5$ GeV, recent experiments (Voss 1985) suggest $\Lambda = 0{\cdot}1$ GeV. We consider the possibilities with $\Lambda = 0{\cdot}1, 0{\cdot}3$ and $0{\cdot}5$ GeV and study their consequences. (Dhar A and Godbole R B 1982, unpublished) suggested that the x and Q^2 dependence of Λ is due to higher order effects of the Alterelli-Parisi evolution equation (Alterelli and Parisi 1977) with the following form

$$\Lambda(x, Q^2) = \Lambda(\ln Q^2/\Lambda^2)^{-0{\cdot}37}\exp[\tfrac{1}{4}\ln(1/1 - x)], \qquad (12)$$

where $\Lambda = 0{\cdot}4$. We study the phenomenological consequence of (12) in our analysis.

3. Results

We now use our formalism to study the inclusive distribution

$$\frac{d\sigma}{dx}(e^+e^- \to \pi + x)$$

and

$$\frac{d\sigma}{dx}(e^+e^- \to K + x)$$

at the PETRA regime (Wu 1984) for typical values of $\sqrt{S} = 30, 34$ and $30, 35$ GeV respectively.

We use $\sqrt{S} = 5{\cdot}25$ GeV for pion and $\sqrt{S} = 7{\cdot}2$ GeV for kaon data as inputs to calculate the lower bounds of the distributions at higher energies. In figures 1 and 2 we show the predictions for the pion data while in figures 3 and 4 we show those for the kaon data. The shaded regions correspond to the variation of the estimated

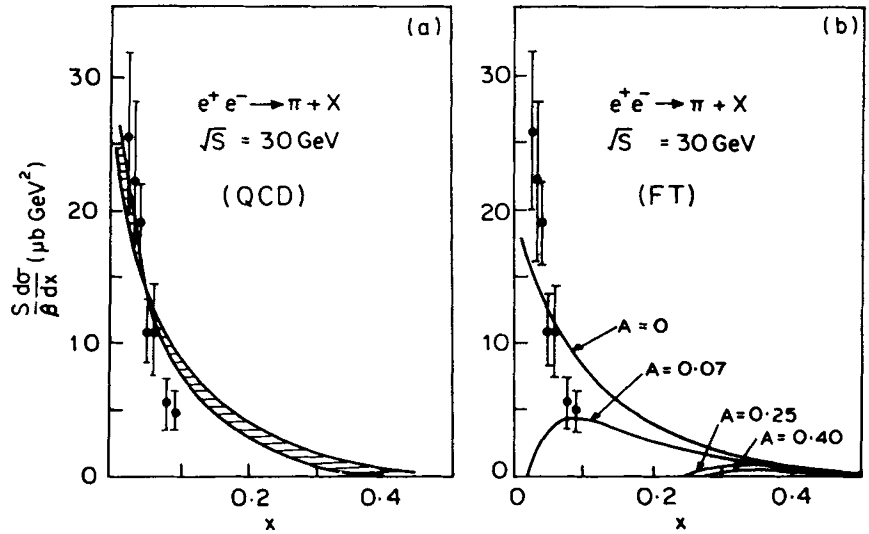

Figure 1. Prediction and comparison with data at $\sqrt{S} = 30\,\mathrm{GeV}$ for $e^+e^- \rightarrow \pi + X$: (a) QCD; (b) fixed point theory (F.T.)

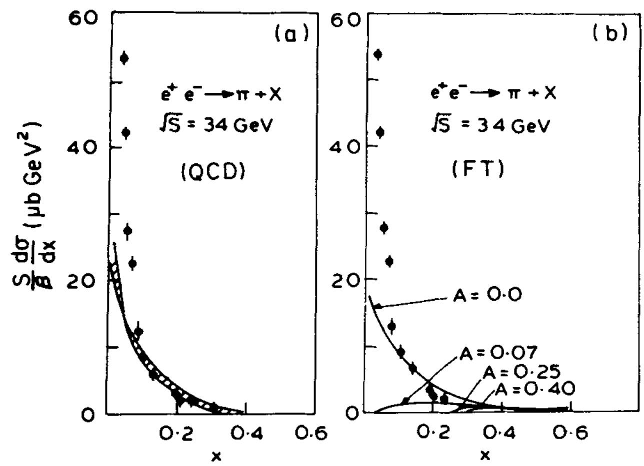

Figure 2. Predictions and comparison with data at $\sqrt{S} = 34\,\mathrm{GeV}$ for $e^+e^- \rightarrow \pi + X$: (a) QCD; (b) F.T.

cross-sections with cut-off parameters $\Lambda(\Lambda = 0{\cdot}1, 0{\cdot}2, 0{\cdot}3, 0{\cdot}4$ and $0{\cdot}5)$ and the x and Q^2 dependent function, equation (12). An overall study of the figures shows that lower bounds based on QCD are closer to the data than those of the fixed point theory. Because of the power law fall, prediction of the fixed point theory falls far below the experiment. Such a feature is true for both $A = 0{\cdot}4$ and $A = 0{\cdot}25$. Even if we find a best fit for the pion data at the lowest $\sqrt{S}$ value ($\sqrt{S} = 12\,\mathrm{GeV}$) with $A = 0{\cdot}07$, it falls below the data at higher $\sqrt{S}$. Only the value of $A = 0$ seems to be nearer to data which is nothing but its free field limit.

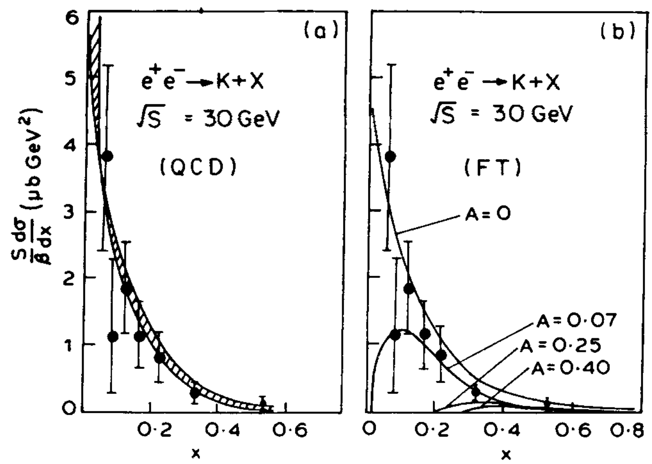

Figure 3. Predictions and comparison with data at $\sqrt{S} = 30$ GeV for $e^+e^- \to K + X$: (a) QCD; (b) F.T.

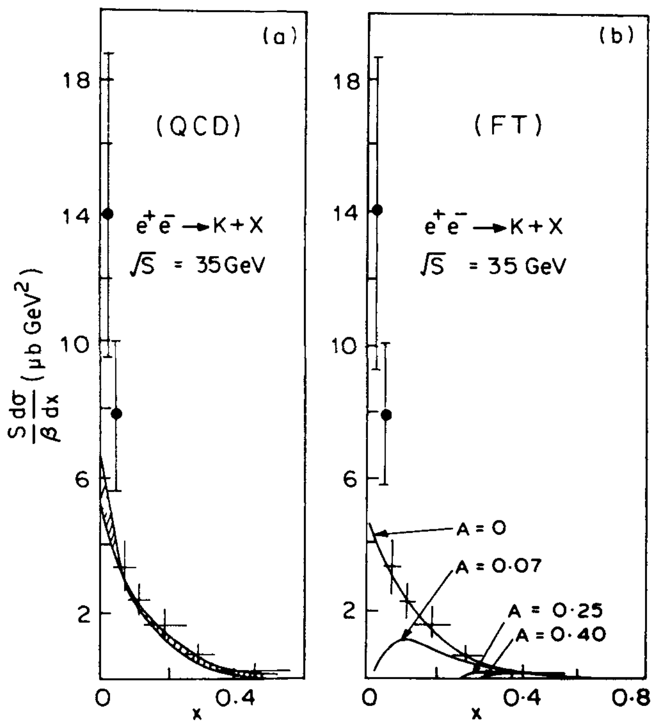

Figure 4. Predictions and comparison with data at $\sqrt{S} = 35$ GeV for $e^+e^- \to K + X$: (a) QCD; (b) F.T.

Let us now discuss how the QCD prediction varies with the change of the cut-off parameter Λ. Our analysis shows that the variation of $\Lambda = 0{\cdot}1$ to $0{\cdot}5$ or its explicit x and Q^2 dependence (equation 12) does not significantly alter the predictions. These are given in the shaded regions of figures 1 to 4.

Finally since SLC and LEP will be in operation shortly (Llewellyn Smith C H 1981, unpublished) we predict QCD with $\Lambda = 0{\cdot}1$ and $\Lambda = 0{\cdot}5$ at $\sqrt{S} = 100\,\text{GeV}$ (figure 5). Data should be above this limit, if preconfinement picture is to hold good at this energy range.

4. Conclusions

In the present work, we have studied the inclusive processes using the moment method (Callan and Gross 1972; Parisi 1972; Tung 1975; Eilam and Glück 1976; Choudhury and Vanryckghem 1978) and the Gribov-Lipatov phenomenological inequality (Brandelik *et al* 1979; Choudhury and Misra 1987) valid up to the PETRA range. We have suggested preconfinement mechanism to be its theoretical origin. Next to leading QCD effects do not necessarily imply such an inequality. At a formal level, since preconfinement hypothesis implies breakdown of KLN theorem (Kawabe 1981), the inequality equation (10) might have deeper significance than envisaged here.

In the present analysis, we observe that the inclusive distribution for pion and kaon is closer to the lower bound obtained from the inequality equation (10). Using our earlier analysis (Choudhury and Misra 1987), we then infer

$$R^{\pi}(x, Q^2), R^{K}(x, Q^2) > R^{\bar{p}}(x, Q^2). \tag{13}$$

Equation (13) implies that the violation of Gribov-Lipatov relation is greater for the proton than for pion or kaon. Since basic preconfinement mechanism is the same for both, such a difference is expected to be at the hadronization level. Since the hadronization mechanism of $3q$ system (baryon) becomes more non-perturbative than that of the $q\bar{q}$ system (meson), it is reasonable to conjecture that the violation of Gribov-Lipatov relation will be much stronger for baryon than for meson, as we have

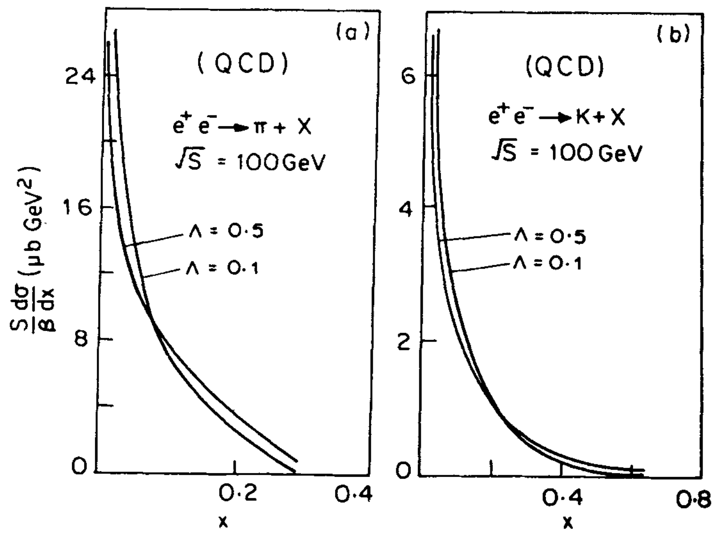

Figure 5. Predictions of QCD at $\sqrt{S} = 100\,\text{GeV}$: (**a**) $e^+e^- \to \pi + X$; (**b**) $e^+e^- \to K + X$ with $\Lambda = 0{\cdot}1\,\text{GeV}$, $0{\cdot}5\,\text{GeV}$.

observed. Even then, the preconfinement condition, equation (9), seems to hold invariably. Further, our analysis rules out a fixed point theory in e^+e^- inclusive processes, since such a theory conforms only to data in its free field limit ($A = 0$). We note that a fixed point survives as an alternative to QCD as far as the R problem (Choudhury and Misra 1983) is concerned. The present analysis however rules out such a possibility altogether.

Acknowledgement

One of us (DKC) gratefully acknowledges financial support from DSTE, Government of Assam.

References

Ali A *et al* 1980 *Phys. Lett.* **B93** 155
Altarelli G and Parisi G 1977 *Nucl. Phys.* **D126** 298
Amati D and Veneziano G 1979 *Phys. Lett.* **B83** 87
Anderson B *et al* 1983 *Phys. Rep.* **97** 31
Brandelik *et al* 1979 *Nucl. Phys.* **B148** 189
Buras A J and Gaemers K J F 1978 *Nucl. Phys.* **B132** 249
Callan C and Gross D 1972 *Phys. Rev.* **D8** 4383
Chang V and Hwa R C 1980 *Phys. Rev. Lett.* **44** 439
Choudhury D K and Vanryckeghem 1978 *Phys. Rev.* **D17** 1766
Choudhury D K and Misra A K 1983 *Phys. Rev.* **D28** 2154
Choudhury D K and Misra A K 1987 *Pramāṇa – J. Phys.* **28** 23
Dokshitzer Yu L 1977 *Sov. Phys. JETP* **46** 641
Eilam G and Glück M 1976 *Phys. Lett.* **B61** 85
Field R D and Feynman R P 1978 *Nucl. Phys.* **B136** 1
Floratos E G, Lacoze R and Kaunnas 1981 *Phys. Lett.* **B98** 89
Fox G C 1977 *Nucl. Phys.* **B131** 107
Gottschalk T D 1983 Lectures at the 19th International School of Elementary Particle Physics Kupari-Dubromik, Yugoslavia
Gottschalk T D 1984 *Nucl. Phys.* **B239** 325, 349
Gribov V N and Lipatov L N 1971 *Phys. Lett.* **B37** 78
Gribov V N and Lipatov L N 1972a *Sov. J. Nucl. Phys.* **15** 438
Gribov V N and Lipatov L N 1972b *Yad. Fiz.* **15** 781
Hoyer P *et al* 1979 *Nucl. Phys.* **B161** 349
Kato K, Munehisa T, Okada H and Shimizu Y 1983 *The perturbative QCD and e^+e^- single particle inclusive reactions*, University of Tokyo preprint (UT-387)
Kawabe T 1981 *Prog. Theor. Phys.* **65** 1973
Kinoshita J 1962 *J. Math. Phys.* **3** 650
Konishi K, Ukaya A and Veneziano G 1978 *Phys. Lett.* **B78** 24
Kubota T 1980 *Nucl. Phys.* **B165** 277
Lee T D and Nauenberg M 1964 *Phys. Rev.* **D133** 1549
Parisi G 1972 *Phys. Lett.* **B42** 114
Peterson C, Schlatter D, Schmitt I and Zerwas P M 1983 *Phys. Rev.* **D27** 105
Politzer H D 1974 *Phys. Rep.* **14** 129
Riordan E M, Bodek A and Coward D H 1974 *Phys. Lett.* **B52** 249
Roy D P, Paranjape S, Pandita P N and Choudhury D K 1978 *Pramāṇa – J. Phys.* **11** 619
Tung W K 1975 *Phys. Rev.* **D12** 3613
Voss R 1985 "Structure function: Deep inelastic scattering on nucleus and nuclei" CERN-EP/85-204.
Webber B R 1984 *Nucl. Phys.* **B238** 492
Wu S L 1984 *Phys. Rep.* **107** 59