Pramāṇa – J. Phys., Vol. 29, No. 6, December 1987, pp. 577–581. © Printed in India.

# Spectrum of the high spin neutron hole states of $^{205}$Pb

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MS received 28 April 1987; revised 13 August 1987

**Abstract.** The high resolution ($^3$He, $\alpha$) reaction on $^{206}$Pb shows the distribution of the $2f_{7/2}$, $1h_{9/2}$ and $1i_{13/2}$ neutron states of $^{205}$Pb within the 6 MeV excitation energy of $^{205}$Pb. The spectrum of these three-hole states is obtained within the hole-core vibrational coupling scheme. The shell model energies of the neutron hole states arising from the core-polarization effect are compared with the Bansal-French energy weighted sum rule. The possible implication of the present neutron hole energies has been discussed in the light of the deduced shell model wave functions of the collective states of $^{206}$Pb.

**Keywords** Neutron hole strengths; quasi particles; core-polarization; Bansal-French sum rule.

**PACS No.** 21·10

## 1. Introduction

In a high resolution ($^3$He, $\alpha$) reaction at 100 MeV incident energy, Guillot *et al* (1980) have detected the presence of weak fragments of the high spin $2f_{7/2}$, $1h_{9/2}$ and $1i_{13/2}$ neutron states of $^{205}$Pb. Within a spread of 1 to 6 MeV excitation energy, the entire shell model strengths of the three neutron-hole states are obtained. We have reported here the spectrum of the excited $7/2^-$, $9/2^-$ and $13/2^+$ states of $^{205}$Pb within the hole-core coupling model scheme. The prime motivation is to examine the nature of the collectivity of the vibrational states of $^{206}$Pb together with the possible changes in the zero-order shell model energies of the neutron states of $^{205}$Pb. As a consequence, the shell model wave functions of the excited vibrational states of $^{206}$Pb will alter from the reproduced values of Ma and True (1973). This has profound influence on the study of the magicity in the superheavy region which has been discussed through our most recent work on all the odd ($A \pm 1$) nuclei around $^{208}_{82}$Pb (Majumdar 1987). Moreover we had earlier observed the effect of quadrupole and octupole vibrations of $^{206}$Pb on the fragmentation of the $1j_{15/2}$ neutron particle state of $^{207}$Pb (Majumdar 1985). As there are three neutron holes in $^{205}$Pb over the doubly magic good vibrator $^{208}$Pb, the effect of pairing interaction is to be incorporated into the interaction Hamiltonian of the core particle model. Also the energies of the shell model states are to be replaced by the corresponding energies of the quasi-particle states. We also see that both the quadrupole and octupole vibrations of $^{206}$Pb are required to understand the structure of these states in the core particle model scheme.

## 2. Results

The model in the present work has been taken from our recent work on $^{207}$Pb (Majumdar 1985). The Hamiltonian of the physical system can be stated as,

$$H = H_{\mathrm{vib}} + H_n + H_{\mathrm{int}}, \tag{1}$$

where $\langle H_{\mathrm{vib}} \rangle$ is the vibrational energy of the $^{206}$Pb core. $\langle H_n \rangle$ is the energy of the neutron hole states in the average shell model potential and $H_{\mathrm{int}}$ represents the hole-core interaction Hamiltonian,

$$H_{\mathrm{int}} = -K(r) \sum_{\lambda\mu} \alpha_{\lambda\mu} Y_{\lambda\mu}(\theta, \phi). \tag{2}$$

Here $K(r)$ is associated with the radial part of the potential in which the extra nucleon moves.

To evaluate the eigenvalues and eigenvectors for the Hamiltonian (1) we write the wavefunction for $J$ spin state, with its projection $M$ along the $Z$-axis, at an energy $E^{(\alpha)}$ as

$$\psi_J = |E^{(\alpha)}; JM\rangle = \sum_{Rlj} a^{(\alpha)}_{Rlj} |\{(N_2 R_2, N_3 R_3) R; (nl\tfrac{1}{2}) j\}^{J,\pi}_{M}\rangle \tag{3}$$

where $j$ is the angular momentum of the shell model hole state, $R_2$ and $R_3$ are the phonon angular momenta for the quadrupole phonon number $N_2$ and octupole phonon number $N_3$, $R$ is the coupled angular momentum of $R_2$ and $R_3$. In (3) $\mathbf{J} = \mathbf{R} + \mathbf{j}$. Expanding $\alpha_{\lambda\mu}$ in terms of the creation and annihilation operators $b^*_{\lambda\mu}$ and $b_{\lambda\mu}$ we can write the expression for the matrix elements of $H_{\mathrm{int}}$ as

$$\begin{aligned}
&\langle N_2 R_2, N_3 R_3; R, j; J | H_{\mathrm{int}} | N'_2 R'_2, N'_3 R'_3; R', j'; J \rangle \\
&= \sum_{\lambda} X_\lambda \hbar\omega_\lambda (-1)^{R+j'-J} [2(2R+1)(2j+1)(2R'+1)(2R_\lambda+1)]^{1/2} \\
&\quad W(R'Rj'j; \lambda J) \begin{pmatrix} j & \lambda & j' \\ \tfrac{1}{2} & 0 & \tfrac{1}{2} \end{pmatrix} [\delta_{\lambda,2} W(R'RR'_\lambda R_\lambda; \lambda R'_3) \\
&\quad (-1)^{R'_\lambda - R - R'_3 - \lambda} + \delta_{\lambda,3} W(R'RR'_\lambda R_\lambda; \lambda R'_2)(-1)^{R' + R_\lambda - R'_2 - \lambda}]. \\
&\quad \delta_{N_n N'_n} \delta_{R_n R'_n} [\delta_{N_\lambda, N'_\lambda + 1} \langle N_\lambda R_\lambda \| b^*_\lambda \| N'_\lambda R'_\lambda \rangle \\
&\quad + \delta_{N'_\lambda, N_\lambda + 1} (-1)^{R_\lambda - R'_\lambda} \langle N'_\lambda R'_\lambda \| b^*_\lambda \| N_\lambda R_\lambda \rangle] (U_j U'_j - V_j V'_j).
\end{aligned} \tag{4}$$

Here $X_\lambda = \left[\dfrac{(2\lambda+1)}{4\pi\hbar\omega_\lambda C_\lambda}\right]^{1/2} \dfrac{\langle k(\gamma) \rangle}{2}$ (Van den Berghe and Heyde 1971)

$\hbar\omega_\lambda$ is the energy of the phonon state for the $\lambda$-mode vibration of the collective $^{208}$Pb core. $W$ is the Racah's coefficient and $U_j$ and $V_j$ are the non-occupation and occupation probabilities of the $j$-state. The subscripts in $N_n$ and $R_n$ assume the value 2 when $\lambda = 3$ and vice versa. The diagonal matrix elements of $H$ (equation 1) are the sum of the quasiparticle energy $E_j$ and the energy of the core $E_c$ which is given by

$$E_c = N_2 \hbar\omega_2 + N_3 \hbar\omega_3. \tag{5}$$

The quasiparticle energies $E_j$ are calculated from the single particle energies $\varepsilon_j$ using the relation

$$E_j = [(\varepsilon_j - \lambda)^2 + \Delta_n^2]^{1/2}, \tag{6}$$

where $\Delta_n$ is a measure of the pairing gap and is also estimated from the neutron separation energies (Wapstra and Andi 1985). The value of $\Delta_n$ is 0·890 MeV. $\lambda$ is the Fermi energy (7·409 MeV) and is calculated from the neutron separation energies of $^{205}$Pb and $^{206}$Pb (Maxhaux and Sartor 1985). In the present case, we have taken $\lambda$ = ground state $\varepsilon_{2f_{5/2}}$ (Straume and Burke 1977). The value of $V_j^2$ is calculated by the relation

$$V_j^2 = \frac{1}{2}\left[1 - \frac{(\varepsilon_j - \lambda)}{E_j}\right]. \tag{7}$$

From the diagonalization of the Hamiltonian matrices for the $J^\pi = 7/2^-$, $9/2^-$ and $13/2^+$ states, the $a_{0j}^2$, the squared amplitude of the zero-phonon coupled state is obtained. This is weighted with $V_j^2$, the occupation probability of the $j$-shell model state and the corresponding values are compared with the experimental spectroscopic factors. The $5/2^-$, $7/2^-$, $9/2^-$ and $13/2^+$ spin states are set up by coupling $3p_{1/2}$, $3p_{3/2}$, $2f_{5/2}$, $2f_{7/2}$, $1h_{9/2}$, $1h_{11/2}$ and $1i_{13/2}$ states with the quadrupole one, two, three and octupole one phonon states of $^{206}$Pb. Both the $2_1^+$ and $3_1^-$ states of $^{206}$Pb possess a good degree of collectivity as the $B(E2, 2_1^+ \to 0^+)$ and $B(E3, 3_1^- \to 0^+)$ values are 6.2 WU (Stein *et al* 1968) and 14·2 WU (Alster 1967) respectively. WU stands for the Weisskopf single particle unit. Also the $^{206}$Pb is a good vibrator as the wavefunction of the $2_1^+$ states $^{206}$Pb from Ma and True (1973) is

$$\begin{aligned}\psi_{2_1^+}(^{206}\text{Pb}) = {} & 0{\cdot}708|p_{1/2}^{-1} f_{5/2}^{-1}\rangle - 0{\cdot}539|p_{1/2}^{-1} p_{3/2}^{-1}\rangle \\ & + 0{\cdot}269|f_{5/2}^{-2}\rangle - 0{\cdot}168|f_{3/2}^{-1} p_{3/2}^{-1}\rangle \\ & + 0{\cdot}205|p_{3/2}^{-2}\rangle + 0{\cdot}197|p_{3/2}^{-1} f_{7/2}^{-1}\rangle \end{aligned} \tag{8}$$

which contains a large amount of configuration mixed states.

First of all, the energies $\varepsilon_j$ of the shell model neutron states are taken from the energies of the experimental excited states having more than 50% of the single hole strengths (Lanford 1975; Guillot *et al* 1980). $\varepsilon_j$ is then converted to $E_j$. The quasiparticle energies of all the neutron states except that of the $2f_{7/2}$, $1h_{9/2}$ and $1i_{13/2}$ states are kept fixed at the adopted values. The energies $E_j$ of these three states and the coupling parameters $X_2$ and $X_3$ are optimized around their input values to locate the main experimental fragments of the $5/2^-$ (ground state), $7/2^-$ (1·762 MeV), $9/2^-$ (2·695 MeV) and $13/2^+$ (1·011 MeV) states (Guillot *et al* 1980). The results are displayed in table 1. All the fragmented hole states of $7/2^-$, $9/2^-$ and $13/2^+$ below 3 MeV excitation energy have been located. The weak fragments arise from the

**Table 1.** Fragmentation of the $2f_{7/2}$, $1h_{9/2}$ and $1i_{13/2}$ neutron states of $^{205}$Pb. $E$, the energy eigenvalues in MeV; $a_{0j}^2$, the squared amplitude of the zero-phonon coupled state weighted with $V_j^2$. The figure within the bracket indicates experimental result Guillot *et al* 1980.

| $f_{7/2}$ state | | $h_{9/2}$ state | | $i_{13/2}$ state | |
|---|---|---|---|---|---|
| $E$ | $a_{0j}^2$ | $E$ | $a_{0j}^2$ | $E$ | $a_{0j}^2$ |
| 1·14 | 0·36 | 0·61 | 0·01 | 1·03 (1·011) | 0·52 (0·59) |
| 1·44 (1·042) | 0·04 (0·056) | 1·28 | 0·01 | 1·74 | 0·01 |
| 1·54 (1·615) | 0·25 (0·11) | 1·35 | 0·01 | 4·58 (4·57) | 0·02 (0·03) |
| 1·65 (1·762) | 0·10 (0·63) | 2·11 (2·695) | 0·36 (0·365) | | |
| 4·48 (3·96) | 0·01 (0·05) | 2·60 | 0·01 | | |
| 4·52 | 0·16 | 2·53 (2·903) | 0·09 (0·120) | | |
| 5·20 | 0·01 | 2·87 | 0·36 | | |
| | | 2·99 | 0·01 | | |
| | | 3·69 | 0·01 | | |
| | | 4·00 (4·79) | 0·02 (0.03) | | |

**Table 2.** Comparison of the neutron shell-model energies (MeV) with respect to the ground state energy of $^{208}$Pb.

| *nlj*: | $3p_{1/2}$ | $3p_{3/2}$ | $2f_{5/2}$ | $2f_{7/2}$ | $1h_{9/2}$ | $1h_{11/2}$ | $1i_{13/2}$ |
|---|---|---|---|---|---|---|---|
| Present: | 23·01 | 23·12 | 22·19 | 24·16 | 24·69 | 29·80 | 23·54 |
| Lanford 1975 : | 22·214 | 22·523 | 22·219 | 23·898 | 24·93 | — | 23·218 |

various hole coupled excited vibrational states. Seven fragments of $f_{7/2}$ state have been obtained. Of these the four states at 1·14, 1·44, 1·54 and 4·52 MeV bear measurable strengths, although the experiment fails to reproduce the state at 1·14 MeV. Ten fragments of $h_{9/2}$ states have been calculated within 4 MeV. Only three weak states of $i_{13/2}$ have been obtained within 5 MeV. Beyond 3 MeV excitation energy, the experimental results show the signature of the presence of these three spin states, albeit the identification have been tentatively assigned. Table 1 shows that in addition to the location of the discrete states in the low energy region, the 4·48 MeV state of $f_{7/2}$, the 4·00 MeV state of $h_{9/2}$ and the 4·58 MeV state of $i_{13/2}$ have been confirmed with the corresponding experimental observations.

Regarding the realistic estimates of the adopted quasiparticle energies, we have compared our deduced theoretical values with the energies of the neutron states from the Bansal-French energy weighted sum rule (Bansal and French 1965) that can be utilized to list the squared amplitudes of the ground state wave function of $^{206}$Pb. On the whole the three neutron holes of $^{205}$Pb are bound by 22·19 MeV with respect to the zero energy of $^{208}$Pb. Adding up the $E_j$ with 22·19 MeV, we get the neutron hole state energies of $^{205}$Pb with respect to the ground state of $^{208}$Pb. These are shown in table 2. The consistency in the energy from table 2 with that of Bansal-French estimates (Lanford 1975, page 823) proves the fidelity of the adopted values.

## 3. Conclusion

To summarize, the present study advocates the nature of the weak fragments of $7/2^-$, $9/2^-$ and $13/2^+$ states of $^{205}$Pb from the hole-core vibrational model calculation. The zeroth order shell model energies of the neutron hole states in $^{205}$Pb are correctly assigned from the results of the present work on clean one-step pick-up reaction on $^{206}$Pb. The shell model wave functions of the neutron hole states will naturally alter due to the shell model energies of the neutron states as a result of core-polarization. As the mixing of the several neutron hole states generate the wave function of the various spin states of $^{206}$Pb, it will be interesting to observe how far the complex shell model calculations of Ma and True (1973) for the $^{206}$Pb will prove the validity of the similar results arising from core-polarization effect. Further it is concluded that $^{206}$Pb is a good vibrator like $^{208}$Pb as the hole core coupling model works well in $^{205}$Pb.

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