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A study of hydrodynamic turbulence using laser transmission

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Abstract. Using laser transmission, the characteristics of hydrodynamic turbulence is studied following one of the recently developed technique in nonlinear dynamics. The existence of deterministic chaos in turbulence is proved by evaluating two invariants viz. dimension of attractor and Kolmogorov entropy. The behaviour of these invariants indicates that above a certain strength of turbulence the system tends to more ordered states.

Keywords. Fluctuation phenomena; turbulence; attractor dimension; Kolmogorov entropy.

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1. Introduction

Turbulence is a highly nonlinear phenomenon obeying Lorentz equations (Lorentz 1963) and show instabilities which are formally equivalent with thermodynamical phase transitions if they are continuously driven away from thermal equilibrium. Due to fundamental fluctuation-dissipation relations, stochastic forces are imposed on the dynamical behaviour of nonequilibrium systems. These forces give rise to fluctuations which are purely of stochastic character. Furthermore, in certain parameter ranges, nonlinear systems can behave in a way which is known as deterministic chaos if they have at least three degrees of freedom (Newhouse *et al* 1978).

Experimentally there are two standard methods to get clear evidence for chaotic behaviour. One is to observe one of the commonly known transition scenarios to chaos. The other method is to determine the invariants of the system which directly characterize the chaotic behaviour. Two such invariants are easily extractable from the experimental data. They are (i) the dimension of the attractor of the system in phase space and (ii) the entropy which is connected with the evolution of the system in phase space. These invariants may change if some control parameter of the system is varied. Recently, Atmanspacher and Scheingraber (1986) described a method to determine the dimension of an attractor and corresponding entropy from the measurements of time series of a single variable of the system in the context of the study of dynamical instabilities in multimode laser. This technique was applied recently to study the dynamics of neutral systems from the analysis of EEG of human brain (Varghese *et al* 1987).

The aim of the present work is to extend this technique to study the nature of the attractor underlying the hydrodynamic turbulence. The behaviour of the invariants with the increase in the strength of turbulence is studied.

2. Calculation procedure

The dimension of the attractor and the corresponding second order entropy are estimated from a measure of time series of a single variable of the system using the method reported by Atmanspacher and Scheingraber (1986). In the present case, the measured variable is the intensity of a laser beam propagated through a turbulent liquid medium. The refractive index of the medium will have spatial and temporal fluctuations due to the turbulent motion of the liquid. Therefore the propagated laser beam will have temporal fluctuations in the intensity. The nature of these fluctuations will contain certain information about the dynamics of the turbulence.

Let $\{X_0(t)\}$ be the original time series. Then d additional data sets ($d=1, 2, 3, \ldots$) are obtained by introducing a time delay ($d\Delta t$). From the resulting data sets, d dimensional phase space is constructed such that d is greater than the dimension of the actual phase space. From the analysis, as described in the following section, one can obtain the number of independent degrees of freedom which will describe the dynamics.

If each data set contains N values spaced by a time increment τ, then the following data sets can be obtained for various values of d.

$$\begin{array}{l} X_0(t_1) \ldots X_0(t_N) \\ X_0(t_1+\Delta t) \ldots X_0(t_N+\Delta t) \\ \quad \vdots \qquad\qquad \vdots \\ X_0(t_1+d\Delta t) \ldots X_0(t_N+d\,\Delta t) \end{array}$$

such that $t_i = t_1 + (i-1)\tau$.

Within the vector representation,

$$\mathbf{X}_i = (X_0(t_i) \ldots X_0(t_i + d\,\Delta t)), \tag{1}$$

and the total set becomes

$$\mathbf{X}_1 \ldots \mathbf{X}_N$$

where $\mathbf{X}_i$ is a point in the constructed d dimensional space and we can determine the distance $|\mathbf{X}_i - \mathbf{X}_j|$ which is the usual Euclidean norm.

Then the correlation function can be calculated as

$$C(r) = \lim_{N\to\infty} \frac{1}{N^2} \sum_{i,j=1}^{N} H(r - |\mathbf{X}_i - \mathbf{X}_j|), \tag{2}$$

where $H(X)$ is the Heaviside function. $H(X)=0$ for $x<0$ and $H(X)=1$ for $X>0$. The function $C(r)$ counts the number of pairs of those points with a distance $|\mathbf{X}_i - \mathbf{X}_j|$ smaller than r. When the distances between all the pairs of points are less than r, then $C(r)=1$.

Grassberger and Procaccia (1983) introduced a second order dimension

$$D_2 = \lim_{r\to 0} \frac{\log C(r)}{\log r}, \tag{3}$$

D_2 is a very useful parameter for estimating the information dimension and the fractal dimension of an attractor. When D_2 is an integer the system is regular, and when it is fractal the system is chaotic while when $D_2 \to d$, the dimension of the constructed space, the system behaviour is stochastic.

This dimension of the attractor is a static invariant since they do not depend on any time scale. But the entropy of a system is always a quantity which has to be specified per unit time. Therefore it is a dynamic invariant describing the properties of the considered process.

Similar to the second-order dimension D_2, a second-order entropy K_2 can be defined as

$$K_2 = \lim_{r \to 0} \lim_{d \to \infty} \frac{1}{\tau} \log \frac{C_d(r)}{C_{d+1}(r)}, \tag{4}$$

where logarithm is to the base 2.

$K_2 = 0$ characterizes a regular system and $K_2 > 0$, corresponds to a chaotic system while $K_2 \to \infty$ describes completely stochastic system.

3. Experimental

A 5 mW He-Ne laser beam expanded to 15 mm was passed through a glass water tank (50 × 20 × 15 cm) which was uniformly heated from below. The upper layer of water was cooled by circulating cold water through a set of parallel copper tubes placed just below the water surface. The temperature gradient so formed, generates turbulent motion in the medium. The resulting temperature fluctuations give rise to refractive index variations, which affects the intensity of the propagating laser beam.

The temporal intensity fluctuations of the propagated laser beam was detected by a photodiode and fed to a data logger. The strength of turbulence was varied by changing the heater power. Measurements were taken for a path length of 200 cm by multipass arrangements. Intensity fluctuations were sampled at 10 ms interval for lower turbulence strengths (up to a heater power −847 W) and at 5 ms interval for higher turbulence strengths.

Figures 1 and 2 show typical examples of the time series over 2·5 s, for two heater powers 210 W and 1323 W respectively. Recently we had shown that the intensity

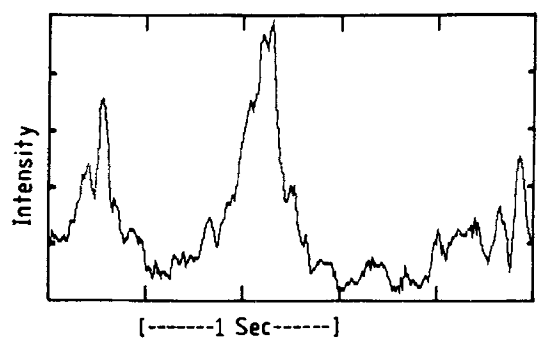

Figure 1. Temporal evolution of the intensity of the laser beam propagated through the turbulent medium. Heater power = 210 W.

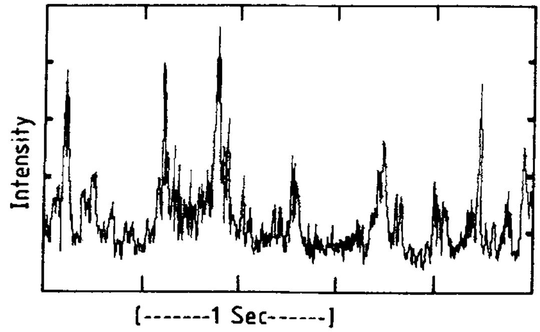

Figure 2. Same as in figure 1. Heater power = 1323 W.

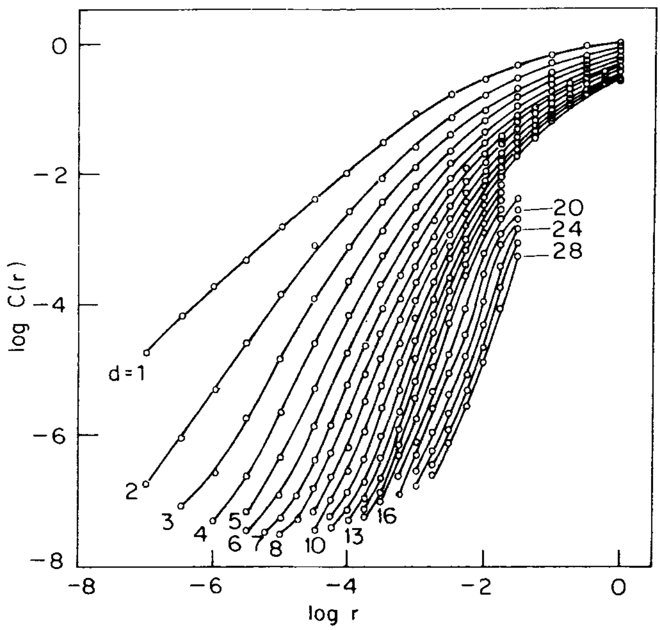

Figure 3. Log-log plot of correlation integral $C(r)$ vs distance r, for different dimensions d, for the time series shown in figure 1.

fluctuations of the transmitted laser beam agrees quite well with the results for atmospheric scintillations (Reghunath and Nampoori 1986).

The correlation function $C(r)$ calculated was at $\Delta t = \tau = 10$ ms and $N = 210$ points for lower turbulence strengths, while it was at $\tau = \Delta t = 5$ ms and $N = 450$ points for higher turbulence strengths.

The time series obtained was first normalized to the maximum intensity within the considered time interval.

4. Results and discussion

The curves of log $C(r)$ vs log r for different d are shown in figure 3 for the case of lowest heater power. As the dimension d of the constructed phase space increases, the slope ν

converges to a limiting value. This is illustrated in figure 4. The converging value of the slope is the dimension D_2 of the attractor. The line $\nu = d$ denotes complete stochastic behaviour. The fractal or noninteger values of D_2 is a sufficient criterion for contribution of deterministic chaos to the behaviour of the system.

From the value of D_2 it is possible to extract the number of degrees of freedom of the system. The dimension d of the constructed phase space is usually larger than the actual number of degrees of freedom of the system. The obtained value $D_2 = 2{\cdot}69$ indicates that the investigated process needs only next higher integer number of degrees of freedom (i.e. three) to be successfully modelled.

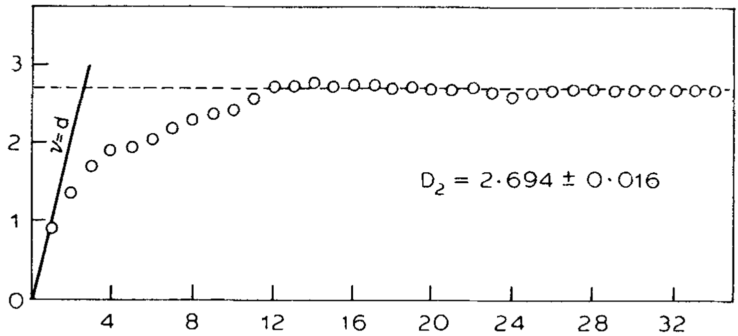

Figure 4. Slope ν in the linear range of the different curves in figure 3, as a function of the dimension d of the constructed phase space. The limiting value of slope gives the second order dimension $D_2 = 2{\cdot}694 \pm 0{\cdot}016$.

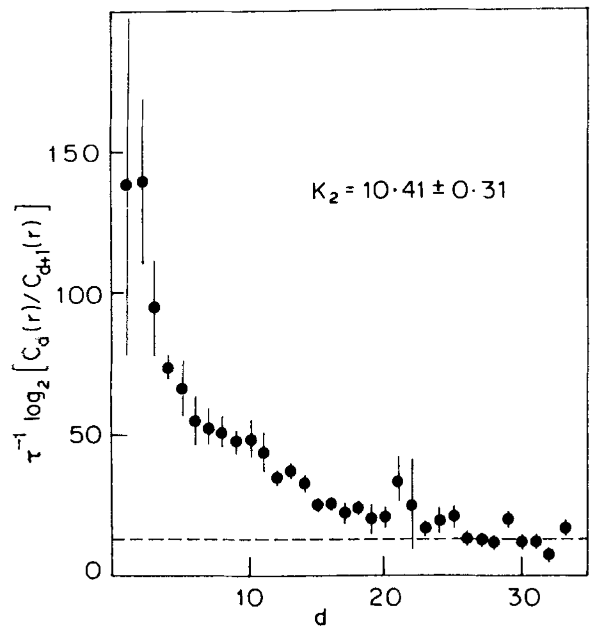

Figure 5. Mean value of $1/\tau \log(C_d(r)/C_{d+1}(r))$ as a function of d, obtained from the linear range of curves in figure 3. The limiting value for large d gives the second order entropy $K_2 = 10{\cdot}42 \pm 0{\cdot}31$.

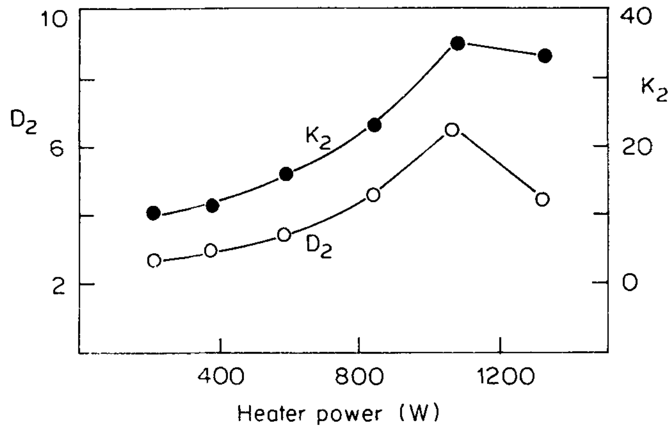

Figure 6. The behaviour of the second order dimension D_2 and second order entropy K_2 for different heater powers (turbulence strengths).

In certain cases, the statistical error prevents the discrimination between an integer or a fractal value D_2. This was observed for the case where the heater power was 375 W. In such cases, the second order entropy turned out to be useful since it provides an additional sufficient condition for chaotic behaviour, namely $K_2 > 0$.

K_2 is estimated from the vertical distance (identical r) between curves belonging to successive dimensions d. In figure 5 it is shown how K_2 approaches a limiting value for high dimension d. The indicated values for each particular d have been calculated for the mean value of $C_d(r)/C_{d+1}(r)$ over the linear range of r. Compared with the case for D_2, a considerably higher value of d is needed for the convergence of K_2.

The obtained values of D_2 and K_2 for different turbulence strengths are calculated and are shown in figure 6. The D_2 increased steadily with the strength of turbulence up to a heater power of 1072 W, after which it showed a decrease. A similar behaviour was observed for K_2, though the decrease in its value above 1072 W was not as pronounced as that for D_2.

5. Conclusions

The two invariants viz. the dimension of the attractor of the system and the entropy connected with the evolution of the system are studied for different turbulence strengths. The noninteger values of attractor dimension D_2 and nonzero values of entropy K_2 confirm the contribution of deterministic chaos to the behaviour of the system.

The steady increase of D_2 with turbulence strengths shows that the minimum number of variables which are needed to successfully model the system is increasing. Evidently the number of degrees of freedom of the system is increased. In other words the complexity of the system increases. This is substantiated by corresponding increase in second order entropy K_2.

The decrease of D_2 after a particular heater power (turbulence strength) implies a simplification of the turbulence, by reducing its degrees of freedom. Similar results were obtained by Atmanspacher and Scheingraber (1986) for the studies on dynamical instabilities in a multimode CW dye laser. The reduction in the complexities of the system is further confirmed by the decrease in K_2. This leads to the conclusion that beyond a critical turbulence strength, a hydrodynamic turbulent system can be represented in a lesser dimensional space. At this region the complexity of the system is reduced. In other words the system tends to more ordered states above a critical turbulence strength.

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