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An analysis of a quantum chromodynamic structure function

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Abstract. We obtain an approximate solution of Altarelli–Parisi equations yielding a sample of quantum chromodynamic structure function. The SLAC-MIT data are analysed with it. Possible effects of intrinsic charm and higher twist are also included. Agreement is found to be good for $x \geqslant 0{\cdot}25$.

Keywords. Quantum chromodynamic structure function; Altarelli–Parisi equation.

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1. Introduction

Phenomenological analysis of an approximate form of QCD structure function is reported. It is obtained as an approximate solution of Altarelli–Parisi equations (Altarelli and Parisi 1977). We investigate if such an approximate form can reasonably explain the SLAC-MIT data (Bodek *et al* 1979). The Q^2 dependence of the approximate form being simpler than the other QCD models (Buras and Gaemers 1978; Abbott *et al* 1980; Duke and Owens 1984), we explore the sensitivity of our results with the QCD cut off parameter Λ and the renormalization point Q_0^2. We also investigate the roles of higher-twist (Abbott *et al* 1980; Aubert *et al* 1981; Bollini *et al* 1981; Godbole and Roy 1982; Eisele *et al* 1982) and intrinsic charm (Brodsky *et al* 1980; Roy 1981) in our analysis.

2. The formalism

2.1 *Altarelli–Parisi equations*

The Q^2 evolution of the leading twist contribution to $F_2(x, Q^2)$ is predicted in lowest order in QCD by the following differential equations (Altarelli and Parisi 1977):

$$Q^2 \frac{\partial}{\partial Q^2} F_2^{NS}(x, Q^2) = \frac{\alpha_s(Q^2)}{3\pi} \bigg[\{3 + 4\ln(1-x)\} F_2^{NS}(x, Q^2) + \int_x^1 dw \frac{2}{(1-w)_+} \left\{ (1+w^2) F_2^{NS}\left(\frac{x}{w}, Q^2\right) - 2F_2^{NS}(x, Q^2) \right\} \bigg]. \tag{1}$$

$$Q^2\frac{\partial}{\partial Q^2}F_2^S(x, Q^2)=\frac{\alpha_s(Q^2)}{3\pi}\left[\{3+4\ln(1-x)\}F_2^S(x, Q^2)\right.$$
$$+\int_x^1 \mathrm{d}w\left\{\frac{2}{(1-w)_+}\left((1+w^2)F_2^S\left(\frac{x}{w}, Q^2\right)-2F_2^S(x, Q^2)\right)\right.$$
$$\left.\left.+\frac{3}{2}N_f(w^2+(1-w)^2)G\left(\frac{x}{w}, Q^2\right)\right\}\right] \tag{2}$$

where N_f is the number of quark flavour (taken to be four) and

$$\alpha_s(Q^2)=\frac{12\pi}{(33-2N_f)\ln\frac{Q^2}{\Lambda^2}}. \tag{3}$$

Defining $t=\ln\frac{Q^2}{\Lambda^2}$, taking $N_f=4$ one has

$$\frac{\partial F_2^{NS}}{\partial t}(x, t)=\frac{4}{25}\frac{1}{t}\left[\{3+4\ln(1-x)\}\, F_2^{NS}(x, t)\right.$$
$$\left.+\int_x^1 \mathrm{d}w\frac{2}{(1-w)_+}\left\{(1+w^2)F_2^{NS}\left(\frac{x}{w}, t\right)-2F_2^{NS}(x, t)\right\}\right] \tag{4}$$

and

$$\frac{\partial F_2^S}{\partial t}(x, t)=\frac{4}{25}\frac{1}{t}\left[\{3+4\ln(1-x)\}F_2^S(x, t)\right.$$
$$+\int_x^1 \mathrm{d}w\left\{\frac{2}{(1-w)_+}\left((1+w^2)F_2^S\left(\frac{x}{w}, t\right)-2F_2^S(x, t)\right)\right.$$
$$\left.\left.+6(w^2+(1-w)^2)G\left(\frac{x}{w}, t\right)\right\}\right]. \tag{5}$$

Approximate analytical solutions of (4) and (5) can be obtained if we neglect the variations of $F_2^S(x, t)$, $F_2^{NS}(x, t)$ and $G(x/w, t)$ on their right hand side. One can then have

$$F_2^{NS}(x, t)=F_2^{NS}(x, t_0)+\frac{4}{25}\ln\left(\frac{t}{t_0}\right)\left[\{3+4\ln(1-x)\}F_2^{NS}(x, t_0)\right.$$
$$\left.+2\int_x^1 \mathrm{d}w\frac{(1+w^2)}{(1-w)_+}F_2^{NS}\left(\frac{x}{w}, t_0\right)-4\int_x^1\frac{\mathrm{d}w}{(1-w)_+}F_2^{NS}(x, t_0)\right] \tag{6}$$

$$F_2^S(x, t)=F_2^S(x, t_0)+\frac{4}{25}\ln\left(\frac{t}{t_0}\right)\left[\{3+4\ln(1-x)F_2^S(x, t_0)\right.$$
$$+2\int_x^1 \mathrm{d}w\frac{(1+w^2)}{(1-w)_+}F_2^S\left(\frac{x}{w}, t_0\right)-4\int_x^1\frac{\mathrm{d}w}{(1-w)_+}-F_2^S(x, t_0)$$
$$\left.+6\int_x^1 \mathrm{d}w(w^2+(1-w)^2)G\left(\frac{x}{w}, t_0\right)\right] \tag{7}$$

with $t_0=\ln Q_0^2/\Lambda^2$, Q_0^2 being the normalization point. In the above equations, the distribution $(1-x)_+^{-1}$ is defined for a function $f(x)$ to be

$$\int_x^1 dx \frac{f(x)}{(1-x)_+}=\int_x^1 \frac{f(x)-f(1)}{(1-x)} \tag{8}$$

while in quantitative calculation, we take

$$\int_x^1 \frac{dy}{y}\frac{f(y)}{(1-x/y)_+}=f(x)\ln\frac{1-x}{x}+\int_x^1 \frac{dy}{y}\frac{f(y)-f(x)}{1-x/y}. \tag{9}$$

The structure function $F_2(x, Q^2)$ is defined in terms of the following combinations of $F_2^{NS}(x, Q^2)$ and $F_2^S(x, Q^2)$:

$$F_2(x, Q^2)=\frac{5}{18}F_2^S(x, Q^2)+\frac{3}{18}F_2^{NS}(x, Q^2). \tag{10}$$

In order to evaluate (6) and (7) numerically one needs the shapes of $F_2^S(x, t_0)$, $F_2^{NS}(x, t_0)$ and the gluon distribution $G(x, t_0)$. To this end, we take the initial distributions as (Abbott *et al* 1980)

$$F_2^{NS}(x, t_0)=C_1x^{C_2}(1-x)^{C_3}$$

$$F_2^S(x, t_0)=C_4(1+C_5x)(1-x)^{C_6}$$

$$G(x, t_0)=A(1-x)^{C_G}. \tag{11}$$

where A is fixed by the momentum sum rule

$$\int_x^1 dx\{F_2^S(x, t_0)+G(x, t_0)\}=1. \tag{12}$$

Abbott *et al* (1980) obtains numerically

$$C_1=0{\cdot}5991, \quad C_2=0{\cdot}853, \quad C_3=2{\cdot}68, \quad C_4=1{\cdot}85$$

$$C_5=1{\cdot}004, \quad C_6=3{\cdot}14, \quad C_G=5$$

for $Q_0^2=30{\cdot}5\ \text{GeV}^2$, $\Lambda=0{\cdot}628$ GeV

while (12) yields

$$A\approx 2{\cdot}8.$$

In order to study the sensitivity of our results with input function $F_2(x, Q_0^2)$, we also take the parametrization given by Barger and Phillips (1974) with $Q_0^2=4\ \text{GeV}^2$ and $\Lambda=0{\cdot}2$ GeV

$$v_u=0{\cdot}594x^{-1/2}(1-x^2)^3+0{\cdot}461x^{-1/2}(1-x^2)^5+0{\cdot}621(1-x^2)^7x^{-1/2}$$

$$v_d=0{\cdot}072x^{-1/2}(1-x^2)^3+0{\cdot}206x^{-1/2}(1-x^2)^5+0{\cdot}621x^{-1/2}(1-x^2)^7$$

$$\xi=0{\cdot}14x^{-1}(1-x)^9 \tag{13}$$

using the SLAC-MIT data (Riordan *et al* 1974) on F_2^{ep} and F_2^{en}

2.2 *Higher twist* (*HT*) *contribution*

There are several efforts in recent times (Abbott *et al* 1980; Aubert *et al* 1981; Bollini *et al* 1981; Godbole and Roy 1982; Eisele *et al* 1982; Choudhury and Misra 1987) on higher twist effect. Recent parametrization is by Aubert *et al* (1985):

$$F_2(x, Q^2) = F_2^{QCD}(x, Q^2)\left(1 + \frac{\mu_4^2 x^\alpha}{(1-x)Q^2}\right) \tag{14}$$

with $\alpha = 3{\cdot}1 \pm 0{\cdot}5$ and $\mu_4^2 = 1{\cdot}7 \pm 0{\cdot}05\ \mathrm{GeV}^2$. While keeping the form of (14) in our work, we will vary μ_4 and α to find the best fit.

2.3 *Intrinsic charm* (*IC*) *contribution*:

Brodsky *et al* (1981) have suggested that proton wave function contains an intrinsic charm component $|uudc\bar{c}\rangle$ with 1–2% probability. Explicity they take

$$c(x) = \bar{c}(x) = 18x^2\left[\frac{1}{3}(1-x)(1-10x+x^2) - 2x(1+x)\ln\frac{1}{x}\right], \tag{15}$$

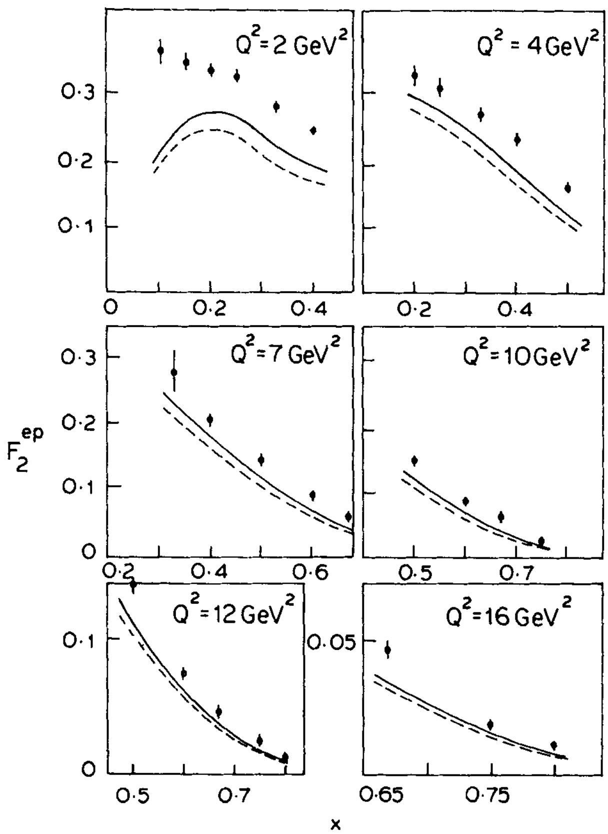

Figure 1. x vs $F_2^{ep}(x, Q^2)$ for $Q^2 = 2, 4, 7, 10, 12$ and $16\ \mathrm{GeV}^2$ respectively with input from Abbott *et al* (1980). Dashed curve represent the prediction without HT and IC while solid one includes these effects. Data are taken from Bodek *et al* (1979).

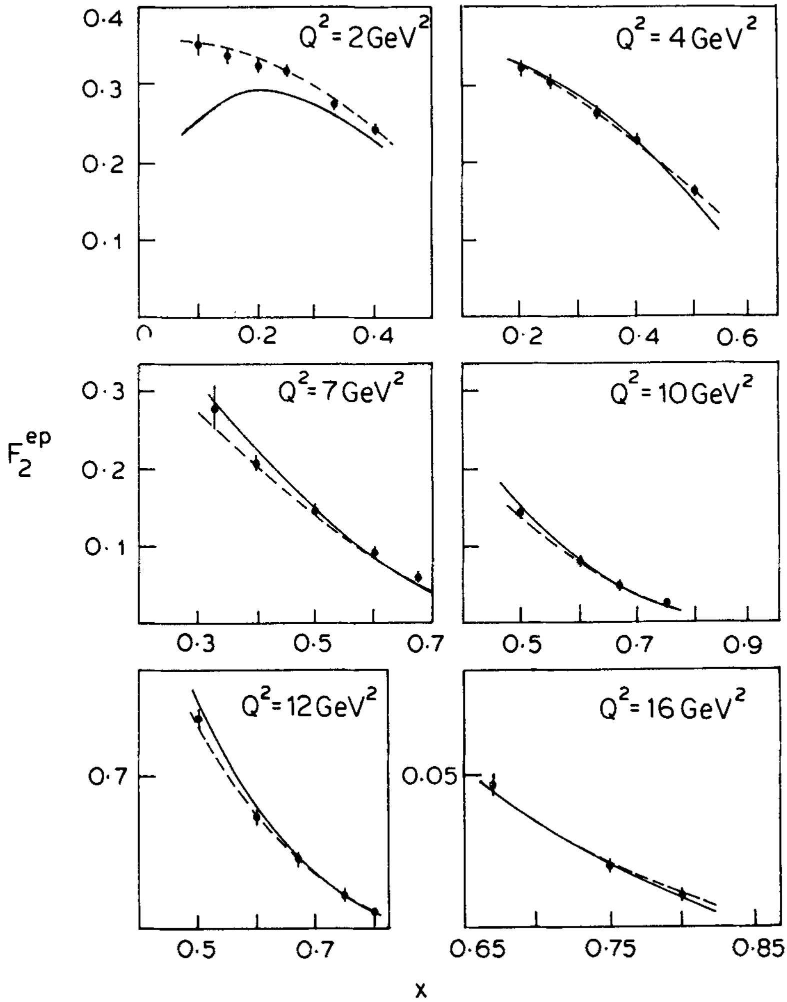

Figure 2. x vs $F_2^{ep}(x, Q^2)$ for $Q^2 = 2, 4, 7, 10, 12$ and $16\ \mathrm{GeV}^2$ respectively with inputs from Barger and Phillips (1974). Data are as in figure 1. Dotted dashed curve represent the prediction of Buras and Gaemers (1978).

so that one adds $4/9x(C(x) + \bar{C}(x))$ to (10) or (14). We will incorporate IC contribution in our work.

3. Results

In order to test the approximate structure function discussed in §2, we use SLAC-MIT data (Bodek *et al* 1979).

In figures 1 and 2, we plot $F_2(x, Q^2)$ vs x for $Q^2 = 2, 4, 7, 10, 12$ and $16\ \mathrm{GeV}^2$ with $Q_0^2 = 30{\cdot}5\ \mathrm{GeV}^2$ and $\Lambda = 0{\cdot}628$ GeV and $Q_0^2 = 4\ \mathrm{GeV}^2$ and $\Lambda = 0{\cdot}2$ GeV using $F_2(x, Q_0^2)$ as given by Abbott *et al* (1980) and Barger and Phillips (1974) respectively. In figure 1 we also show the effects of intrinsic charm and higher twist, while in figure 2, we compare the prediction of the model of Buras and Gaemers (1978).

Our analysis shows that the agreement with data is better with parametrization of Barger and Phillips (1974) rather than Abbott *et al* (1980). Best fit with higher twist parameters of (14) yields $\mu_4^2 = 2{\cdot}1$ GeV^2 and $\alpha = 3$ with parameters of Abbott *et al* (1980). Such effects seem not necessary in the case of Barger and Phillips (1974).

This last observation conforms to the expectation that the necessity to go beyond the perturbative QCD evolution equation in the forms of higher twist or intrinsic charm effects arises invariably only when one tries to cover the large Q^2 range from SLAC to CERN data. Since our fit is confined to Q^2 range of SLAC experiments only, the approximate solution, (6) and (7) with suitable form of inputs at $Q^2 = Q_0^2$ is adequate by itself to describe the Q^2 dependence in this range.

To conclude, we obtain a simple form of the structure function as an approximate solution of the QCD evolution equation. It can be represented by

$$F_2(x, Q^2) = F_2(x, Q_0^2) + \ln \frac{\ln(Q^2/\Lambda^2)}{\ln(Q_0^2/\Lambda^2)} H(x, Q_0^2) \tag{16}$$

as is evident from (6) or (7). We have compared our prediction with more rigorous one of Buras and Gaemers (1978). Our analysis indicates that it is nearly as good as that of Buras and Gaemers at least for high x region ($x \gtrsim 0{\cdot}25$ and $Q^2 \gtrsim 4$ GeV^2). It is indeed interesting to note that such a simple QCD structure function can reasonably fit the SLAC-MIT data without significant higher twist and intrinsic charm contributions.

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References

Abbott L F, Atwood W B and Barnett R M 1980 *Phys. Rev.* **D22** 582
Altarelli G and Parisi G 1977 *Nucl. Phys.* **B126** 298
Aubert J J *et al* 1981 *Phys. Lett.* **B105** 315
Aubert J J *et al* 1985 *Nucl. Phys.* **B259** 189
Barger V and Phillips R J N 1974 *Nucl. Phys.* **B73** 269
Bodek A *et al* 1979 *Phys. Rev.* **D20** 1471
Brodsky S J *et al* 1980 *Phys. Lett.* **B93** 451
Bollini D *et al* 1981 *Phys. Lett.* **B104** 403
Buras A J and Gaemers K J F 1978 *Nucl. Phys.* **B132** 249
Choudhury D K and Misra A K 1987 *Pramana–J. Phys.* **28** 23
Duke D W and Owens J F 1984 *Phys. Rev.* **D30** 49
Eisele F *et al* 1982 *Phys. Rev.* **D26** 41
Godbole R M and Roy D P 1982 *Z. Phys.* **C15** 39
Riordan E M, Bodek A and Coward D H 1974 *Phys. Lett.* **B52** 249
Roy D P 1981 *Phys. Rev. Lett.* **47** 213