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Economic models for Dirac neutrinos in grand unified theories

ANJAN S JOSHIPURA and UTPAL SARKAR
Institute of Mathematical Sciences, Madras 600 113, India

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Abstract. It is shown that dimension five non-renormalizable interactions can produce light Dirac neutrinos in an extension of the minimal SU(5) GUT containing additional SU(5) singlets and global U(1) symmetries.

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1. Introduction

The limits (Avignone *et al* 1983) on neutrinoless double beta decay and the claim (Boris *et al* 1983) of the ITEP group for the limit on neutrino mass from the data on ^{3}H decay suggest (Roy and Shanker 1984; Joshipura *et al* 1984; Shanker 1985; Roncadelli and Wyler 1984) that the neutrinos are Dirac particles. To accommodate any light Dirac neutrinos in grand unified theories two distinct ideas were implemented previously. In the first approach (Roy and Shanker 1984) one introduces several additional singlet fermionic fields and several Higgs scalars, and obtain a typical 4×4 mass matrix for the neutral fermions for every generation. Diagonalizing the mass matrix then results in a light Dirac neutrino and a heavy Dirac neutrino for every generation. In the second approach (Shanker 1985; Joshipura 1985) neutrinos of different generations combine together and form Dirac particles. In the present article we show that it is possible to generate Dirac neutrinos for every generation in a much more economic model if dimension five non-renormalizable interactions are considered.* We do this by considering grand unified theories based on non-supersymmetric SU(5) (§ 2) and locally supersymmetric SU(5) (§ 3). Section 4 contains the concluding remarks.

2. Dirac neutrino and non-supersymmetric SU(5)

We start with minimal SU(5) GUT, in which the matter fields belong to the $\bar{5}$-plet (ψ^i) and 10-plet (χ_{ij}) and the Higgs scalars are a 5-plet (ϕ^i) and a 24-plet (Σ^i_j). In addition we introduce a singlet fermionic field η, which can combine with the left-handed neutrino to form a Dirac particle. We can then write the most general Lagrangian with these

* The effect of dimension five non-renormalizable interactions has been considered in a different context earlier (Ellis and Gaillard 1979; Panagiotakopoulos and Shafi 1984; Nandi and Sarkar 1986).

fields. Since we are interested in the masses of the fermionic fields we shall only write the Yukawa part of the Lagrangian. In addition to the conventional renormalizable terms we shall also consider the non-renormalizable dimension five interactions, which will be characterized by inverse power of Planck scale. We shall not worry about the occurrence of these non-renormalizable terms and assume that when higher order calculations are performed there is a cut-off in the loop momenta of the order of the next higher mass scale, i.e. M_p, and such interactions have a scale parameter of order M_p. These interactions can also be induced by gravity. We then have,

$$L_{\text{Yukawa}} = \lambda_u \varepsilon^{ijklm} \chi_{ij} \chi_{kl} \varphi_m^* + \lambda_d \psi^i \chi_{ij} \phi^j + \lambda_\nu \psi^i \eta \varphi_i^* + m_\eta \eta^2 + \frac{1}{M_p} \{ \gamma_u \varepsilon^{ijklm} \chi_{ij} \chi_{kl} \Sigma_m^n \varphi_n^* + \gamma_d \psi^i \chi_{ij} \Sigma_k^j \phi^k + \gamma_\eta \eta^2 \Sigma_j^i \Sigma_i^j + \gamma_\nu \psi^i \eta \Sigma_i^j \phi_j^* \}. \quad (1)$$

The first and fourth terms contribute to the up-quark mass and second and fifth terms contribute to the down-quark and electron masses. The rest of the terms contribute to the mass matrix of the neutral particles. We demand that the up and down quarks get masses through renormalizable interactions, when the ϕ-field acquire vev, i.e. λ_u and λ_d are non-zero and the non-renormalizable off-diagonal term contributes to the neutrino mass, i.e. $\gamma_\nu \neq 0$. This immediately implies that this theory can permit either or both of the following two U(1) symmetries:

$$\mathrm{U}(1)_A: \quad \psi^i \to e^{-\frac{3}{2} i\alpha} \psi^i, \quad \chi_{ij} \to e^{\frac{i}{2}\alpha} \chi_{ij}, \quad \phi^i \to e^{i\alpha} \phi^i, \quad \eta \to e^{\frac{5}{2} i\alpha} \eta$$

and

$$\mathrm{U}(1)_B: \quad \eta \to e^{i\beta} \eta, \quad \Sigma_j^i \to e^{-i\beta} \Sigma_j^i.$$

The $\mathrm{U}(1)_A$ symmetry implies m_η and γ_η to be zero, while $\mathrm{U}(1)_B$ implies vanishing of λ_ν, m_η, γ_u and γ_d. In particular, if we impose both these symmetries, then the up and down quarks and the electron will get masses from the renormalizable interactions only when ϕ^i acquires vev. The neutrino mass matrix will now be purely off-diagonal and will be given by [in the basis (ν, η)],

$$m_\nu = \begin{bmatrix} 0 & \frac{1}{M_p} \gamma_\nu \langle \Sigma \rangle \langle \phi \rangle \\ \frac{1}{M_p} \gamma_\nu \langle \Sigma \rangle \langle \phi \rangle & 0 \end{bmatrix}, \quad (2)$$

diagonalizing this matrix will result in a Dirac neutrino of mass $(1/M_p) \gamma_\nu \langle \Sigma \rangle \langle \varphi \rangle \sim 10^{-5} m_e$ for suitable choice of parameters, which can explain both the experiments on neutrino mass (Avignone *et al* 1983; Boris *et al* 1983).

It is to be noted that the generators of the $U(1)_A$ symmetry is related to the weak hypercharge (Y) and the $(B-L)$ quantum numbers of the particles by,

$$(B-L)=\tfrac{2}{5}(A+Y) \tag{3}$$

where the weak hypercharge is related to the electric charge by $Q=T_{3L}+\frac{1}{2}Y$. Thus when ϕ^i acquires vev the global $U(1)_A$ symmetry breaks along with $SU(2)_L$ and U(1), but the global $(B-L)$ symmetry survives. Thus there is no Goldstone boson corresponding to the broken global $U(1)_A$ symmetry. However, when Σ^i_j acquires vev $U(1)_B$ will be broken and there will be Goldstone bosons. This symmetry and its cosmological consequences have been considered by Roncadilli and Wyler (1983). They point out that the corresponding Goldstone boson couples to matter weakly and becomes invisible.

In the absence of non-renormalizable terms η decouples from the rest of the fields. So, if $m_\eta=0$ to start with, it will not be generated even if $U(1)_B$ is broken by $\langle\Sigma\rangle$. In the presence of non-renormalizable terms, radiative corrections can be evaluated by cutting off integrals at M_p beyond which effective theory is not valid. In such case m_η will be protected by $U(1)_A$, but $\lambda'_\nu\psi^i\eta\phi^*_i$ will be generated with $\lambda'_\nu\sim(1/M_p)\gamma_\nu\langle\Sigma\rangle\sim10^{-5}\gamma_\nu$. This is comparable to the tree level term and will not upset the smallness of the Dirac mass of the neutrino.

3. Dirac neutrino in supersymmetric SU(5)

We shall now demonstrate how the above model can be embedded in a $N=1$ supersymmetric theory. Since we are considering the dimension five terms (suppressed by Planck scale) which can be of gravitational origin a consistent model should be based on supergravity rather than on global supersymmetry. We consider the following model which is a slight modification of an earlier supergravity model (Joshipura *et al* 1985) based on geometrical hierarchy ideas (Witten 1981; Ovrut and Raby 1983a, b). The technicalities associated with the minimization of the scalar potential are identical to the ones presented in (Joshipura *et al* 1985). Hence we omit them and concentrate on the features which are new and deal with Dirac mass for the neutrino.

The Higgs superfields of the model are: A and Z transforming as adjoint representation of SU(5), H, H' ($\bar{H}$, $\bar{H}'$) transforming as $5(\bar{5})$ and singlet superfields X, Y, σ and σ'. The matter superfields consists of ψ ($\bar{5}$-plet), χ (10-plet) and a singlet η. The superpotential is given by,

$$W=W_1+W_2+W_3$$

$$W_1=\alpha_1X(\mathrm{tr}A^2-\sigma\sigma')+\alpha_2Y(\sigma\sigma'-\mu^2)+\alpha_3ZA^2+\beta\mu^2M$$

$$W_2=\lambda_1\bar{H}'(A+m)H+\lambda_2\bar{H}(A+m)H'$$

$$W_3=h_1\psi\chi\bar{H}+h_2\chi\chi H+\frac{h_3}{M_p}\psi\eta H\sigma.$$

The mass parameters m and μ are of the order of supersymmetry breaking scale ($\sim10^{10}$ GeV) and $M=M_p/\sqrt{8\pi}$. W_1 breaks supersymmetry through the

O'Raifeartaigh mechanism. The vevs of Y and Z become proportional to that of X which gets related to M, and β is fine tuned to make the cosmological constant vanish. W_2 is same as in Joshipura *et al* (1985), Ovrut and Raby (1983a, b).

The part W_3 predicts the quark masses and charged lepton mass in the usual way. The neutral particle η combines with the usual left handed neutrino through the higher order term to form a Dirac particle with mass $\sim(h_3\langle\sigma\rangle/M_p)\langle H\rangle$. This becomes ~ 10 eV when $\langle\sigma\rangle$ is of the order of supersymmetry breaking scale μ for suitable choice of h_3.

Now proceeding in a similar way as in (Joshipura *et al* 1985) it can be shown that the conditions of minimization of the scalar potential (which is completely determined by W if we restrict ourselves to the minimal coupling) and vanishing of the cosmological constant can be satisfied with,

$$\langle\sigma\rangle=\langle\sigma'\rangle=\frac{(30\alpha_1^2\alpha_2^2+\alpha_2^2\alpha_3^2)^{1/2}}{(30\alpha_1^2\alpha_2^2+\alpha_2^2\alpha_3^2+\alpha_3^2\alpha_1^2)^{1/2}}\mu$$

$$+O(\mu^2/M)\ \text{and higher order terms.}$$

The Yukawa part of the superpotential W_3 possesses two global U(1) symmetries,

$$\mathrm{U}(1)_A:\quad \psi\to e^{-\frac{3i}{2}\alpha}\psi \qquad \chi\to e^{\frac{i}{2}\alpha}\chi$$

$$\eta\to e^{\frac{5i}{2}\alpha}\eta \qquad H\to e^{-i\alpha}H$$

$$\bar{H}\to e^{i\alpha}\bar{H}$$

$$\mathrm{U}(1)_B:\quad \eta\to e^{i\beta}\eta \qquad \sigma\to e^{-i\beta}\sigma$$

$$\sigma'\to e^{i\beta}\sigma'.$$

The generator of $\mathrm{U}(1)_A$ is related to the weak hypercharge Y and $(B-L)$ through (3) and will not pose any problem when broken by vev of H, $\bar{H}$. $\mathrm{U}(1)_B$ is broken when σ and σ' acquires vev, but as before the corresponding Goldstone boson will be invisible and the model will be consistent with phenomenology as well as cosmology.

4. Conclusions

We have demonstrated yet another way of obtaining a small Dirac mass for the neutrino in grand unified theories. This seems to be the most economical among schemes that are so far proposed. The basic ingradient namely the presence of the non-renormalizable interaction becomes natural in supergravity theories. The model of § 3 could thus provide a realistic and economic scenerio for generating a small Dirac neutrino. It should be pointed out that the neutrino mass in the present scheme bears no relation to the masses of its family members as in other models e.g. (Roy and Shanker 1984).

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