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Fluctuations in $^{12}C+{}^{24}Mg$ elastic and inelastic scattering

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Abstract. The data on the $\theta_{c.m.}=180°$ excitation functions of $^{12}C+{}^{24}Mg$, $^{12}C^*$ (4·43 MeV) $+{}^{24}Mg$ and $^{12}C+{}^{24}Mg^*$(1·36 MeV) from 12·27 to 22·80 MeV, 16·53 to 27·47 MeV, and 11·33 to 26·40 MeV(c.m.) respectively have been subjected to statistical analysis. The effect of averaging interval, employed for data reduction, on the coherence widths as obtained from the autocorrelation function has been studied. The fluctuating features of the cross-sections turn out to be consistent with the statistical model expectations.

Keywords. Statistical analysis; probability distributions; cross-correlation-coefficients; coherence widths.

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1. Introduction

The structures observed in the 180° excitation functions of $^{12}C+{}^{28}Si$ and $^{16}O+{}^{28}Si$ elastic and inelastic scattering turned out to be of the statistical origin (Barrette *et al* 1979; Braun-Munzinger and Barrette 1982; Singh 1983). In the recent past an investigation of $^{12}C+{}^{24}Mg$ elastic and inelastic scattering in the energy range $12 < E_{c.m.} < 27$ MeV indicated the presence of intermediate structure that was speculated to be the manifestation of resonant behaviour of the scattering process (Mermaz *et al* 1981). According to this investigation an ^{36}Ar dinuclear system could be formed as a result of $^{12}C+{}^{24}Mg$ collision that gives rise to large values of the cross-channel correlation coefficients and large average value of the coherence width ($\Gamma=710$ keV). However, on the basis of a systematic study, Braun-Munzinger and Barrette (1982) noted that such a large value of coherence width was obtained, at least in part, by a large step size of the data and by inappropriately accounting for the presence of gross structure in the excitation functions. Very recently the $^{12}C+{}^{24}Mg$ system has been found to exhibit intermediate structure in back-angle cross-sections in the energy range between $E_{c.m.}=30$ and 38 MeV (Glaesner *et al* 1986). Of course, according to Glaesner *et al* (1986) the largest value of the width was obtained to be (460 ± 100) keV. On the other hand Mermaz *et al* (1982) earlier noted that a fluctuation type of behaviour of $^{12}C+{}^{24}Mg$ elastic and inelastic scattering at higher energies $(24\leqslant E_{c.m.}\leqslant 43$ MeV) could not be ruled out.

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In order to have a more clear idea of the $^{12}C + ^{24}Mg$ scattering behaviour we have carried out a statistical analysis of $^{12}C + ^{24}Mg$, $^{12}C^*(4\cdot43 \text{ MeV}) + ^{24}Mg$, and $^{12}C + ^{24}Mg^*(1\cdot36 \text{ MeV})$ excitation functions at $\theta_{c.m.} = 180°$ in the energy ranges $12\cdot27 \leqslant E_{c.m.} \leqslant 22\cdot80$, $16\cdot53 \leqslant E_{c.m.} \leqslant 27\cdot47$, and $11\cdot33 \leqslant E_{c.m.} \leqslant 26\cdot40$ MeV respectively. The analysis, which follows the approach of Ericson (1963), and Brink and Stephen (1963), consists of calculations of the percentage deviations of the fluctuating part of the cross-sections from unity, probability distributions and probability limits, cross-channel correlation coefficients and the coherence widths. The effect of the averaging interval, that is used to remove the energy-dependent gross structure from the excitation functions, on the coherence widths as obtained from the autocorrelation functions has also been examined.

2. Analysis

The data, obtained from Saclay (Mermaz *et al* 1981), consisted of $\theta_{c.m.} = 180°$ excitation functions for elastic scattering in the energy range $12\cdot27 \leqslant E_{c.m.} \leqslant 22\cdot80$ MeV, and for inelastic scattering ($^{12}C^*$ (4·43) + ^{24}Mg and $^{12}C + ^{24}Mg^*$ (1·36)) in the energy ranges $16\cdot53 \leqslant E_{c.m.} \leqslant 27\cdot47$ MeV and $11\cdot33 \leqslant E_{c.m.} \leqslant 26\cdot40$ MeV respectively, measured in steps of 133 keV (c.m.) by employing a ^{12}C target having a thickness of about 150 keV. The overall energy resolution was 350 keV.

In order to be able to compare the predictions of the statistical model with the behaviour of the experimental cross-sections it is necessary to remove the energy-dependent gross structure from the excitation functions. This was done, as usual (Pappalardo 1964), by dividing the cross-sections $d\sigma(E)$ at individual data points by the running average $\langle d\sigma(E)\rangle$ taken over an energy interval of $\Delta E_{c.m.} = 2\cdot13$ MeV (such that $\Gamma_{fine} \ll \Delta E_{c.m.} \ll \Gamma_{gross}$). The resulting data (reduced data) $X = d\sigma(E)/\langle d\sigma(E)\rangle$, was subjected to the analysis. The percentage deviations of the reduced data from unity for all the three excitation functions, shown in figure 1, indicate that there is hardly any gross structure left.

The distribution of the fluctuating cross-sections in the presence of the direct reactions is given by (Meyer-Kuckuk 1964)

$$P(X) = \left(\frac{N}{1-Y_d}\right)^N X^{N-1} \exp\left(-N\frac{X+Y_d}{1-Y_d}\right) \frac{I_{N-1}[2N(XY_d)^{1/2}/(1-Y_d)]}{[N(XY_d)^{1/2}/(1-Y_d)]^{N-1}}, \quad (1)$$

where as mentioned earlier $X = d\sigma(E)/\langle d\sigma(E)\rangle$, Y_d is the fraction of cross-section contributed by the direct reaction process, N the number of effective channels and I_{N-1} the modified Bessel function of order $N-1$. Since the excitation functions were measured at $\theta_{c.m.} = 180°$, N should be unity. We have estimated the value of Y_d from the relation

$$Y_d = (1 - N\,C(0))^{1/2}, \quad (2)$$

where $C(0)$, the normalized variance of the reduced data, is given by

$$C(0) = (\langle X^2\rangle/\langle X\rangle^2) - 1. \quad (3)$$

The experimental and theoretical (equation (1)) distributions of the cross-sections are given in figure 2. It can be seen that there is good agreement between the experimental

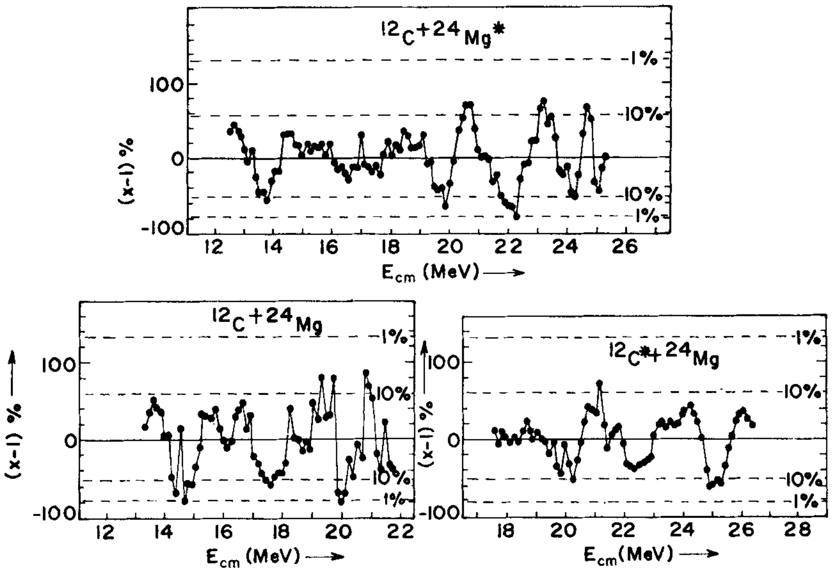

Figure 1. The percentage deviations from unity of the reduced data $(d\sigma(E)/\langle d\sigma(E)\rangle - 1)$ for $^{12}C + ^{24}Mg$, $^{12}C + ^{24}Mg^{*}(1{\cdot}36)$ and $^{12}C^{*}(4{\cdot}43) + ^{24}Mg$ excitation functions at $\theta_{c.m.} = 180°$. The dashed lines marked 1% and 10% indicate deviations from the average for which the probability of finding larger deviations are 1% and 10% respectively.

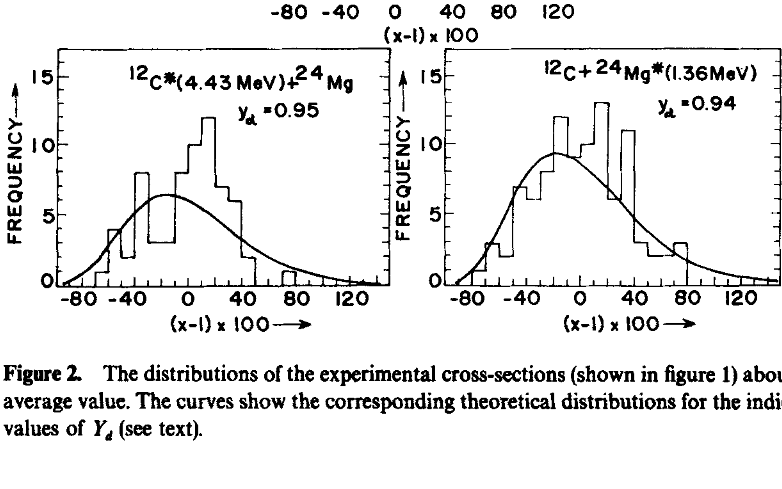

Figure 2. The distributions of the experimental cross-sections (shown in figure 1) about the average value. The curves show the corresponding theoretical distributions for the indicated values of Y_d (see text).

and theoretical distributions. Here, it may be mentioned that while comparing the experimental and theoretical distributions of cross-sections for inelastic scattering it should be kept in mind that since the data were taken by using the full solid angle of the QDDD spectrometer which had a horizontal aperture of $\Delta\theta = 6{\cdot}9°$, there can be contributions from $m \neq 0$ substates and, therefore, N may not be exactly equal to unity as we have assumed (see Singh 1983). In addition, in the case of $^{12}C^*(4{\cdot}43\ MeV) + {}^{24}Mg$ excitation function there can be contributions to the cross-sections from the inelastic excitation of ^{24}Mg to 4·12 MeV(4^+) and 4·23 MeV (2^+) states (Mermaz *et al* 1981). Therefore, the agreement between the experimental and theoretical distributions is not expected to be very good. It may be pointed out that similar values of Y_d were found in the analysis of $^{12}C + {}^{28}Si$ and $^{16}O + {}^{28}Si$ (at $\theta_{c.m.} = 180°$) elastic and inelastic scattering (Singh 1983) at somewhat higher energies. The probability of observing a cross-section fluctuation which is larger than a certain value X' is given by (Dayras *et al* 1976)

$$Q(X') = \int_{X'}^{\alpha} P(X)\,dX. \tag{4}$$

Looking at the 1% and 10% probability limits (Singh 1983; Singh *et al* 1980) in figure 1 one can figure out that nothing appears to be unlikely to be a statistical fluctuation.

The cross-channel correlation coefficients were calculated by using the formula (Hugi *et al* 1982).

$$C_{ij} = \langle (X_i - 1)(X_j - 1) \rangle / (C_i(0)C_j(0))^{1/2}, \tag{5}$$

which gives the value of the correlation coefficient between ith and jth channels. The bracket $\langle\ \rangle$ indicates the average taken over the entire excitation function. The values of the C_{ij}'s are given in table 1. The indicated errors include the contributions due to finite range of data as well as from the errors in the cross-sections. The latter ranged

Table 1. The coherence widths as obtained from the autocorrelation functions by using different values of the averaging intervals ($\Delta E_{c.m.}$) to account for the gross structure in the excitation functions. Lower portion of the table gives the values of the cross-channel correlation coefficients between the indicated excitation functions (see text).

	Γ(keV)		
$\Delta E_{c.m.}$ (MeV)	$^{12}C + {}^{24}Mg$	$^{12}C^* + {}^{24}Mg$	$^{12}C + {}^{24}Mg^*$
1·60	182 ± 77	269 ± 113	211 ± 76
2·13	270 ± 115	264 ± 110	323 ± 117
3·19	270 ± 115	431 ± 180	370 ± 134
4·26	378 ± 161	431 ± 180	396 ± 143
5·06	324 ± 138	377 ± 158	423 ± 153

Cross-channel correlation coefficients

g.s. − $^{12}C^* + {}^{24}Mg$	g.s. − $^{12}C + {}^{24}Mg^*$	($^{12}C^* + {}^{24}Mg$) − ($^{12}C + {}^{24}Mg^*$)
0·07 ± 0·28	0·25 ± 0·23	0·20 ± 0·19

between 5 and 15%. It is clear that large uncertainties in the correlation coefficients render them meaningless.

The empirical estimates of the coherence widths were made by employing the formula (Ericson and Meyer-Kuckuk 1966; Stokstad 1974)

$$\Gamma = 14 \exp(-4{\cdot}69\sqrt{A/E_x})\ \text{MeV}, \tag{6}$$

where A is the compound nuclear mass and E_x is its excitation energy in MeV. For elastic channel the value of Γ varied from 72 to 155 keV, for $^{12}C^*(4{\cdot}43) + ^{24}Mg$ from 103 to 199 keV and for $^{12}C + ^{24}Mg^*(1{\cdot}36)$, it varied from 66 to 188 keV. The peak counting method (Brink and Stephen 1963; Singh *et al* 1980) gave an average value (appropriately corrected for target thickness and finite energy resolution (Van der Woude 1966)) of $\Gamma = 275 \pm 50$ keV.

By autocorrelation analysis, Mermaz *et al* (1981, 1982) obtained average widths of 710 and 300 keV at the excitation energies of 35·8 and 50 MeV in ^{36}Ar respectively. The presence of the gross structure was accounted for by using the averaging intervals of 4·13 and 5·36 MeV respectively in these analyses (Mermaz *et al* 1981, 1982). We have investigated the effect of the size of averaging interval on the value of Γ as obtained by

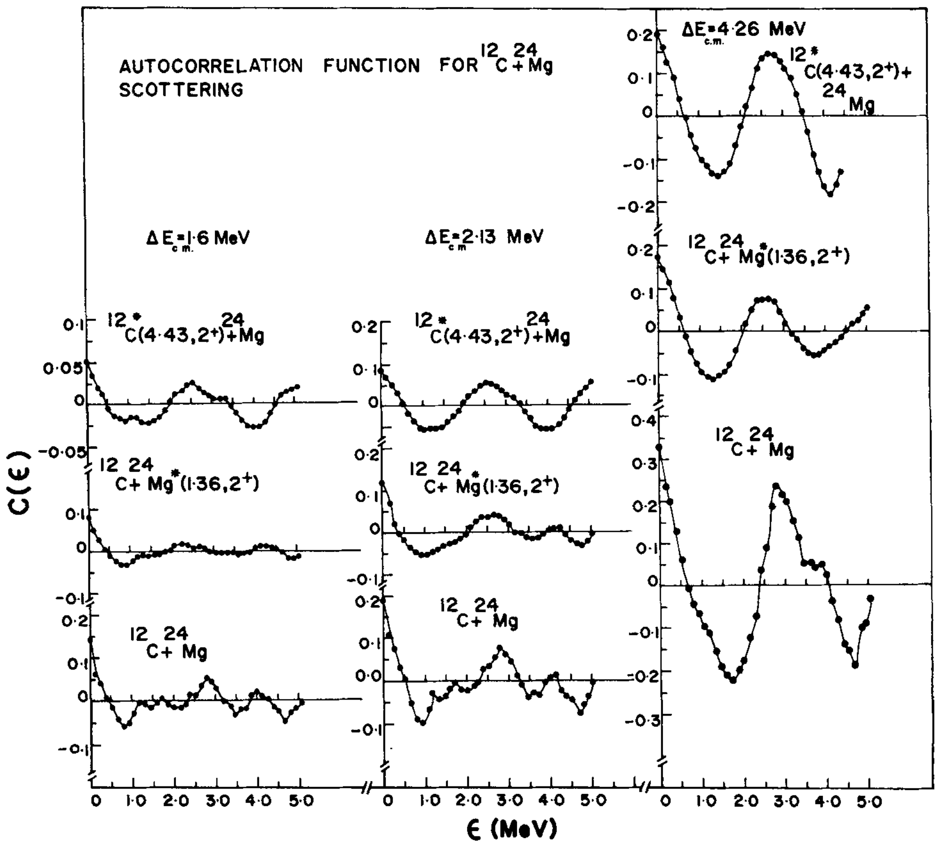

Figure 3. Autocorrelation functions for the three excitation functions for the given different values of the averaging intervals ($\Delta E_{c.m.}$) employed to account for the presence of gross structure.

autocorrelation function technique (Ericson and Meyer-Kuckuk 1966)

$$C(\varepsilon)=\langle X(E)\cdot X(E+\varepsilon)\rangle/\langle X(E)\rangle\langle X(E+\varepsilon)\rangle-1$$
$$=C(0)/(1+\varepsilon^2/\Gamma^2). \tag{7}$$

We have used five different averaging intervals from 1·6 to 5·06 MeV. Some typical plots are shown in figure 3 and all the values of Γ, appropriately corrected for the finite range of data (Halbert *et al* 1967) are listed in table 1. The errors are due to the finite range of data (Richter 1974). The values of Γ obtained this way (for $\Delta E_{\text{c.m.}}=2{\cdot}13$ MeV) seem to be somewhat higher but are not in disagreement with the ones reported in literature (Shapira *et al* 1974; Eberhard and Richter 1972; Hugi *et al* 1982; Braun-Munzinger and Barrette 1982; Mermaz *et al* 1982) and accepted to be within the statistical model picture. Table 1 shows that the values of Γ tend to increase as the averaging interval increases (at least upto 4·26 MeV). Noting the gross structure widths in the excitation functions the averaging intervals used earlier (Mermaz *et al* 1981, 1982) were a bit too large.

3. Conclusion

The experimental and theoretical distributions of the cross-sections agree well with each other, and the 1% and 10% probability limits lie higher than most of the fluctuating cross-sections. It is therefore clear that the structures in the excitation functions are compatible with the statistical model picture. The coherence widths do not really point to the presence of any intermediate structure. Large uncertainties in the values of the cross-channel correlation coefficients reduce them to insignificance. Thus according to the present analysis the observed structures can be understood within the framework of the statistical model.

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