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Confinement models for gluons

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Abstract. Confinement model for gluons using a 'colour super current' is formulated. An attempt has been made to derive a suitable dielectric function corresponding to the current confinement. A simple inhomogeneous dielectric confinement model for gluons is studied for comparison. The model Hamiltonians are second quantized and the glueball states are constructed. The spurious motion of the centre of confinement is accounted for. The results of the current confinement scheme is found in good agreement with experimental candidates.

Keywords. Confinement model; gluons; dielectric function; glueball states.

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1. Introduction

It has been well accepted that quantum chromodynamics is a prime candidate for the theory of strong interactions. This is a nonabelian gauge theory governing the colour dynamics of quarks and gluons. The gluons which are the quanta of the colour field carry colour charges and they interact among themselves. Evidence for such interactions is the very existence of glueballs (the colour singlet bound states of multigluons). Thus the study of glueballs and its experimental confirmation is very crucial to the validity of quantum chromodynamics. Since these coloured gluons are many in number (SU3-octet) the coupled equations obeyed by them are too complex to solve simultaneously. However an important desirable feature of any theory describing colour dynamics should be the confinement of colour. The only indication for the confinement is from the lattice simulations apart from its experimental confirmations. Hence to understand the physical reality of its microscopic structure, one has to go for phenomenological models (De Rujula 1975; Isgur and Carl 1977).

Here we consider all the eight gluons described by the Yang-Mills field tensor to be of equal strength and the coupled nonlinear term in the field tensor corresponds to a source. It can be seen that this source in general is a function of the field potentials and its derivatives. This source is treated as a colour supercurrent in analogy with Ginzburg-Landaus theory of superconductivity. In this picture we consider the gluon field as quasi Maxwellian fields. The details are given in §2. As a special case a simple confinement scheme for gluons similar to that of relativistic harmonic (vector + scalar) potential model (RHM) for quarks (Khadkikar and Gupta 1983) is discussed in detail in §3. The confined quasigluon modes are obtained in the general frame of Lorentz gauge with a subsidiary condition called the oscillator gauge condition

(Khadkikar 1985). These confined modes are quantized and the energies are calculated. In §4 we obtain a dielectric function corresponding to the source current so that the scheme corresponds in line with the dielectric confinement model proposed by Lee (1979). The dielectric function that we have obtained is in general non-local and inhomogeneous. As the dynamical dependence is neglected the function reduces to that of a simple inhomogeneous dielectric medium. The confinement model for the quasigluons in such a dielectric medium is discussed in §5.

The phenomenology of glueballs is described in §6 and the di-gluon and tri-gluon colour singlet states are constructed which account for the spurious motion of the centre. Finally the parameter is fixed by fitting iota(0^{-+}) 1440 MeV as a di-gluon glueball candidate and all other low-lying di-gluon and tri-gluon states are predicted. Finally we discuss our results in §7 with the present status of the experimental results and compare them with the naive bag model results for glueballs.

2. Gluons as quasi-Maxwellian field potentials

For a pure colour gluon field the Lagrangian density is given by the Yang-Mills field tensor,

$$\mathscr{L} = -1/4 F^l_{\mu\nu} F^l_{\mu\nu}, \tag{1}$$

where
$$F^l_{\mu\nu} = f^l_{\mu\nu} + G^l_{\mu\nu}, \tag{2}$$

with
$$f^l_{\mu\nu} = \partial_\mu A^l_\nu - \partial_\nu A^l_\mu, \tag{3}$$

and
$$G^l_{\mu\nu} = g f^{lmn} A^m_\mu A^n_\nu, \tag{4}$$

the colour indices (l, m, n) carry 1, 2, ..., 8 and (μ, ν) are the four-vector indices. By variational principle the equation of motion for the field is obtained as

$$\partial_\mu F^l_{\mu\nu} + g f^{lmn} A^m_\mu F^n_{\mu\nu} = 0. \tag{5}$$

Substituting for $F^n_{\mu\nu}$ from (2) to (4) we get

$$\begin{aligned}\partial_\mu f^l_{\mu\nu} = &-g f^{lmn}[A^m_\mu \partial_\mu A^n_\nu + \partial_\mu A^m_\mu A^n_\nu + A^m_\mu \partial_\mu A^n_\nu - A^m_\mu \partial_\nu A^n_\mu \\ &+ g f^{nl'm'} A^m_\mu A^{l'}_\mu A^{m'}_\nu],\end{aligned} \tag{6}$$

g is the coupling constant; f^{lmn} is the SU(3) structure constant. The right side of the equation can be considered as a supercurrent of the colour gluons in analogy with the Ginzburg Landaus theory of superconductivity. In an external gluon field the low momentum approximation, say, A^l_μ corresponding to the confinement, this colour supercurrent is assumed as

$$J^l_\nu = \theta_{\mu\nu} A^l_\mu \tag{7}$$

similar to that of the London equation in superconductivity. With this picture we treat the gluon field A^l_μ as a quasi-Maxwellian field potentials satisfying the dynamical equation

$$\partial_\mu f^l_{\mu\nu} = -J^l_\nu. \tag{8}$$

3. Current confinement model for gluons (CCM)

In this section we consider the gluon fields in a quasi-Maxwellian theory with a confinement current assumed to be similar to that in (7)

$$J_\mu = \theta_{\mu\nu} A_\nu \tag{9}$$

where $A_\nu = (\mathbf{A}, \Phi)$.

For simplicity and in analogy with the relativistic harmonic confinement model (RHM) for quarks (Khadkikar and Gupta 1983) we choose

$$\theta_{\mu\nu} = -\delta_{\mu\nu}\theta_\mu \tag{10}$$

and $$\theta_\mu = 2\alpha\delta_{\mu 0} - \alpha^2 r^2. \tag{11}$$

The Lagrangian density for this quasi-Maxwellian gluons can now be written as

$$\mathscr{L} = -\tfrac{1}{4} f_{\mu\nu} f_{\mu\nu} + \tfrac{1}{2}\theta_{\mu\nu} A_\mu A_\nu + \tfrac{1}{2}(\partial_\mu A_\mu)^2, \tag{12}$$

where $$f_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{13}$$

is the quasi-colour gluon field tensor. Here the colour labels are omitted since all the fields carry the same colour index.

From the variational principle,

$$\delta \int \mathscr{L}\, \mathrm{d}^3 r\, \mathrm{d}t = 0, \tag{14}$$

we find the field equation for the vector potential $\mathbf{A}$ as

$$-\nabla^2 \mathbf{A} + \theta \mathbf{A} + \ddot{\mathbf{A}} = 0 \tag{15}$$

with the Lorentz gauge condition

$$\partial_\mu \mathrm{A}_\mu = \nabla \cdot \mathbf{A} + \dot{\Phi} = 0 \tag{16}$$

where the number of dots above the variable represents the order of time derivatives.

To carry out the quantization in the Lorentz gauge we observe that Φ is a dependent variable. The conjugate momentum of $\mathbf{A}$ is

$$\mathbf{P} = \dot{\mathbf{A}} + \nabla\Phi \tag{17}$$

and that of Φ is

$$\pi = \nabla \cdot \mathbf{A} + \dot{\Phi} = 0 \tag{18}$$

as given by (16). Consequently all the time derivatives of (18) also vanish, i.e.

$$\dot{\pi} = \frac{\partial \mathscr{L}}{\partial \Phi} = -\nabla \cdot (\dot{\mathbf{A}} + \nabla\Phi) + \theta' \Phi = 0. \tag{19}$$

Thus from (17) and (19), we have

$$\Phi = (\nabla \cdot \mathbf{P})/\theta', \tag{20}$$

where θ and θ' are the vector and scalar components of θ_μ associated with $\mathbf{A}$ and Φ of A_μ respectively. Eliminating Φ from (16) and (17),

$$\nabla \cdot \mathbf{A} + (\nabla \cdot \mathbf{P})/\theta' = 0 \tag{21}$$

and
$$\mathbf{P} = \dot{\mathbf{A}} + \nabla(\nabla \cdot \mathbf{P})/\theta'. \tag{22}$$

In order to derive the Hamiltonian, we regard $\mathbf{A}$ and $\mathbf{P}$ as independent variables but Φ as a function of $\mathbf{P}$ through (20). Thus the Hamiltonian

$$H = \tfrac{1}{2}\int d^3r\,[\mathbf{P}^2 + (\nabla \cdot \mathbf{P})^2/\theta' + \theta \mathbf{A}^2 + (\nabla \cdot \mathbf{A})^2 - \mathbf{A} \cdot \nabla^2 \mathbf{A}]. \tag{23}$$

The Hamilton's equations of motion for $\mathbf{A}$ and $\mathbf{P}$:

$$\dot{\mathbf{A}} = \partial H/\partial \mathbf{P} = \mathbf{P} - \nabla(\nabla \cdot \mathbf{P})/\theta', \tag{24}$$

$$\dot{\mathbf{P}} = -\partial H/\partial \mathbf{A} = \nabla^2 \mathbf{A} - \theta \mathbf{A} - \nabla(\nabla \cdot \mathbf{A}). \tag{25}$$

Taking the second time derivatives of $\mathbf{A}$ and $\mathbf{P}$, we get

$$\ddot{\mathbf{A}} = \nabla^2 \mathbf{A} - \theta \mathbf{A} + \nabla[\nabla \cdot \theta \mathbf{A} - \theta' \nabla \cdot \mathbf{A}]/\theta', \tag{26}$$

$$\ddot{\mathbf{P}} = \nabla^2 \mathbf{P} - \theta \mathbf{P} + [\theta \nabla - \nabla \theta'](\nabla \cdot \mathbf{P})/\theta'. \tag{27}$$

Using (25) the Lorentz condition given by (21) leads to

$$\nabla \cdot \theta \mathbf{A} - \theta' \nabla \cdot \mathbf{A} = 0. \tag{28}$$

This equation with the choice of θ and θ' given in (10) and (11) leads to the oscillator gauge condition (Khadkikar 1985)

$$[\nabla + \alpha \mathbf{r}] \cdot \mathbf{A} = 0. \tag{29}$$

It has been seen that the conservation requirement of the induced four-vector current also demands the oscillator condition. In terms of oscillator operators defined by

$$\mathbf{a} = (2\alpha)^{-1/2}(\nabla + \alpha \mathbf{r}) \tag{30}$$

and its Hermitian conjugate

$$\mathbf{a}^\dagger = (2\alpha)^{-1/2}(-\nabla + \alpha \mathbf{r}) \tag{31}$$

equation (29) now reduces to

$$\mathbf{a} \cdot \mathbf{A} = 0. \tag{32}$$

Similarly the equation for $\mathbf{A}$ taking the time variations of the field as $\exp(-i\omega t)$, becomes

$$(\mathbf{a} \cdot \mathbf{a}^\dagger + \mathbf{a}^\dagger \cdot \mathbf{a})\,\mathbf{A} = \omega^2 \mathbf{A}. \tag{33}$$

The scalar oscillator wave function corresponding to this equation is given by

$$\Psi_{n'lm}=N_{n'l}[\mathbf{a}^{\dagger}\cdot\mathbf{a}^{\dagger}]^{n'}\,Y_{lm}(\mathbf{a}^{\dagger})\Psi_0, \tag{34}$$

where

$$\Psi_0=[\alpha^{1/2}\pi^{-1/2}]^{\frac{3}{2}}\exp(-\tfrac{1}{2}\alpha^2r^2) \tag{35}$$

and

$$N_{n'l}=[4\pi/2n'!!\,(2n'+2l+1)!!]^{1/2} \tag{36}$$

with oscillator eigenvalues

$$E_n=\omega_n^2=(2n+3)\alpha;\quad n=2n'+1, \tag{37}$$

where $Y_{lm}(\mathbf{a}^{\dagger})$ is the solid spherical harmonics. Hence the solution for $\mathbf{A}$ can be written as

$$\mathbf{A}=\mathbf{e}\Psi_{nlm}, \tag{38}$$

$\mathbf{e}$ represents the direction of polarization of the field $\mathbf{A}$. In the oscillator space, we choose

$$\mathbf{e}^1\propto i\mathbf{a}\times\mathbf{a}^{\dagger};\quad \mathbf{e}^2\propto\mathbf{a}\times(\mathbf{a}\times\mathbf{a}^{\dagger});\quad \mathbf{e}^3\propto\mathbf{a}^{\dagger}. \tag{39}$$

Then the oscillator gauge condition classically gives the third component of $\mathbf{A}$

$$\mathbf{A}_3=0. \tag{40}$$

Then the remaining transverse components of the fields are given by

$$\mathbf{A}^1_{nJm}=N^1_{nJ}i(\mathbf{a}\times\mathbf{a}^{\dagger})\Psi_{nJm}\exp(-i\omega_nt), \tag{41}$$

$$\mathbf{A}^2_{nJm}=N^2_{nJ}[\mathbf{a}-\mathbf{a}^{\dagger}(1/\mathbf{a}\cdot\mathbf{a}^{\dagger})\mathbf{a}\cdot\mathbf{a}]\Psi_{n+1Jm}\exp(i\omega_nt) \tag{42}$$

and the corresponding conjugate momenta

$$\mathbf{P}^1_{nJm}=-M^1_{nJ}(\mathbf{a}\times\mathbf{a}^{\dagger})\Psi_{nJm}\exp(i\omega_nt), \tag{43}$$

$$\mathbf{P}^2_{nJm}=M^2_{nJ}i[\mathbf{a}-\mathbf{a}^{\dagger}(1/\mathbf{a}\cdot\mathbf{a}^{\dagger})\mathbf{a}\cdot\mathbf{a}]\Psi_{n+1Jm}\exp(i\omega_nt), \tag{44}$$

and

$$\mathbf{P}^3=\nabla_3[(1/(\theta'-\nabla^2))\nabla\cdot\dot{\mathbf{A}}^2], \tag{45}$$

where the normalization constants are

$$\begin{aligned}
N^1_{nJ}&=[J(J+1)2\alpha^{1/2}(2n+3)^{1/2}]^{-1/2},\\
N^2_{nJ}&=[J(J+1)/(n+1)\cdot2\alpha^{1/2}(2n+3)^{1/2}\\
&\qquad(1-J(J+1)/\{2(2n+3)(n+1)\})]^{-1/2},\\
M^1_{nJ}&=[\alpha^{1/2}(2n+3)^{1/2}/2J(J+1)]^{1/2},\\
M^2_{nJ}&=[\alpha^{1/2}(2n+3)^{1/2}(n+3)/\{2J(J+1)\}\\
&\qquad(1-J(J+1)/\{(2n+3)(n+1)\})]^{1/2},
\end{aligned} \tag{46}$$

where $\mathbf{J}=\mathbf{L}+\mathbf{S}$ is the total angular momentum of the fields, $\mathbf{L}$ is the angular momentum and $\mathbf{S}$ is the spin operator.

For quantization we expand the quasi-gluon fields in terms of the above eigen basis to get the gluon energy in terms of the frequency ω_n

$$\mathbf{A}=\sum_{nJM\lambda}[C_{nJM\lambda}A^{\lambda}_{nJM}+C^{\dagger}_{nJM\lambda}A^{\lambda *}_{nJM}]. \tag{47}$$

where λ refers to the type of the mode (magnetic/electric). $C_{nJM\lambda}$ and $C^{\dagger}_{nJM\lambda}$ are the annihilation and creation operators satisfying the commutation relations

$$[C_{nJM\lambda}, C^{\dagger}_{n'J'M'\lambda'}]=\delta_{nn'}\,\delta_{JJ'}\,\delta_{MM'}\,\delta_{\lambda\lambda'}$$

and

$$[C_{nJM\lambda}, C_{n'J'M'\lambda'}]=[C^{\dagger}_{nJM\lambda}, C^{\dagger}_{n'J'M'\lambda'}]=0. \tag{48}$$

The condition on $\mathbf{A}_3$ here is now replaced by

$$\mathbf{A}_3|\text{physical}\rangle=0 \tag{49}$$

and that on $\mathbf{P}^3$:

$$[\mathbf{P}^3-\nabla_3(1/(\theta'-\nabla^2)\nabla\cdot\dot{A}^2]\,|\text{physical}\rangle=0. \tag{50}$$

However the Hamiltonian is independent of $\mathbf{P}^3$. Finally the Hamiltonian from (23) becomes

$$H=\sum_{nJM\lambda}\omega_n(C^{\dagger}_{nJM\lambda}C_{nJM\lambda}+\tfrac{3}{2}). \tag{51}$$

From (37) the frequency

$$\omega_n=(2n+3)^{1/2}\alpha^{1/2}, \tag{52}$$

where $C^{\dagger}_{nJM}\,C_{nJM}$ is the number operators in the quantum (nJM) of the modes $\lambda(=1,2)$. The low-lying digluon and trigluon glueball states are calculated in §6.

4. Expression for a dielectric function

Classically Maxwell's displacement current $\mathbf{D}$ can be written as

$$\mathbf{D}=\mathbf{E}+\mathbf{P}, \tag{53}$$

where $\mathbf{E}$ is the electric field causing a polarization and $\mathbf{P}$ is the polarization vector current. For a source-free case

$$\nabla\cdot\mathbf{D}=0 \tag{54}$$

and

$$\nabla\times\mathbf{B}=\partial\mathbf{D}/\partial t. \tag{55}$$

Thus we get the induced polarization charge

$$\rho=-\nabla\cdot\mathbf{P} \tag{56}$$

and the corresponding induced polarization current density

$$\mathbf{J} = \partial \mathbf{P}/\partial t. \tag{57}$$

Now considering the current $\mathbf{J}$ defined in §§2 and 3 as a polarization current in an external gluon field A_μ then

$$J_\mu = (\partial \mathbf{P}/\partial t;\ -\nabla \cdot \mathbf{P}), \tag{58}$$

$$\mathbf{P} = -\mathbf{J}/i\omega, \tag{59}$$

from (9) to (12)

$$\mathbf{P} = -\theta A/i\omega \tag{60}$$

using the expression for $\mathbf{E}$

$$\mathbf{E} = i\omega \mathbf{A} - \nabla\Phi,$$

and

$$\nabla \cdot \mathbf{E} = i\omega\, \nabla \cdot \mathbf{A} - \nabla^2\Phi. \tag{61}$$

Using the Lorentz condition, we eliminate $\mathbf{A}$ from (61);

$$\nabla \cdot \mathbf{E} = -\omega^2\Phi - \nabla^2\Phi. \tag{62}$$

Hence,

$$\Phi = -[1/(\omega^2 + \nabla^2)]\nabla \cdot \mathbf{E}. \tag{63}$$

Then the equation for $\mathbf{P}$ reduces to

$$\mathbf{P} = (\theta/\omega^2)\,[\mathbf{E} + \nabla\Phi]. \tag{64}$$

Substituting Φ from (63)

$$\mathbf{P} = (\theta/\omega^2)[\mathbf{E} - \nabla 1/(\omega^2 + \nabla^2)\nabla \cdot \mathbf{E}] \tag{65}$$

$$= (\theta/\omega^2)[1 - \nabla 1/(\omega^2 + \nabla^2)\nabla \cdot \mathbf{E}]. \tag{66}$$

Thus from (53)

$$\mathbf{D} = \varepsilon(\theta, \nabla)\mathbf{E}, \tag{67}$$

where the dielectric function is now a non-local operator given by

$$\varepsilon(\theta, \nabla) = 1 + \theta/\omega^2 \cdot [1 - \nabla\{1/(\omega^2 + \nabla^2)\}\nabla]. \tag{68}$$

Thus to get an identical confinement scheme as the CCM one has to use the dielectric function

$$\varepsilon(r, \nabla) = 1 - \alpha^2 r^2/\omega^2 \cdot [1 - \nabla\{1/(\omega^2 + \nabla^2)\}\nabla]. \tag{69}$$

5. Dielectric confinement model for gluons (DCM)

Here we consider the dielectric function obtained in equation (69) of §4 as a confinement medium for gluons, as proposed by Lee (1979). Neglecting the non-locality this function reduces to a simple inhomogeneous function,

$$\varepsilon(r) = 1 - a^2 r^2, \tag{70}$$

where a is a non-zero constant parameter and r is the spatial coordinate. For any value of $r \neq 0$, $\varepsilon(r) < 1$ gives the antiscreening property as required by QCD vacuum (Lee 1979); however, when $r \to 0$, $\varepsilon(r) \to 1$ corresponds to the asymptotic freedom. And as $r \to 1/a$, $\varepsilon(r) \to 0$ corresponds to the perfect dielectric nature of the medium; therefore the colour electric field is pushed inside the region leading to confinement. The parameter a can be chosen in this limit. In the case of bag models $\varepsilon(r)$ is taken as a step function with $\varepsilon(r) = 1$ inside the bag and $\varepsilon(r) = 0$ outside the bag surface. Here we have a smooth radially varying function avoiding such sharp transition.

In this quasi-classical macroscopic picture of the medium, we have to solve Maxwell's type of equation to get the quasi-gluon fields, characterized by the quasi-gluon electric field $\mathbf{E}$ and the magnetic field $\mathbf{B}$;

$$\begin{aligned} \nabla \times \mathbf{E} &= -\partial \mathbf{B}/\partial t, \\ \nabla \cdot \mathbf{B} &= 0, \\ \nabla \times \mathbf{B} &= \partial D/\partial t, \\ \nabla \cdot \mathbf{D} &= 0, \end{aligned} \tag{71}$$

where $$\mathbf{D} = \varepsilon(r)\mathbf{E}. \tag{72}$$

These field strengths can be expressed in terms of the quasi-gluon potential (A, Φ) as

$$\begin{aligned} \mathbf{E} &= -\partial A/\partial t - \nabla\Phi, \\ \mathbf{B} &= \nabla \times \mathbf{A}. \end{aligned} \tag{73}$$

Assuming the time variation of the field as $\exp(-i\omega t)$, we obtain the stationary wave equation for $\mathbf{A}$:

$$\nabla^2 \mathbf{A} + \omega^2 \varepsilon(r)\mathbf{A} = \nabla(\nabla \cdot \mathbf{A}) - i\omega\, \varepsilon(r) \nabla\Phi. \tag{74}$$

Because of the spatial dependence of the $\varepsilon(r)$, equation (74) for $\mathbf{A}$ cannot be made homogeneous as in the case of bag model cavity eigenmodes for $\mathbf{A}$ (Kuti 1977). To obtain a homogeneous equation for simple eigenmodes we require a gauge condition such that.

$$\nabla(\nabla \cdot \mathbf{A}) - i\omega\varepsilon(r)\nabla\Phi = 0. \tag{75}$$

Obviously, the Lorentz condition cannot provide this requirement for a general Φ. The above condition leads to the Lorentz condition as $r \to 0$ (asymptotic regions). As these quasi-gluons move away from the asymptotic region to confinement region where $r \to 1/a$,

$$\nabla \cdot \mathbf{A} = 0. \tag{76}$$

Then in this region the two transverse eigenmodes can be defined as

$$\mathbf{A}^{\mathrm{TE}} = \mathbf{L}\psi_{nlm} \tag{77}$$

and

$$\mathbf{A}^{\mathrm{TM}} = \nabla \times \mathbf{L}\, \Psi_{nlm}, \tag{78}$$

where TE and TM represent the transverse electric and magnetic modes. **L** is the angular momentum operator. Ψ_{nlm} is the solution satisfying the homogeneous scalar wave equation given by

$$\Psi_{nlm} = N_{nl}(\alpha_n r)^l \exp(-\tfrac{1}{2}\alpha_n^2 r^2) L^{l+\frac{1}{2}}_{(n-l)/2}(\alpha_n^2 r^2)\, Y_{lm}(\theta, \phi). \tag{79}$$

The size parameter α_n is

$$\alpha_n = (a\omega_n)^{1/2}, \tag{80}$$

where

$$\omega_n = (2n+3)a. \tag{81}$$

$L_n^l(Z)$ is the associated Laguerre polynomial. The normalization factor is

$$N_{nl} = [2\alpha_n^3 (n-1)/2)!/4\pi\, \gamma(n+3+1)/2]^{1/2}. \tag{82}$$

In terms of the vector spherical harmonics

$$A^{\mathrm{TM}}_{nJM} = (2\omega_n \varepsilon(r))^{-1/2}[(J/(2J+1))^{1/2} R_{nJ+1}(r)\, Y_{JJ+1M}(\hat{n}) + ((J+1)/(2J+1))^{1/2} R_{nJ-1}(r)\, Y_{JJ-1M}(\hat{n})] \exp(-i\omega_n t) \tag{83}$$

with parity

$$P = (-1)^J \tag{84}$$

and

$$A^{\mathrm{TE}}_{nJM} = (2\omega_n \varepsilon(r))^{-1/2} R_{nJ}(r)\, Y_{JJM}(\hat{n}) \exp(-i\omega_n t) \tag{85}$$

with parity

$$P = (-1)^{J+1} \tag{86}$$

and the vector spherical harmonic satisfies

$$\int Y^*_{lJM}\, Y_{lJm}\, \mathrm{d}\Omega = \delta_{ll'}\delta_{JJ'}\delta_{MM'}, \tag{87}$$

where $\mathbf{J} = \mathbf{L} + \mathbf{S}$ (88)

and $J^2 Y^{(\hat{n})}_{lJM} = J(J+1)\, Y_{lJM}(\hat{n});\quad J_z\, Y_{lJM}(\hat{n}) = M\, Y_{lJM}(\hat{n}).$ (89)

The Hamiltonian for this quasi-gluon field is given by

$$H = \tfrac{1}{2}\int \mathrm{d}^3 r[\mathbf{E}\cdot\mathbf{D} + \mathbf{B}^2]. \tag{90}$$

This Hamiltonian is second quantized as in §3 using the eigenmodes defined in (83) and (85) to get the energy of the confined quasi-gluons in terms of its frequency.

6. Construction of the glueball states

The fact that no coloured objects are seen free in nature allows only colour singlet states to exist with finite energies. In addition to $q\bar{q}$, qqq states one expect the colour singlet states of colour gluons also to exist (Jaffe and Johnson 1976). Such states are referred to as gluonium or glueballs. Strong experimental evidences for such particles exist (Sharre *et al* 1980; Lindenbaum *et al* 1985; Konigsmann 1986). Construction of digluon and trigluon colour singlet low-lying glueball states in the case of CCM and DCM is discussed here.

The lowest gluon modes we obtained are $J^{PC}=1^{--}$ for the transverse magnetic (TM/E) and $J^{PC}=1^{+-}$ for the transverse electric (TE/M). J is the total angular momentum of the gluon state, P is the parity and C represents the colour charge conjugation. Since glueballs are bosonic hadrons the total wave function including colour should be symmetric. The colour part is governed by the combinations of Gell-Mann's λ-matrices as

$$\begin{aligned}[\lambda_l, \lambda_m] &= 2if_{lmn}\lambda_n,\\ \{\lambda_l, \lambda_m\} &= \tfrac{4}{3}\delta_{lm}+2d_{lmn}\lambda_n,\end{aligned} \tag{91}$$

where (lmn) are the colour indices, f_{lmn} are completely antisymmetric in its indices while d_{lmn} are completely symmetric. For digluon states the colour coupling is of the form δ_{lm} giving $C=+1$, whereas in the case of trigluon states the colour symmetric coupling of the type d_{lmn} gives $C=-1$ and that of the colour anti-symmetric coupling of the type f_{lmn} gives $C=+1$. The colour singlet glueball states with orbital, spin and colour symmetries can have the following combinations:

Orbital	*Spin*	*Colour*
S	AS	AS
S	S	S
AS	AS	S
AS	S	S
(MS/MAS	MS/MAS)$_S$	AS
(MAS/MS	MS/MAS)$_{AS}$	AS

where S and AS refer to symmetric and antisymmetric respectively while MS and MAS refer to mixed symmetric and mixed antisymmetric respectively.

Accordingly low-lying states of the digluon systems are obtained as $E^2(0^{++}, 2^{++})$, $M^2(0^{++}, 2^{++})$ and $EM(0^{-+}, 2^{-+})$. And the low-lying states of the trigluon systems with colour coupling of the type d_{lmn} are $1^{+-(--)}\,3^{+-(--)}$ and the colour coupling of the type f_{lmn} are $0^{++(-+)}$ for $M^3\,(E^3)$ combinations. The wave functions corresponding to these states are given by

$$\begin{aligned}\Psi_{J=3,1} &= d_{lmn}\chi_{\overline{123}}^{J=1,3}\Phi_{\overline{123}},\\ \Psi_{J=0} &= f_{lmn}\,\chi_{\underline{123}}^{J=0}\,\Phi_{\underline{123}},\end{aligned} \tag{92}$$

where χ_{123} and Φ_{123} are the spin and orbital wave functions respectively. The bar over the indices represents symmetric and below the indices represents the antisymmetric combinations between the particle indices. A detailed account of the M^3 and M^2E glueball states is given by Senba and Tanimoto (1984). Similarly we have obtained the low-lying ME^2 glueball states as $(0^{++}, 2^{++}, 1^{+-}, 3^{+-})$ and M^2E glueball states as $(0^{-+}, 2^{-+}, 1^{--}, 3^{--})$. The calculations of hyperfine splitting of these state are beyond the scope of this paper.

The spurious motion of the centre of the glueball containing A-gluons should also be taken into account while constructing the glueball states. This can be done in a simplified manner by keeping the centre always at the lowest possible eigenstate. Finally the intrinsic energy of the gluon in A-gluon system can be obtained as follows. Let particles 1, 2, 3, ... A are confined around a common centre C at a distance R. $r_1, r_2, \ldots r_A$ are the distances of each particle measured from the centre. $x_1, x_2, \ldots x_A$ are the position vectors of these particles. Then

$$\mathbf{R} = \sum_{i=1}^{A} \mathbf{x}_i/A,$$

$$\mathbf{R} + \mathbf{r}_i = \mathbf{x}_i. \tag{93}$$

Now the oscillator type of equations obtained in §§2 and 3 in the relative coordinate with respect to the centre of confinement can be resolved in terms of $\mathbf{R}$ and $\mathbf{x}_i$ using (93). In the case of CCM

$$\left[-\sum_{i=1}^{A} \nabla_i^2 + \sum_{i=1}^{A} \alpha^2 x_i^2 - (-\nabla_R^2 + \alpha^2 R^2)\right]\Psi = \omega_n^2 \Psi, \tag{94}$$

where

$$(-\nabla_R^2 + \alpha^2 R^2)\Psi = E_N^C \Psi \tag{95}$$

and

$$E_N^C = (2N+3)\alpha. \tag{96}$$

We construct the states with the centre of confinement in the lowest oscillator state $E_0^C = 3\alpha$. Assuming an equal contribution of E_0^C/A due to each gluon at the centre and ε_n is the intrinsic energy of each gluon, then

$$(-\nabla_i^2 + \alpha^2 x_i^2)\Psi_i = (\varepsilon_n^2 + E_0^C/A)\Psi_i, \tag{97}$$

where

$$\varepsilon_n^2 + E_0^C/A = (2n+3)\alpha. \tag{98}$$

Thus

$$\varepsilon_n^{\mathrm{CCM}} = (2n+3-3/A)^{1/2}\alpha^{1/2}. \tag{99}$$

Similarly in the case of DCM,

$$\varepsilon_n^{\mathrm{DCM}} = (2n+3-3/A)a. \tag{100}$$

We calculate the energies of low-lying digluon and trigluon states in DCM and CCM. The expressions for the gluon energy quanta and the intrinsic gluon energy expressions for the lower modes are given in table 1. The single parameter a in DCM and $\alpha^{1/2}$ in CCM are calculated by fitting the iota (1440 MeV) 0^{-+} state as a digluon glueball. Without considering the spurious motion of the centre and not subtracting the zero point energy, the glueball energies become just addition of the respective ω_n's. In this case the DCM parameter a and the CCM parameter $\alpha^{1/2}$ are obtained as 180 MeV and 363 MeV respectively. The energies are tabulated in table 2. While considering the spurious motion of the centre and removing the average zero point energy the parameters are obtained as $a = 288$ MeV and $\alpha^{1/2} = 466$ MeV respectively by fitting the same iota (0^{-+}) as a digluon state, using the intrinsic energy expression

Table 1. Energy expressions for the low-lying gluon states.

Quanta n	DCM $\omega_n = (2n+3)a$	DCM $\varepsilon_n = (2n+3-3/A)a$ $A=2$	$A=3$	CCM $\omega_n = (2n+3)^{1/2}\alpha^{1/2}$	CCM $\varepsilon_n = (2n+3-3/A)^{1/2}\alpha^{1/2}$ $A=2$	$A=3$
0	$3a$	$\frac{3}{2}a$	$2a$	$\sqrt{3}\alpha^{1/2}$	$\sqrt{\frac{3}{2}}\alpha^{1/2}$	$\sqrt{2}\alpha^{1/2}$
1	$5a$	$\frac{7}{2}a$	$4a$	$\sqrt{5}\alpha^{1/2}$	$\sqrt{\frac{7}{2}}\alpha^{1/2}$	$2\alpha^{1/2}$
2	$7a$	$\frac{11}{2}a$	$6a$	$\sqrt{7}\alpha^{1/2}$	$\sqrt{\frac{11}{2}}\alpha^{1/2}$	$\sqrt{6}\alpha^{1/2}$
3	$9a$	$\frac{15}{2}a$	$8a$	$3\alpha^{1/2}$	$\sqrt{\frac{15}{2}}\alpha^{1/2}$	$\sqrt{8}\alpha^{1/2}$

Table 2. Low-lying glue-ball energies without considering the spurious motion and without subtracting the zeropoint energy.

Coupled modes	J^{PC}	DCM (MeV)	CCM (MeV)
EE	$0^{++}\ 2^{++}$	1080	1257
EM	$0^{-+}\ 2^{-+}$	1440	1440
MM	$0^{++}\ 2^{++}$	1800	1623
EEE	$0^{-+}\ 1^{--}\ 3^{--}$	1620	1886
EEM	$0^{++}\ 2^{++}\ 1^{+-}\ 3^{+-}$	1980	2069
EMM	$0^{-+}\ 2^{-+}\ 1^{--}\ 3^{--}$	2340	2251
MMM	$0^{++}\ 1^{+-}\ 3^{+-}$	2700	2434

and taking the possible linear combinations of the low-lying levels to ensure the centre of confinement remains at the ground state. The calculated energies for the digluon and trigluon low-lying states are given in table 3, comparing with the naive bag model results (Kuti 1977) and some of the experimental candidates.

7. Comparison with experiment and discussion

The lightest glueballs are expected to have masses ranging from 1–3 GeV and spin-parities 0^{++}, 0^{-+} and 2^{++}. Observations of such particles are very crucial to QCD. This mass range is accessible in radiative J/Ψ decays and these states are expected to dominate this decay. The first candidate iota (1440 MeV) 0^{-+} was found by Mark II in the decay mode $\iota \to K_s^0\, K^{+-}\, \pi^{-+}$ (Sharre *et al* 1980). Being an oldest glueball candidate we fit our parameters and predicted all other glueball states which are found to be in good agreement with other existing candidates (see table 3). Although the latest

Table 3. Calculated low-lying glue-ball energy states in DCM and CCM with removal of the spurious motion of the centre in comparison with bag model results (Kuti 1977) and with experimental candidates.

Coupled modes	J^{PC}	Calculated energies (in MeV) DCM	CCM	BAG	Experimental candidates J^{PC}	Energy (in MeV)
EE	$0^{++}\,2^{++}$	864	1137	1796	0^{++}	1240 (g_s)
EM	$0^{-+}\,2^{-+}$	1440	1440	1446	0^{-+}	1440 (l)
MM	$0^{++}\,2^{++}$	2016	1703	1096	2^{++}	1700(θ)
EEE	$0^{-+}1^{--}3^{--}$	1728	1971	2694	—	—
EEM	$0^{++}\,2^{++}1^{+-}\,3^{+-}$	2304	2246	2344	2^{++}	(2120, 2220, 2360) (g_t)
EMM	$0^{-+}\,2^{-+}\,1^{--}\,3^{--}$	2880	2489	1990	—	—
MMM	$0^{++}1^{-}{}^{-}3^{+-}$	3456	2720	1644	—	—

experimental results of J/Ψ decay give very strong evidences for the state iota 0^{-+} (1459 ± 5 in Mark III) to be gluonic, the present situation is such that it is not unambiguously possible to identify the state due to the strong mixing of the $q\bar{q}$ pseudoscalar mesons in this energy range (Konigsmann 1986). The other glueball candidate which showed very good agreement is the $\theta\,2^{++}$ (1700 MeV) discovered by the crystal ball group in the channel $J/\Psi\to\gamma\eta\eta$ (Edwards *et al* 1982). A detailed analysis of this state conclusively showed it to be a gluonic meson. But its decay patterns cause slight problems and hence these states have to be thoroughly studied before they are confirmed. Another glueball candidate is the three resonances $g_T(2120)$, $g'_T(2220)$ and $g''_T(2360)$ with $J^{PC}=2^{++}$ obtained in the reaction $\pi^-P\to\phi\phi n$ which breaks down the OZI suppression. The analysis of these resonances as a three-gluonic combinations explained all their features in a clear-cut and simple manner by Lindenbaum (1985). Some recent differences regarding the degree of OZI forbiddeness of this reaction have been resolved by them and it is concluded that they are produced by glueball and strongly argued that alternative explanations are incorrect and do not fit the experimental facts (Lindenbaum *et al* 1985). Our results for the 2^{++} trigluon state in this energy range are obtained from the EEM coupled modes whose energy is very close to the average energy of the g_T resonances (2233 MeV). The other candidate is the g_s $(1240)0^{++}$ obtained in the reaction $\pi^-p\to K_s^0K_s^0n$ (Etkin *et al* 1982). The characteristics of this state satisfied that expected by a digluon state. This state shows poor agreement with our results, when the zero point energy is subtracted from the energy quanta. But it is in good agreement with the CCM lowest glueball energy (table 2) without the correction due to the spurious motion of the centre. However, all other states are in good agreement with the experimental candidates only when the spurious motion of the centre is taken into account. Thus, as in the case of RHM, the success of CCM is also closely linked with the accounting for the spurious centre of motion. The DCM results are not satisfactory even though the EEM 2^{++} state is close to g_T. The discrepancy as seen from table 3 between the CCM and the naive bag model energies of the coupled modes is due to the fact that the lowest gluon energy state in our case is the electric mode with $l=0$, $J=1^-$ state while that of the bag model is the magnetic mode with $l=1, J=1^+$. Most of the other potential models also provide the $l=0$ solution but they are neglected for massless spin-1 fields. We feel that it will be incorrect to neglect such solutions in phenomenological models like DCM or CCM where the inhomogeneous medium may provide an effective mass to the field as the interaction grows when it moves away from the 'centre of confinement'. The $\theta_{\mu\nu}$ tensor in §2 and then in §3 can be considered as a dynamical gluon mass for low momentum modes (Celenza and Shakin 1986). A new gauge condition named the oscillator gauge obtained in §3 is another feature of our model which helps us to get a consistent confinement modes similar to RHM. This gauge is found to be a linear combination of the Coulomb gauge and the axial gauge.

The non-local dielectric function obtained in §4 corresponding to the CCM is momentum-dependent and the asymptotic freedom at lower distances or at high momentum transfer is built in it. There are indications from QCD about the momentum dependence for the dielectric function (Baker *et al* 1983) for confinement models. It remains to be seen how exactly the nonlinearity of the QCD can be viewed through phenomenological models with a proper gauge condition. And also interested to see the link between the description of the gluon condensate and the dielectric medium. There are much more low-lying glueball states of which very few are the experimental candidates. It is very crucial to investigate to distinguish $q\bar{q}, q\bar{q}g, gg$ and ggg states among the vast experimental data for the exotic states in the energy range 1–3 GeV. It is also very important to see how these states lead to the understanding of the strong interaction between nucleons at a more fundamental level.

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