Supersymmetric $SU_C(4) \times SU_L(2) \times SU_R(2)$ and spontaneous breakdown of symmetries

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Abstract. A supersymmetric version of the left right symmetric partial unification group $SU_C(4) \times SU_L(2) \times SU_R(2)$ is presented. The spontaneous breakdown of gauge symmetry in a favourable chain of descent has been studied in detail. The mass spectra have been calculated. The method of O’Raifeartaigh has been used to break supersymmetry. The lifting of degeneracy of mass levels between physical multiplets has been shown to occur due to radiative corrections.

Keywords. Gauge symmetry; supersymmetry; radiation correction.

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1. Introduction

Pati and Salam (1973) proposed to unify the strong, electromagnetic and weak interaction by gauging a symmetry group $SU_C(4)$ associated with a multiplet containing both quarks and leptons. The semi-simple group $SU_C(4) \times SU_L(2) \times SU_R(2)$ called the Pati-Salam group with fractionally charged quarks is the only rank 5 partial unification group and one of the most attractive extensions of the standard electroweak model. The model is left right symmetric and is formulated so that parity has to be a spontaneous broken symmetry.

To include both bosons and fermions in the same multiplet one has to supersymmetrize the theory which also gives a natural solution to hierarchy problem in grand unified theories. However if supersymmetric gauge theories are to explain low energy phenomenon both supersymmetry and gauge symmetries are to be broken. Even though spontaneous breakdown of gauge symmetry can be easily induced, supersymmetry is not easily broken.

The breakdown of supersymmetry in the case of standard model $SU_C(3) \times SU_L(2) \times U(1)$ has been studied by Dine and Fischler (1982). But the supersymmetric version of the Pati-Salam group and the subsequent supersymmetric breakdown to $SU_C(3) \times U_{EM}(1)$ has not yet been worked out.

Breakdown of gauge invariance in supersymmetric SO(10) has been studied by Gipson and Marshak (1984) who considered the chain of symmetry breaking

$$SO(10) \rightarrow SU_C(4) \times SU_L(2) \times SU_R(2) \rightarrow SU_C(3) \times SU_L(2) \times SU_R(2) \times U_{B-L}(1)$$

$$\rightarrow SU_C(3) \times SU_L(2) \times U_Y(1) \rightarrow SU_C(3) \times U_{EM}(1),$$
by the multiplet $\chi(15, 1, 1), \Delta_L(10, 3, 1) \bar{\Delta}_R(10, 1, 3)$ and $\Phi(1, 2, 2)$. In this chain, the left right symmetry breaking occurs in the GUT scale. To get an intermediate scale, one has to break partially. However it has been found (Marshak and Mohapatra 1984) that the most favourable chain breaking including two-loop correction is

$$\begin{align*}
SO(10) & \xrightarrow{M_R} SU_c(4) \times SU_L(2) \times SU_R(2) \\
(1, 1, 1) & \\
\xrightarrow{M_L} SU_c(3) \times U_L(1) \times SU_L(2) \times U(1) \\
(15, 1, 3) & \\
\xrightarrow{M_R'} SU_c(3) \times SU_L(2). \\
(10, 1, 3) &
\end{align*}$$

What we have attempted here is supersymmetrizing at the left right symmetric stage which might have descended from any grand unified group and followed the above most favourable chain of gauge symmetry breaking. The calculation of supersymmetry breaking and gauge symmetry breaking is actually highly involved and will be given in the text. In §2, we have studied the spontaneous breakdown of gauge symmetry $SU_c(4) \times SU_L(2) \times SU_R(2)$ to $SU_c(3) \times U_L(1) \times U(1)$ through intermediate stages, preserving supersymmetry. In §3, we discuss supersymmetry breaking by the O'Raifeartaigh mechanism. The supersymmetry breaking is not directly coupled to the physical light world. The O’Raifeartaigh sector of superheavy supermultiplets which are responsible for supersymmetry breaking is coupled to the light world. This produces indirect couplings. Due to mass splitting of the supermultiplets their differing contribution as intermediate state breaks the supersymmetry of the physical world.

2. Spontaneous breakdown of gauge symmetry

For the left right symmetric model there will be three gauge superfields. They belong to adjoint representation of $SU_c(4)$, $SU_L(2)$ and $SU_R(2)$. The colour gauge field is a fifteenplet and the left right gauge fields are triplet in nature.

A gauge superfield $V = V^\dagger$ in the Wess Zumino gauge (Wess and Zumino 1974)

$$V(y\theta \bar{\theta}) = -\theta \sigma^m \bar{\sigma} V_m(y) + i \theta \theta \bar{\theta} \bar{\chi}(y) - i \bar{\theta} \theta \bar{\chi}(y) + \frac{1}{2} \theta \theta \bar{\theta} \bar{\chi}(y) \right].$$

The associated non-abelian gauge transformation is $\exp(gv) = \exp(i\Lambda^a)\exp(gv)\exp(i\Lambda^a)$. Both $\Lambda$ and $v$ are matrices. $\Lambda = T^a \Lambda^a$, $V = T^a V_a$. The gauge and superinvariant Lagrangian $L_g$ for the gauge multiplet is of the form

$$L_g = Tr[ -\frac{1}{4} (V^{\mu} V_{\mu} - i \sigma^m \delta m \bar{\lambda} + \frac{1}{2} D^2],$$

where

$$V_{\mu \nu} = \partial_{\mu} V_{\nu} - \partial_{\nu} V_{\mu} + ig[V_{\mu}, V_{\nu}],$$

and

$$D_m \bar{\chi} = \partial_m \bar{\chi} + ig[V_m \bar{\chi}].$$

We have made the replacement $V_{\mu} \rightarrow 2g V_{\mu}$, $\lambda \rightarrow 2g \lambda$, $D \rightarrow 2g D$. Now we study the
spontaneous breakdown of $SU_c(4) \times SU_L(2) \times SU_R(2)$ as mentioned in the introduction. Introducing the chiral multiplets $\varphi_c(15,1,3) \Phi_1(10,1,3)$, $\Phi_2(10,1,3)$, $D_1(1,2,2)$ as well as the three singlets $X$, $Y$, $Z$ the correct gauge symmetry breaking can be implemented. The component fields of $\varphi_c$, $\Phi_R$ and $\Phi$ are $(A_c \Psi_c F_c)$, $(A_R \Psi_R F_R)$ and $(A \Psi F)$; respectively. For the singlets $X$, $Y$, $Z$ $(a_x \Psi_x f_x)$, $(a_y \Psi_y f_y)$ and $(a_z \Psi_z f_z)$ respectively are the component fields.

The invariant Lagrangian is taken to be

$$L_H = [\text{Tr}(\Phi_1^T \exp(V_L + V_R) \Phi_1) + \Phi_2^T \exp(V_L + V_R) \Phi_2] - \text{Tr}[\{a(\phi_c^2 - M_c^2)X + b(\Phi_2^T \Phi_1 - M_c^2)Y + c(\Phi_2 \Phi^* \tau_2)] - 2 m m' \tau_2 \text{F} + \text{c.c.}]$$

(3)

where $\Phi_1 \equiv \Phi$ and $\Phi_2 \equiv \tau_2 \Phi^* \tau_2$. $M_c$ is the mass scale for lepton colour symmetry breaking and is usually of the order of $10^{15}$ to $10^{16}$ GeV. $M_R$ is the left right symmetry-breaking scale and can range from $10^4$ GeV upwards. $m$ is of the electroweak scale of $10^2$ GeV. The equation of motion for the auxiliary fields are

$$F_1^c = 2 a A_c a_c, \quad f_1^c = \text{Tr}(A_c^2 - M_c^2)$$
$$F_1^R = b A_1 a_1, \quad f_1^R = \text{Tr}(A_1 a_1^2 - M_1^2).$$
$$F_2^R = b A_1 a_1, \quad f_2^R = \text{Tr} c A_2 a_2^2 - c 2 m m',$$
$$F_1^t = c \tau_2 a_2 a_2, \quad D_L = - g_L(A_1 A_1^t - A_2 A_2^t)$$
$$F_2^t = c A_1 a_2.$$}

(4)

Here the upper indices refer to colour and lower indices refer to flavour quantum numbers. The potential is

$$U = \text{Tr} \left[ \frac{1}{2} D_1^2 + \frac{1}{4} D_2^2 + \frac{1}{2} F_1^F F_1 + F_2^F F_2 + F_1^F F_c + F_1^R F_1^R$$
$$+ F_2^R F_2 + f_x^* f_x + f_y^* f_y + f_z^* f_z \right].$$

(5)

This potential has supersymmetric minima at

$$U = 0, \quad \langle A_c \rangle = M_c = m_c \text{diag}(1,1,1,-3),$$
$$\langle A_1 \rangle = m, \quad \langle A \rangle = \begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix},$$
$$\langle X \rangle = \langle Y \rangle = \langle Z \rangle = 0.$$
$U$ is a sum of several positive terms. So the value at minimum is zero. The vacuum expectation values (VEVs) for these fields which lead to this minima are obtained by differentiation and by choosing the appropriate values to follow the desired symmetry-breaking pattern. The non-zero VEVs break the gauge symmetry spontaneously. \( <A> \) breaks $SU_c(4)$ to $SU_c(3) \times U_{R-\lambda}(1)$ and $SU_R(2)$ to $U_R(1)$, \( \Phi^{4+}_{11} \) combines $U_{R-\lambda}(1) \times U_R(1)$ to $U_{\lambda}(1)$ and \( <A> \) breaks $SU_L(2) \times U_L(1)$ to $U_{EM}(1)$. Shifting the chiral fields by their VEVs and diagonalizing the mass matrices (Sohnius 1977) we get

$$L_n^m = (64g_1^2m_1^2 + 4g_2^2m_2^2)[|V_{\mu
u}^4|^2 + |P|^2] + (64g_2^2m_2^2 + 4g_3^2m_3^2)^{1/2}[E_1 E_1 - E_2 E_2 + E_3 E_3 - E_4 E_4] + M_1^2(|w_1|^2 + |W_1|^2) + M_1(E_{11} E_{11} + E_{12} E_{12}) + M_2^2(|w_2|^2 + |W_2|^2) + M_2(E_{21} E_{21} + E_{22} E_{22}) + M_3^2(|w_3|^2 + |W_3|^2) + M_4(E_4 E_4 + E_4 E_4) + 96a^2m_2^2[|a|^2 + |\gamma|^2] + (96)^{1/2}am_1(E_{x2} x_{x2} - E_{x1} x_{x1}) + 2b^2m_2^2[|a|^2 + |\gamma|^2] + (2)^{1/2}bm_{R}(E_{x2} x_{x2} - E_{x1} x_{x1}) + 2c^2(m^2 + m'^2)[|a|^2 + |\gamma|^2] + 2(m^2 + m'^2)^{1/2}c(E_{x2} x_{x2} - E_{x1} x_{x1}). \quad (7)$$

Notations with details are given in Appendix A. The mass spectrum is given in table 1. This shows that for each mass there is an equal number of fermionic as well as bosonic degrees of freedom. Thus supersymmetry is preserved even though the gauge symmetry breaks. In the process of gauge breaking the gauge particles get mass. The mass of the lepto quarks \( V_{\mu
u}^4 \) of the fifteenplet is of the order of $m_c$ in the limit $m_c \gg m_R$. The mass of the neutral component $S^0(V_{\mu
u}^4$ of the fifteenplet) is of the order of $m_R$ as it gets mass from the VEVs of $\Phi_R$. In $SU_L(2)$ and $SU_R(2)$ group the $W_0$ and $W_{\pm}$ mesons get mixed due to VEVs of $\Phi(122)$. However in the limit of $m_c \gg m_R \gg m$ and $m'$, $W_{\pm}$ are decoupled. The mass of one of them comes to be very large and is of the order of $m_c$ whereas the other one has a low mass of the order of $(m^2 + m'^2)^{1/2}$. Thus we see two sets of $W$ mesons, $W_R$ which is very heavy and $W_L$ which is much lighter. By calculating the

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**Table 1.** Mass spectrum of bosons and fermions when gauge symmetry $SU_c(4) \times SU_L(2) \times SU_R(2)$ breaks to $SU_c(3) \times U_{EM}(1)$.  

<table>
<thead>
<tr>
<th>Particle (bosons)</th>
<th>Mass</th>
<th>Particle (fermions)</th>
<th>Mass</th>
</tr>
</thead>
</table>
| $V_{\mu
u}^4$, $P$ | $2g_1(m_R^2 + 16m_2^2)$ | $E_{c1}, E_{c2}, E_{c3}, E_{c4}$ | $2g_2(m_R^2 + 16m_2^2)$ |
| $w_1$, $W_1$ | $M_1$ | $E_{11} E_{11}$ | $M_1$ |
| $w_2$, $W_2$ | $M_2$ | $E_{21} E_{21}$ | $M_2$ |
| $\tilde{w}_1$, $\tilde{W}_1$ | $M_3$ | $E_3$, $E_3$ | $M_3$ |
| $\tilde{w}_2$, $\tilde{W}_2$ | $M_4$ | $E_4$, $E_4$ | $M_4$ |
| $a_1$, $w_1$ | $(96)^{1/2}am_c$ | $E_{x1}$, $E_{x2}$ | $(96)^{1/2}am_c$ |
| $a_2$, $w_2$ | $(2)^{1/2}bm_{R}$ | $E_{x1}$, $E_{x2}$ | $(2)^{1/2}bm_{R}$ |
| $a_3$, $w_3$ | $2(m^2 + m'^2)^{1/2}c$ | $E_{x1}$, $E_{x2}$ | $2(m^2 + m'^2)^{1/2}c$ |
mass spectra and eigenvectors it is found that the photon field is

\[ A_\mu = e \left[ \frac{W_{0L}}{g_L} - \frac{W_{0R}}{g_R} + \frac{2}{3} \frac{S^0}{g_c} \right] \]

and the coupling constant \( e \) satisfies the relation

\[ \frac{1}{e^2} = \frac{1}{g_L^2} + \frac{1}{g_R^2} + \frac{2}{3g_c^2}. \]

Furthermore the neutral components \( S^0, W_{0L} \) and \( W_{0R} \) get mixed to give the photon field \( A_\mu \) with zero mass and \( Z^0 \) fields, \( Z_{\mu 1} \) and \( Z_{\mu 2} \) with masses \( M_3 \) and \( M_4 \) respectively. In the above limit \( Z_{\mu 1} \) turns to be very much massive \((M_3 \approx m_R)\) and \( Z_{\mu 2} \) very light.

To discuss quarks and leptons we have to include the matter field. We consider two chiral multiplets \( \Psi_L(4, 2, 1) \) and \( \Psi_R(4, 1, 2) \). The superpotential part of the Lagrangian containing Yukawa couplings is given by Mohapatra and Senjanovic (1981),

\[ L_Y = \text{Tr}[h_1 \Psi_L \Phi_1 \Psi_R + h_2 \Psi_L \Phi_2 \Psi_R + i h_3 (\Psi_R^T C \tau_2 \Phi_R \Psi_L) + \text{h.c.}] \] (8)

\( C \) is the Dirac charge conjugation matrix. The potential has supersymmetric minima at \( \langle \Psi_L \rangle = \langle \Psi_R \rangle = 0 \). It is interesting to study the neutrino sector. The VEVs of \( \Phi_R \) and \( \Phi \) contribute to the following mass terms for \( v_L \) and \( v_R \) sector,

\[ L_v^{\text{mass}} = (h_1 m + h_2 m') (\bar{v}_L v_R) + (\bar{v}_R v_L) + h_3 m_R (v_R^T C v_R + v_R C v_R^T). \] (9)

This can be simplified into

\[ L_v^{\text{mass}} = (h_1 m + h_2 m') v^T C N - h_3 m_R (N^T C N) + \text{h.c.}, \] (10)

where \( v \) and \( N \) are effectively two component complex spinors

\[ v \equiv v_L, \quad N \equiv C(v_R)^T. \]

In terms of the matrix \( M \) it can be written as

\[ L^{\text{mass}} = (v^T, N^T) M C \begin{pmatrix} v \\ N \end{pmatrix} + \text{h.c.}, \]

where

\[ M = \begin{pmatrix} 0 & \frac{1}{2}(h_1 m + h_2 m') \\ \frac{1}{2}(h_1 m + h_2 m') & -h_3 m_R \end{pmatrix}. \] (12)

There will be two eigenstates for neutrinos with masses \( m_1 \) and \( m_2 \)

where

\[ m_1 = \frac{1}{4} \frac{(h_1 m + h_2 m')^2}{h_3 m_R}, \quad m_2 = -h_3 m_R. \]

In the limit \( m_R \gg m \) and \( m', m_1 \) is negligibly small and \( m_2 \) is very large. The states with small mass is defined as \( v_e \) and that with larger mass as \( N_e \). For \( m_R \to \infty \), \( m_e \to 0 \) and \( m_{N_e} \to \infty \). The Yukawa coupling gives rise to the mass of the electron as

\[ m_e = (h_1 m' + h_2 m). \]
Since the superpotential $L_y$ given by (8) is supersymmetric at all energy scales the supersymmetric partners of these fermions have the same mass.

Here we see that SUSY does not break for matter field. Thus in tree level SUSY remains unbroken both for Higgs field and matter field. To break SUSY for Higgs field we shall use the mechanism of O’Raifeartaigh (1975). This breaks the SUSY of Higgs scalars which are singlets. To break SUSY of Higgs multiplets we have to consider the radiative corrections. This will be done in the succeeding section.

3. Breakdown of supersymmetry

In the last section we have used three singlet fields $X$, $Y$ and $Z$ for gauge symmetry breaking. To implement both gauge symmetry and supersymmetry breaking we require two O’Raifeartaigh singlets $A_{01}$ and $A_{02}$. It is to be noted that models with spontaneous broken SUSY at low energy have serious phenomenological difficulties (see e.g. Dimopoulos and Georgi 1981 and Weinberg 1982). So the singlets $A_{01}$ and $A_{02}$ are made to interact with the singlets $X$, $Y$ and $Z$ which in turn are coupled to the low energy sector. This will be in similar spirit to the work of Ellis et al (1982). We construct the superpotential part of the Lagrangian as

$$L = [\lambda + g_{1x}(X - M_x)^2 + g_{1y}(Y - M_y)^2 + g_{1z}(Z - M_z)^2]A_{01} + [m_x(X - M_x) + m_y(Y - M_y) + m_z(Z - M_z)]A_{02} + \text{Tr}[g_{2x}(q_x - M_x)^2] + g_{2y}(q_y - M_y)^2 + g_{2z}(q_z - M_z)^2 \right]A_{01} + \left[ r_{x}(X - M_x) + r_{y}(Y - M_y) + r_{z}(Z - M_z) \right]A_{02} + \text{Tr}[g_{3x}(q_x^2 - M_x^2)(X - M_x) + g_{3y}(q_y^2 - M_y^2)(Y - M_y) + g_{3z}(q_z^2 - M_z^2)(Z - M_z)] + g_{3x}X^3 + g_{3y}Y^3 + g_{3z}Z^3 + h.c.$$

(13)

The mass of the singlet $A_{02}$ is the order of $m_x \simeq m_y \simeq m_z$. One can take this to be superheavy and $M_x \simeq M_y \simeq M_z$ to be of an intermediate energy scale. The treatment for symmetry breaking is, however, quite general. The equations of motion for the auxiliary fields are

$$F_{x}^{\ast} = -\frac{[\lambda + g_{1x}(a_x - M_x)^2 + g_{1y}(a_y - M_y)^2 + g_{1z}(a_z - M_z)^2]}{[m_x(a_x - M_x) + m_y(a_y - M_y) + m_z(a_z - M_z)]}$$

$$F_{y}^{\ast} = -\frac{[2g_{1x}a_{01}(a_x - M_x) + m_xa_{02} + 3g_{3x}a_x^2 + \text{Tr}g_{2x}(A_x^2 - M_x^2)]}{[m_x(a_x - M_x) + m_y(a_y - M_y) + m_z(a_z - M_z)]}$$

$$F_{z}^{\ast} = -\frac{[2g_{1x}a_{01}(a_x - M_x) + m_xa_{02} + 3g_{3x}a_x^2 + \text{Tr}g_{2x}(A_x^2 - M_x^2)]}{[m_x(a_x - M_x) + m_y(a_y - M_y) + m_z(a_z - M_z)]}$$

$$F_{x}^{\ast} = -\frac{[2g_{1y}a_{01}(a_y - M_y) + m_ya_{02} + 3g_{3y}a_y^2 + \text{Tr}g_{2y}(A_y^2 - M_y^2)]}{[m_x(a_x - M_x) + m_y(a_y - M_y) + m_z(a_z - M_z)]}$$

$$F_{y}^{\ast} = -\frac{[2g_{1z}a_{01}(a_z - M_z) + m_za_{02} + 3g_{3z}a_z^2 + \text{Tr}g_{2z}(A_z^2 - M_z^2)]}{[m_x(a_x - M_x) + m_y(a_y - M_y) + m_z(a_z - M_z)]}$$

$$F_{x}^{\ast} = -\frac{2g_{2x}A_x(a_x - M_x)}{[m_x(a_x - M_x) + m_y(a_y - M_y) + m_z(a_z - M_z)]}$$

$$F_{y}^{\ast} = -\frac{2g_{2y}A_y(a_y - M_y)}{[m_x(a_x - M_x) + m_y(a_y - M_y) + m_z(a_z - M_z)]}$$

$$F_{z}^{\ast} = -\frac{2g_{2z}A_z(a_z - M_z)}{[m_x(a_x - M_x) + m_y(a_y - M_y) + m_z(a_z - M_z)]}$$

(14)

Evaluating the potential in terms of the auxiliary fields and minimizing it we obtain

$$\langle a_x \rangle = M_x, \quad \langle a_y \rangle = M_y, \quad \langle a_z \rangle = M_z, \quad \langle a_{01} \rangle = 0$$

$$\langle a_{02} \rangle = -3g_{3x}(M_x^2/m_x) = -3g_{3y}(M_y^2/m_y) = -3g_{3z}(M_z^2/m_z).$$

(15)
The fields of the $R$ invariance breaking terms $X^3, Y^3, Z^3$ are shifted by $M_x, M_y, M_z$ respectively. Bilinear fermionic terms like $M_x\bar{\psi}_x\psi_x$, $M_y\bar{\psi}_y\psi_y$ and $M_z\bar{\psi}_z\psi_z$ arise due to such shifts. Internal fermion lines of Feynmann diagrams (see figures 1a–d) carry this information. On evaluating these diagrams we find that the gluinoes get their masses. These have been explicitly calculated in the subsequent paragraphs.

The equations of motion for $F_x, F_y$ and $F_z$ clearly show that $A_{02}$ is required to make $F^*_x F_x, F^*_y F_y$ and $F^*_z F_z$ part of $U_{min}$ zero by properly choosing $\langle a_{02} \rangle$ and the respective coupling constants. All the auxiliary fields contributes zero to $U_{min}$ except $F^*_0 F_0$. This makes $U_{min} > 0$ and is equal to $\chi^2$ where $\chi$ is the O'Raifeartaigh parameter.

Shifting of the chiral fields about its VEVs gives

$$F^*_0 F_0 = \chi [g_{1x}(a_x^2 + a_x^{*2}) + g_{1y}(a_y^2 + a_y^{*2}) + g_{1z}(a_z^2 + a_z^{*2})].$$

This shows that $\chi$ enters the mass of $a_x, a_y$ and $a_z$ that is the bosonic part of $X, Y$ and $Z$. The products of $F^*_x F_x, F^*_y F_y, F^*_z F_z, F^*_0 F_0$ contain terms like $a_x A_x, a_y A_y, a_z A_z, a_x a_0, a_y a_0$ and $a_z a_0$ with some (mass)$^2$ as coefficients. To obtain the masses of all these participating fields, one has to diagonalize a fairly large mass matrix. What is relevant and important is that the masses of all these objects will contain the O'Raifeartaigh parameter $\chi$. Their fermionic partners, however, will have mass values independent of $\chi$.

As a result SUSY will break down in this sector at the tree level.

To break the SUSY of the remaining components of $\Phi_c, \Phi_R$ and $\Phi$, we consider radiative correction. For this we collect cubic and quadratic terms from Yukawa couplings and draw Feynmann diagrams. Figures 2a, b are the quadratic divergent one-loop diagrams which can contribute additional mass to $A_c$. The infinite contribution of figure 2a cancels with that from figure 2b, as there is a negative sign from

![Figures 1a–d](image-url)
the fermionic closed loop. There is a finite term left over which in the lowest order is given as

$$\delta m^2 = \left(\frac{g_x^2 m_x^2}{4\pi^2}\right) \log\left(\frac{m^2}{m_x^2}\right).$$  \quad (16)$$

Figures 2c, d contribute to the masses of the fermionic partners $\Psi_x$ and the contribution vanishes to the order $g_x^2$. Note that SUSY is unbroken when $m_{\Psi_x} = m_{\Psi_y} = m_x$; similarly figures 2e, f and 3c, d contribute a finite mass to the bosonic components of $\Phi_R$ and $\Phi$ that is $A_R$ and $A$, whereas figures 3a, b and 3e, f contribute zero mass to their fermionic partners.

$$\delta m_{A_R}^2 = \left(\frac{g_y^2 m_y^2}{4\pi^2}\right) \log\left(\frac{m^2}{m_y^2}\right),$$  \quad (17)

$$\delta m_{A}^2 = \left(\frac{g_y^2 m_y^2}{4\pi^2}\right) \log\left(\frac{m^2}{m_y^2}\right).$$  \quad (18)

These bosonic parts of $\varphi_c$, $\Phi_R$ and $\Phi$ are directly coupled to $V_{\mu c}^{14,4i}$, $V_{\mu R}^{ij,2j}$, $V_{\mu L}^{ij,2j}$ whereas the fermionic partners to $\lambda_c^{14,4i}$, $\lambda_R^{ij,2j}$, and $\lambda_L^{ij,2j}$. So it can be easily inferred that the six leptoquarks and their gauginoes in the colour gauge field $V_{\mu c}$ the two $w^\pm$ and their gauginoes in $V_{\mu R}$ and $V_{\mu L}$ acquire different masses, due to the one-loop level correction. Supersymmetry between the remaining gauge bosons and gauginoes is also broken due to radiative correction (Ellis et al 1982). Gauge bosons remain massless.
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Whereas the gauginos acquire mass, which is given by the expression,

$$
(\delta m)^2_g = \frac{3g^2}{4(2\pi)^2} [g_{23}^2 \log(m_{3_a}^2/m_{3_a}^2) + g_{23}^2 \log(m_{3_a}^2/m_{3_a}^2)].
$$

The diagrams which contribute to the mass of gaugino are shown in figures 1a–d. Similar diagrams can be drawn for $\lambda_R$ and $\lambda_L$. There are four Feynmann diagrams for $\lambda_c$. In addition, there will be two more diagrams for $\lambda_R$ because $V_R$ couples with $\Phi_1(15,1,3)$, $\Phi_2(10,1,3)$ and $\Phi_2(1,2,2)$ whereas $V_L$ couples with $\Phi_3$ and $\Phi_R$ only. Furthermore there are only two diagrams for $\lambda_L$ as $V_L$ couples with $\Phi$ only. Contributions $(\delta m)$ to mass of the corresponding gauginos can be given by an expression similar to $(\delta m)_g^2$.

So far we have not considered the SUSY breaking of matter field. When one introduces the super matter field the SUSY can also be broken by radiative corrections. The fermions will still remain massless due to chiral invariance. The bosonic partners, squarks and sleptons, will have loops of gluinos and other gauge bosons. Assume for instance $SU_L(2)$ and $SU_R(2)$ are not broken. Due to imperfect cancellation of the
superpartners, gluons and gluinoes in the intermediate state (loop and tadpole diagrams, figures 4a–c) the squarks and sleptons will acquire mass. In the leading logarithmic order

$$(\delta m^2)_H = \frac{g_2^2 \mu^2}{2\pi^2} \ln \frac{\mu^2}{(\mu + \delta m_R)^2}$$

where $\mu = 8g_rm_c$. Similar expressions can be obtained for sleptons.

4. Conclusion

We have discussed both gauge symmetry breaking and SUSY breaking of the Pati-Salam unification group $SU_C(4) \times SU_L(2) \times SU_R(2)$. The SUSY is broken by the O’Raifeartaigh mechanism. SUSY breaking is not directly coupled to the ordinary light world, but to a world of super heavy supermultiplets, which in turn couple to the light world. Supersymmetry in the light world is broken down as the supersymmetry-broken heavy supermultiplets occur in the intermediate states contributing to the mass shifts. Most of the machinery of the supersymmetry breaking is thus hidden at the super heavy scale.

Appendix A

In this appendix we give different fields with their mass that appear in the expression for $L_R^n$ (equation (7))

\[ p = \frac{1}{2}(A_{\nu 1}^{i4} - A_{\nu 1}^{i4} - A_{\nu 2}^{i4} + A_{\nu 2}^{i4}), \]

\[ E_{c_1} = \frac{1}{(2)^{1/2}}(i\lambda_c^{i4} + \Psi_{c_1}), \quad E_{c_2} = \frac{1}{(2)^{1/2}}(i\lambda_c^{i4} - \Psi_{c_1}), \]

\[ E_{c_3} = \frac{1}{(2)^{1/2}}(-i\lambda_c^{i4} + \Psi_{c_3}), \quad E_{c_4} = \frac{1}{(2)^{1/2}}(-i\lambda_c^{i4} - \Psi_{c_3}), \]

\[ \Psi_{c_1} = \frac{[2(4)g_{c_1}m_{c_1}(\Psi_{c_1}^{i4} - \Psi_{c_1}^{i4}) - 4(2)g_{c_1}m_{c_1}(\Psi_{c_1}^{i4} - \Psi_{c_1}^{i4})]}{(64g_{c_1}^2 m_c^2 + 4g_{c_1}^2 m_R^2)^{1/2}}, \]

\[ \Psi_{c_3} = \frac{[4(2)g_{c_1}m_{c_1}(\Psi_{c_1}^{i4} - \Psi_{c_1}^{i4}) - 2(2)g_{c_1}m_{c_1}(\Psi_{c_1}^{i4} - \Psi_{c_1}^{i4})]}{(64g_{c_1}^2 m_c^2 + 4g_{c_1}^2 m_R^2)^{1/2}}, \]

\[ M_1^2 = \frac{1}{2}[(x + y) + (x - y)^2 + 4z^2]^{1/2}, \quad x = 4g_{c_1}^2 m_R^2 + 9g_{c_1}^2 m_c^2 + 2g_{c_1}^2 (m^2 + m'^2), \]

\[ M_2^2 = \frac{1}{2}[(x + y) - (x - y)^2 + 4z^2]^{1/2}, \quad y = 2g_{c_1}^2 (m^2 + m'^2) \]

\[ z = 2g_{c_1}^2 g_R^2 (m^2 + m'^2), \]

\[ w_1 = [ZV_{\mu 1}^{i2} + (M_1 - x)V_{\mu 1}^{i2}]/[Z^2 + (M_1^2 - x)^2]^{1/2}, \]

\[ w_2 = w_1 \quad \text{(with } M_1 \rightarrow M_2) \]
Supersymmetric \(SU_c(4) \times SU_d(2) \times SU_R(2)

\[ W_1 = N [M_1^2 a'b'c'(B) + c'[M_2^2 (M_1^2 - a'^2) - (M_2^2 - a'^2)(M_1^2 - b'^2) + a'^2b'^2]K] + b'(M_2^2 - a'^2)(M_1^2 - b'^2) - a'^2b'^2w], \quad N = \text{normalization constant} \]

\[ W_2 = W_1 \text{ (with } M_1 \rightarrow M_2), \]

\[ a' = (96)^{1/2} g_R m_R, \quad b' = [2(g_L^2 + g_R^2)(m^2 + m'^2)]^{1/2}, \quad c' = 2g_R m_R, \]

\[ B = [(A_{ci_1}^i - A_{ci_2}^i) - 3(A_{ci_1}^{44} - A_{ci_2}^{44})]/(24)^{1/2}, \]

\[ K = [(mA_1^{12} - mA_2^{12}) + (mA_2^{12} - mA_1^{12})]/[2(m^2 + m'^2)]^{1/2}, \]

\[ w = \frac{1}{2} [(A_{R_{12}}^{44} + A_{R_{21}}^{44} + A_{R_{12}}^{44} + A_{R_{21}}^{44})], \]

\[ E_{1L} = N[(\Psi_1)^4(M_1^2 - A')^{1/2} + (B')^{1/2}M_2^2 \Psi_2 + M_1(M_1^2 - A^2)\Psi_2 = +(A'B')^{1/2}M_1 \Psi_2], \]

\[ E_{2L} = E_{1L} \text{ (with } M_1 \rightarrow M_2), \]

\[ A' = (96g_R^2 m_R^2 + 4g_R^2 m_R^2)^{1/2}, \quad B' = 2g_R^2(m^2 + m'^2), \]

\[ \Psi_1 = [2]^{1/2} g_R m_R(\Psi_1^{44} - \Psi_1^{144}) \]

\[ + (8)^{1/2} g_R m_R(3\Psi_1^{44} - \Psi_1^{144})]/[(4g_R^2 m_R^2 + 96g_R^2 m_R^2)]^{1/2}, \]

\[ \Psi_2 = (m\Psi_1^{12} - m'\Psi_1^{21}]/[(m^2 + m'^2)]^{1/2}, \]

\[ M_3^2 = (B'' + [B'' - 2c'']^{1/2}), \quad M_4^2 = (B'' - [B'' - 2c'']^{1/2}), \]

\[ B'' = 8g_R^2 m_R^2 + 4g_R^2(m^2 + m'^2) + 4g_R^2 2m_R^2 + m^2 + m'^2, \]

\[ C'' = 8g_R^2 m_R^2[4(g_L^2 + g_R^2)(m^2 + m'^2) + 32g_R^2 g_R^2 m_R^2(m^2 + m'^2)], \]

\[ Z_{\mu} = N(a_1 b_1 c_1 V_{\mu c}^{44} + b_1 c_1 d_1 V_{\mu d}^{44} + a_1 d_1 V_{\mu d}^{44}), \]

\[ Z_{\mu 2} = Z_{\mu 1} \text{ (with } M_3 \rightarrow M_4), \]

\[ a_1 = 8g_R g_R m_R^2, \quad b_1 = 2g_R g_R m_R^2 c_1 = \left[ \frac{M_1^2}{2} - 4g_R^2(m^2 + m'^2) \right], \]

\[ d = \left( \frac{M_1^2}{2} - 8g_R^2 m_R^2 \right), \quad e = g_R g_R (m^2 + m'^2) \]

\[ w_3 = N [b_1 x + (M_3^2 - a_1 z)]/[b_1^2 + (M_3^2 - a_1 z)]^{1/2}, \quad w_4 = w_3 \text{ (with } M_3 \rightarrow M_4), \]

\[ b_1 = 4[2(m^2 + m'^2) + g_R^2 m_R^2, \quad a_1 = 4(g_R^2 + g_R^2)(m^2 + m'^2), \]

\[ x = [(mA_1^{11} - mA_1^{12}) + (mA_1^{11} - mA_1^{12})]/[2(m^2 + m'^2)]^{1/2}, \]

\[ Z = \frac{1}{2} (A_{R_{12}}^{44} + A_{R_{21}}^{44} + A_{R_{12}}^{44} + A_{R_{21}}^{44}), \]

\[ E_3 = N[x_1 y_1 (M_3^2 - W^2)]^{1/2} + (M_3^2 - x^2)(M_3^2 - W^2)]^{1/2} + wZ_1 (M_3^2 - x^2) + \frac{1}{4} Z_1 (M_3^2 - x^2)]^{1/2} \]

\[ - M_3 y_1 (M_3^2 - W^2)]^{1/2} \Psi_1^* - M_3 Z_1 (M_3^2 - x^2)]^{1/2}], \]
\[x_1 = 4g_sm_R, \quad y_1 = 4g_rm_R, \quad z_1 = 2[2(m^2 + m'^2)]^\frac{1}{2}g_R,\]

\[w = 2[2(m^2 + m'^2)]^\frac{1}{2}g_L, \quad \Psi_1 = \frac{1}{(2)^{\frac{1}{2}}} (\Psi_{11}^{44} - \Psi_{12}^{44}),\]

\[\Psi_2 = \frac{1}{(m^2 + m'^2)^\frac{1}{2}} (m\Psi_{11}^{11} - m'\Psi_{12}^{11}), \quad w_e = \frac{1}{(24)^{\frac{1}{2}}} (A_{e11}^{ii} - A_{e22}^{ii} - 3A_{e11}^{44} + 3A_{e22}^{44}),\]

\[W_R = \frac{1}{(2)^{\frac{1}{2}}} (A_{11}^{44} + A_{22}^{44}), \quad W = (mA_{11}^{22} + m'A_{11}^{11})/(m^2 + m'^2)^{\frac{1}{2}},\]

\[E_{x1} = \frac{1}{(2)^{\frac{1}{2}}} (\Psi + \Psi_x), \quad \Psi = \frac{1}{(24)^{\frac{1}{2}}} (\Psi_{11}^{uu} - \Psi_{22}^{uu} - 3\Psi_{22}^{44}),\]

\[E_{x2} = \frac{1}{(2)^{\frac{1}{2}}} (\Psi - \Psi_x), \quad \Psi' = \frac{1}{(2)^{\frac{1}{2}}} (\Psi_{11}^{UU} + \Psi_{22}^{UU}),\]

\[E_{y1} = \frac{1}{(2)^{\frac{1}{2}}} (\Psi' + \Psi_y), \quad \Psi'' = \frac{1}{(2)^{\frac{1}{2}}} (m\Psi_{11}^{UU} + m\Psi_{22}^{UU} + m\Psi_{11}^{22} + m\Psi_{22}^{22})/(2(m^2 + m'^2)^{\frac{1}{2}}),\]

\[E_{y2} = \frac{1}{(2)^{\frac{1}{2}}} (\Psi' - \Psi_y),\]

\[E_{z1} = \frac{1}{(2)^{\frac{1}{2}}} (\Psi'' + \Psi_z),\]

\[E_{z2} = \frac{1}{(2)^{\frac{1}{2}}} (\Psi'' - \Psi_z),\]

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