Pramāṇa – J. Phys., Vol. 28, No. 6, June 1987, pp. 607–616. © Printed in India.

Chou-Yang model and predictions for high-energy pion-deuteron scattering

SUNACHAND PATEL and M K PARIDA*

Physics Department, N. A. C. College, Burla, Sambalpur 76817, India

*Physics Department, North Eastern Hill University, Shillong 793 003, India

MS received 6 November 1986; revised 16 March 1987

Abstract. Using fits to the available data on the pion and deuteron electromagnetic form factors and the Chou-Yang model, the computed values of differential cross-sections for high-energy pion-deuteron elastic scattering agree reasonably well with the available experimental data. Whereas only a shoulder is expected to appear up to energies of several hundreds of GeV, a dip and a secondary maximum are predicted to be conspicuous only for $P_{lab} \gg 400$ GeV/C. The position of the dip starts at a much lower value, $|t_d| \simeq 0{\cdot}5$–$0{\cdot}6$ GeV^2, as compared to the corresponding position in the pp scattering. The positions of the first dip are plotted against total cross-section which can be verified by future high-energy experiments. Limitations of the model predictions for ultrahigh energies are pointed out.

Keywords. Chou-Yang model; hadron-nucleus scattering; pion-deuteron scattering.

PACS Nos 11·80; 12·40; 13·85; 25·80

1. Introduction

Until about six years ago, the Chou-Yang model (Chou and Yang 1967, 1968, 1979; Durand and Lipes 1968; Clarke and Lo 1974; Kac 1973; Chan *et al* 1978) with the Hayot-Sukhatme modifications (Hayot and Sukhatme 1974) was considered to provide an excellent description of the elastic hadron-hadron scattering at high energies including the forward diffraction peak and the dip regions. Besides simplicity, another aesthetically appealing feature of the model is its proposed ability to provide total and differential cross-section values in terms of the interaction strength and the electromagnetic charge form factors of the colliding hadrons. However, critical analyses of the available experimental data at CERN ISR and $\bar{p}p$ collider energies have revealed several shortcomings of the model: For pp scattering with $P_{lab} \gg 400$ GeV/C, the proposed factorization hypothesis for the eikonal, as two separate functions of the impact parameter (b) and the centre of mass energy $[(s)^{1/2}]$, breaks down, and the inelastic overlap function differs significantly from the model prediction (Amaldi and Schubert 1980). Also the standard Chou-Yang model fails to account for the observed slope-break at $|t| \simeq 0{\cdot}1$ GeV^2. Similar deficiencies of the model have also been noted in the case of $\bar{p}p$ scattering and new forms of the eikonal have been suggested for

* To whom correspondence should be addressed.

better description of the data at the ISR and collider energies (Chou and Yang 1983; Fearnley 1985).

In spite of these and other shortcomings, the model has still several impressive successes (Chou and Yang 1968, 1979; Durand and Lipes 1968; Clarke and Lo 1974; Kac 1973; Chan *et al* 1978). For $P_{lab} \lesssim 400$ GeV/C, it provides a good description of the elastic differential cross-section data for pp and $\pi^{\pm} p$ scattering existing in the forward peak and the dip regions. At the ISR energies, the pp differential cross-section data for values of $|t|$ somewhat below the dip region ($|t| \lesssim 1{\cdot}1$ GeV^2) are reasonably well represented by the model; besides, as noted by Chou and Yang (1979), the predicted dip position, as a function of total cross-section, agrees well with the experimental data. At the CERN collider energies also, a reasonably good description of the gross features of the differential cross-section data is possible for $\bar{p}p$ scattering within a limited $|t|$ range below the dip. For example, at $(s)^{1/2} = 546$ GeV, gross features are well reproduced for $|t| \lesssim 0{\cdot}5$ $(0{\cdot}75)$ GeV^2 when real part effects are neglected (included) (Fearnley 1985). Expecting similar successes of the model, we make predictions for high-energy hadron-nucleus scattering in this paper so that successes and failures of the model can be assessed, partially, by using the available data at limited values of P_{lab} and $|t|$ and perhaps completely, by future high-energy experiments. In this paper, we use the Chou-Yang model with Hayot-Sukhatme modifications and the well-known fits to the pion and deuteron charge form factors, to predict differential cross-section and its possible structures.

The paper is planned in the following manner: In § 2, we briefly describe the formulas relevant to the model. In § 3, the predictions of the model are reported. In § 4, we discuss our results and their range of validity.

2. Theoretical aspects of the model

Ignoring the real part and spin effects at high energies, the scattering amplitude, $A(s, t)$, for pion-deuteron scattering and its differential cross-section $d\sigma/dt$ can be expressed as

$$d\sigma/dt = \pi |A(s, t)|^2, \tag{1}$$

$$A(s, t) = \int \frac{d^2 b}{2\pi} [1 - S(\mathbf{b})] \exp(i\mathbf{k} \cdot \mathbf{b}) \tag{2}$$

where $$S(\mathbf{b}) = \exp[-\Omega(\mathbf{b})] \tag{3}$$

with $\mathbf{k}^2 = -t$, t being the square of four-momentum transfer. Here $\mathbf{b}$ is the two-dimensional impact parameter and $\Omega(\mathbf{b})$ is the opaqueness function, whose Fourier transform can be related to the pion and the deuteron form factors,

$$\langle \Omega \rangle = \mu F_{\pi}(t) F_d(t). \tag{4}$$

In the above expression μ is the $\pi^- d$ interaction strength which is generally a function of s, the square of centre of mass energy. Using equation (4) in (2), the scattering amplitude can be written as a series in increasing order of the convolution products of form factors,

$$A(s,t)=\mu F_\pi(t)F_d(t)-\frac{\mu^2}{2!}F_\pi(t)F_d(t)*F_\pi(t)F_d(t)$$
$$+\frac{\mu^3}{3!}F_\pi(t)F_d(t)*F_\pi(t)F_d(t)*F_\pi(t)F_d(t). \tag{5}$$

The method of computation is simplified by assuming an exponential form of parametrization for the product of the two-form factors,

$$F_\pi(t)F_d(t)=\sum_{i=1}^{4}a_i\exp(-b_i|t|) \tag{6}$$

leading to the scattering amplitude

$$A(s,t)=\mu\sum_i a_i\exp(-b_i|t|)-\frac{\mu^2}{2!}\sum_{i,j}\frac{a_ia_j}{2Y_{ij}}\exp\left(-\frac{b_ib_j|t|}{Y_{ij}}\right)$$
$$+\frac{\mu^3}{3!}\sum_{i,j,l}\frac{a_ia_ja_l}{2^2Y_{ijl}}\exp\left(-\frac{b_ib_jb_l|t|}{Y_{ijl}}\right)$$
$$-\frac{\mu^4}{4!}\sum_{i,j,l,m}\frac{a_ia_ja_la_m}{2^3Y_{ijlm}}\exp\left(-\frac{b_ib_jb_lb_m|t|}{Y_{ijlm}}\right)+\dots \tag{7}$$

where

$$Y_{ij}=b_i+b_j,$$
$$Y_{ijl}=Y_{ij}b_l+b_ib_j,$$
$$Y_{ijlm}=Y_{ijl}b_m+b_ib_jb_l. \tag{8}$$

Using the optical theorem and the series (7), the total cross-section can be related to the interaction strength at a given energy as

$$\sigma_T^{\pi d}=\mu-\frac{\mu^2}{2!}\sum_{i,j}\frac{a_ia_j}{2Y_{ij}}+\frac{\mu^3}{3!}\sum_{i,j,l}\frac{a_ia_ja_l}{2^2Y_{ijl}}-\frac{\mu^4}{4!}\sum_{i,j,l,m}\frac{a_ia_ja_la_m}{2^3Y_{ijlm}}+\dots. \tag{9}$$

Assuming that the above series converges sufficiently rapidly, so that it approximately terminates after a finite number of terms, μ can be calculated numerically to match the total cross-section data at the given energy. Alternatively, we also calculate the total cross-section values for assumed values of μ at higher energies where neither total nor differential cross-section measurements are available.

3. Predictions on differential cross-sections

Available experimental data on the pion form factor are well represented by the familiar vector dominance formula

$$F_\pi(t)=1/[1+(|t|/m_\rho^2)]. \tag{10}$$

Throughout this paper we use all masses in GeV and t in GeV^2. The square of the deuteron charge form factor has been well approximated by the analytic representation (Parida 1979; Parida *et al* 1983),

$$F_d^2(t)=\frac{e_0\exp(-\alpha z)}{e_0+e_1t+e_2t^2+e_3t^3+h(t)+\frac{4}{3}m_\pi^2/\pi}, \tag{11}$$

where

$$z(t)=\{\ln([(-t/t_a)^{\frac{1}{2}}]+[(-t/t_a)+1]^{\frac{1}{2}})\}^2$$

$$h(t)=\frac{2}{\pi}\frac{k^3}{\sqrt{t}}\ln\left[\frac{k}{m_\pi}+\left(\frac{k^2}{m_\pi^2}+1\right)^{1/2}\right]-\frac{ik^3}{\sqrt{t}}$$

$$k=\left(\frac{t}{9}-m_\pi^2\right)^{1/2}$$

$$\begin{aligned}\alpha&=0\cdot858, & e_0&=0\cdot947\ \mathrm{GeV}^2,\\ e_1&=5\cdot772, & e_2&=111\cdot97\ \mathrm{GeV}^{-2},\\ e_3&=0\cdot188\ \mathrm{GeV}^{-4}, & t_a&=0\cdot033\ \mathrm{GeV}^2.\end{aligned} \tag{12}$$

Using formulas (10)–(12), we first obtain the curve for the function $F_\pi(t)F_d(t)$ as a function of $|t|$ in the spacelike region for $0\leqslant -t\leqslant 10\ \mathrm{GeV}^2$ and then fit this curve by formula (6). The best fit is shown in figure 1 and has the following parameters

$$\begin{aligned}a_1&=0\cdot647, & b_1&=15\cdot98\ \mathrm{GeV}^{-2},\\ a_2&=0\cdot320, & b_2&=7\cdot984\ \mathrm{GeV}^{-2},\\ a_3&=0\cdot018, & b_3&=1\cdot985\ \mathrm{GeV}^{-2},\\ a_4&=2\cdot7\times10^{-4}, & b_4&=0\cdot492\ \mathrm{GeV}^{-2}.\end{aligned} \tag{13}$$

In figure 1 the circles represent the product function $F(t)=F_\pi(t)F_d(t)$ as given by formulas (10)–(12) and the solid-tune is the fit by the formula (6) with the parameters given in (13). Visually, this appears to be an excellent fit. Since the experimental data on the deuteron (pion) form factor is available up to $|t|=6\ \mathrm{GeV}^2$ ($2\ \mathrm{GeV}^2$) and the formula (11) ((10)) fits them very well, we propose that the four-exponential fit given by (6) is a very good approximation to the product function at least upto $t=2\ \mathrm{GeV}^2$. To make the model calculations more reliable, our predictions on the differential cross-sections at various energies are confined to the region $|t|\lesssim 2\ \mathrm{GeV}^2$.

In order to calculate the differential cross-section, we have first calculated the interaction strength μ, for a given value of total cross-section using (9) with values of parameters given by (13). The value of μ and other parameters were then used in (1) and (7) to calculate the differential cross-sections. For our calculations we have retained the first five terms in (8) and (9) as the higher order terms were not found to be significant. For example, at the available laboratory momentum $P_{\mathrm{lab}}=9$ GeV/C (15·2 GeV/C) the

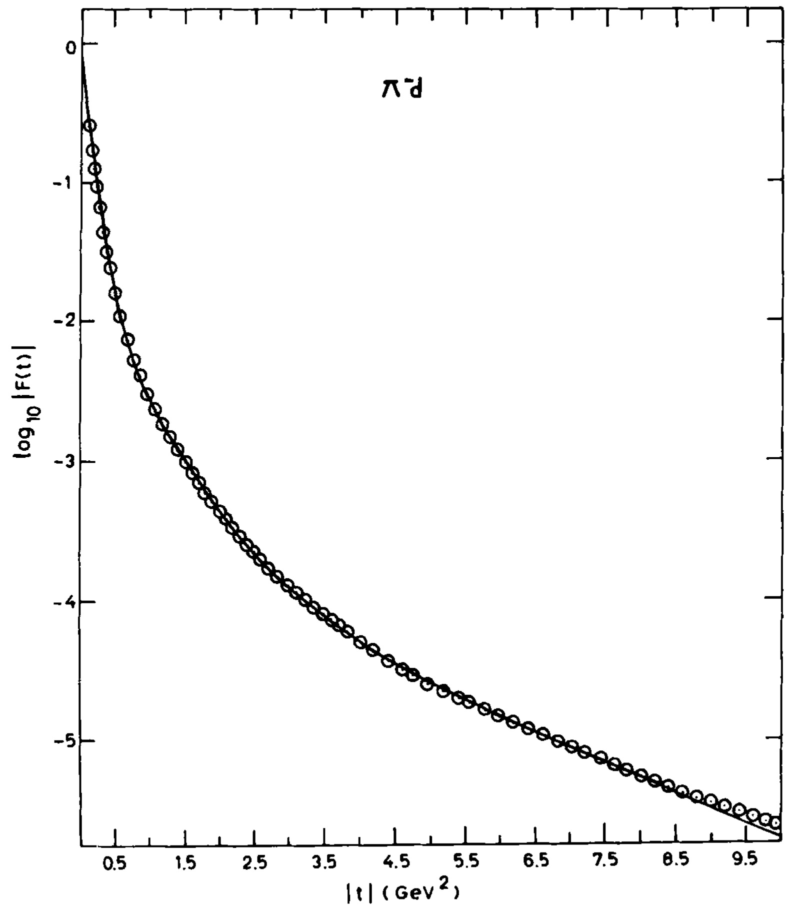

Figure 1. Demonstration of the agreement between the product function of the pion and the deuteron form factors and the four exponential approximation given by equation (6) with the parameters given in (13).

formula (9) yields $\mu = 11{\cdot}58\ \mathrm{GeV}^{-2}$ ($10{\cdot}76\ \mathrm{GeV}^{-2}$) corresponding to the measured value (Bradamante *et al* 1970) of total cross-section, $\sigma_T = 50{\cdot}0$ ($46{\cdot}8$) mb. The predicted values of the differential cross-section at each of these values as a function of $|t|$ are shown in figure 2 by the solid lines. Agreement with the experimental data at larger $|t|$ region with $|t| > 0{\cdot}4\ \mathrm{GeV}^2$ seems to be good. But the predicted values of $d\sigma/dt$ appear to lie 50–100% above the data in the diffraction peak region for $|t| < 0{\cdot}4\ \mathrm{GeV}^2$. Since the differential cross-section at $t = 0$ has been normalized to σ_{tot} via an optical theorem, the predicted values are expected to lie at or below the data points, depending upon whether or not the real part is negligible. Such seemingly contrasting behaviour between the data and the model predictions could be due to the possible presence of a slope-break around $-t \simeq 0{\cdot}1\ \mathrm{GeV}^2$ that the model fails to account. However, since the theory is essentially a high-energy model, we expect better agreement with the future data at higher energies ($P_{lab} \simeq 50$–100 GeV/C).

Calculated values of total cross-section for higher values of interaction strengths using (9) are presented in table 1 for which the available energy and cross-sections have

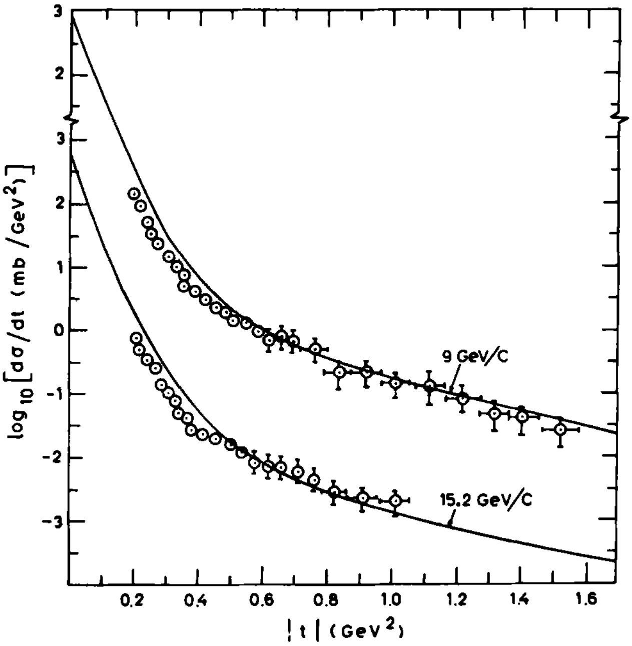

Figure 2. Comparison between Chou-Yang model predictions (solid lines) and the available experimental data (circles with dots) at $P_{lab} = 9$ GeV/C and 15·2 GeV/C. The data points for $|t| \gtrsim 1{\cdot}01$ GeV^2 are at $P_{lab} = 8{\cdot}9$ GeV/C. The unit of $d\sigma/dt$ is in mb/GeV^2, in figures 2–4.

Table 1. Total cross-section for $\pi^- d$ scattering as a function of interaction strength.

μ (GeV^{-2})	σ_T (mb)
10·75	46·80
11·58	50·00
15·00	62.81
17·50	73·44
20·00	80·22
25·00	96·23
30·00	111·03

not been experimentally measured. The predicted values of differential cross-section as a function of $|t|$ are shown in figure 3 for $\mu = 15{\cdot}0$, 17·5, 20·0 and 25 GeV^2, and also in figure 4 for $\mu = 30$ GeV^{-2}. We observe that only a shoulder appears in the differential cross-section for $\mu = 17{\cdot}5$–20 GeV^{-2} in the region $|t| = 0{\cdot}6$–$0{\cdot}8$ GeV^2, but a conspicuous dip structure appears for $\mu \gtrsim 25$ GeV^{-2}. As μ increases from 25 GeV^{-2} to 30 GeV^{-2},

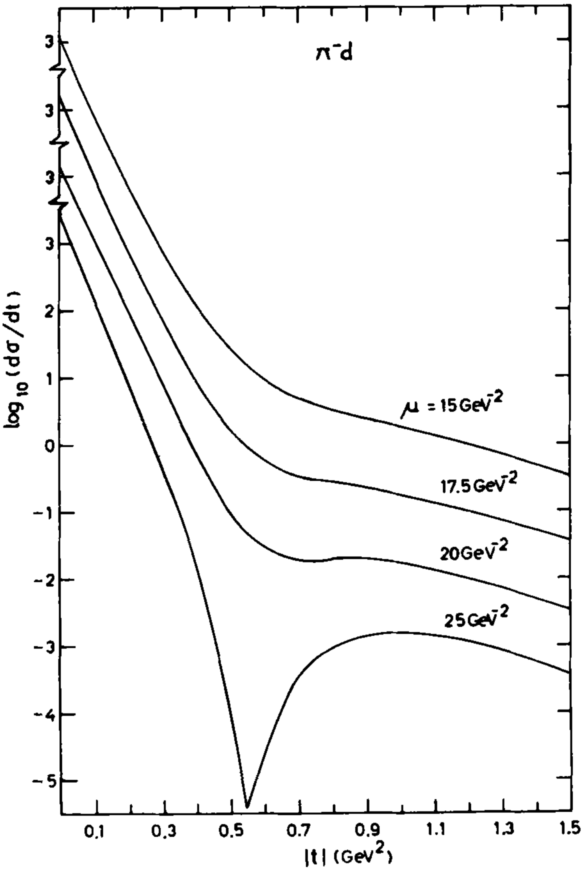

Figure 3. Predictions for the differential cross-sections and positions of shoulders, dips, and higher order maxima.

the diffraction peak shrinks and the dip moves from $|t_d| = 0{\cdot}55\ \mathrm{GeV}^2$ to $|t_d| = 0{\cdot}4\ \mathrm{GeV}^2$. With $\mu = 30\ \mathrm{GeV}^{-2}$, the differential cross-section, besides exhibiting the prominent first dip and the second maximum, also displays the presence of a prominent second dip at $t_d = 0{\cdot}7\ \mathrm{GeV}^2$ and a third maximum in the range $|t| = 0{\cdot}9$–$1{\cdot}1\ \mathrm{GeV}^2$, which is rather broad. As shown in table 1, our calculations for $\pi^- d$ scattering predict the increase of total cross-section from 62·8 mb to 111 mb with μ increasing between 15·0 and $30\ \mathrm{GeV}^{-2}$. A plot of the position of the first dip position as a function of total cross-section is shown in figure 5 which can be verified from the total and differential cross-section measurements at very high energies in future. The occurrence of the first dip at the highest position $|t_d|^{\max} \simeq 0{\cdot}5$–$0{\cdot}6\ \mathrm{GeV}^2$ for $\mu \simeq 22$–$25\ \mathrm{GeV}^{-2}$ is in sharp contrast to such positions in pp and $\bar{p}p$ scattering for which $|t_d|^{\max} \gtrsim 1{\cdot}5\ \mathrm{GeV}^2$. Note that our earlier analysis revealed the Chou-Yang model prediction for pd scattering to be $|t_d|^{\max} \simeq 0{\cdot}35$–$0{\cdot}4\ \mathrm{GeV}^2$ which is also in contrast to pp, $\bar{p}p$ and $\pi^{\pm}p$ scattering and also somewhat lower than the model prediction for $\pi^- d$ scattering. Our predictions of differential cross-sections at such energies for which a dip appears are subject to the limitations, as stated in §§ 1 and 4. Using the analogy of the model predictions for πN and NN scattering with those in πd and pd scattering, it is possible to have a rough idea

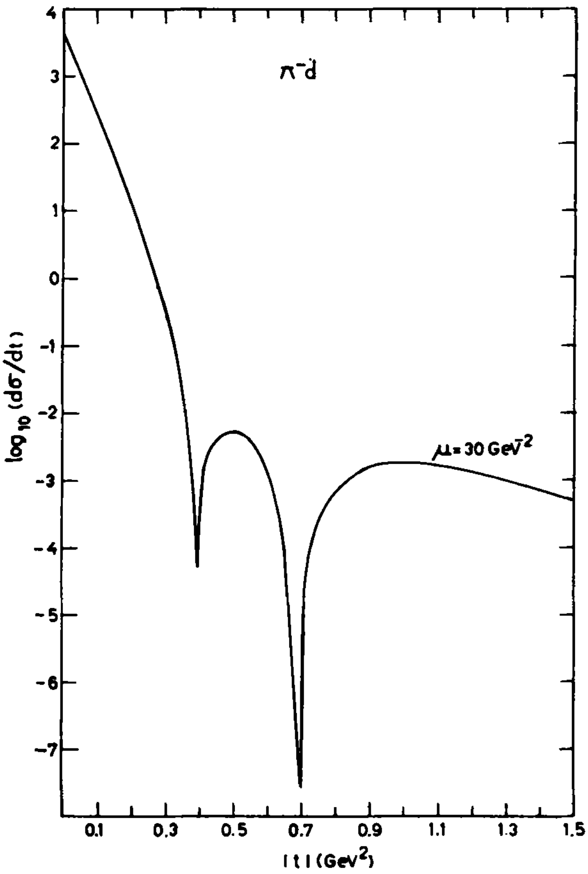

Figure 4. Same as figure 3 but for $\mu = 30\,\mathrm{GeV}^{-2}$.

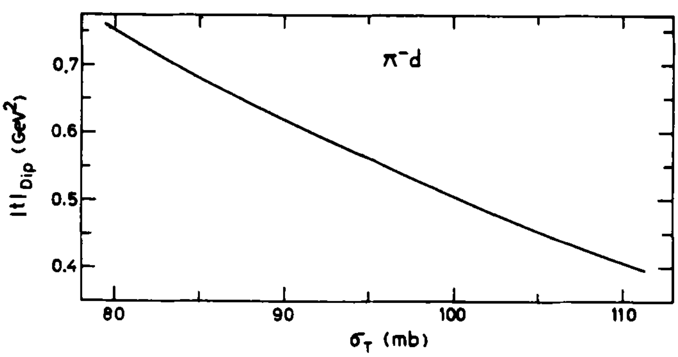

Figure 5. A plot of given values of total cross-sections and predicted first dip positions at very high energies.

about the approximate energy region for which a shoulder would continue to appear in the πd scattering. It is well known that within $|t| \lesssim 2\,\mathrm{GeV}^2$, the dip structure in $\pi^{\pm}p$ scattering does not set in earlier in energy scale than the pp scattering. One of the possible reasons for such a phenomenon in the model may be due to the fact that at a given high energy, the total cross-section in $\pi^{\pm}p$ is less (approximately two-third) than its corresponding value in pp scattering. If we expect a similar analogy to be valid between pd and πd scattering, a shoulder would continue to appear at least until $P_{\text{lab}} \simeq 400$ GeV/C since such a feature has been found among the model predictions for pd scattering carried out by us earlier (Parida and Patel 1981).

4. Summary, discussion and limitations

In this section we briefly summarize and discuss our results and point out limited range of validity of the model predictions in $\pi^{-}d$ scattering, in view of the existing critical analyses in pp and $\bar{p}p$ scattering (Amaldi and Schubert 1980; Fearnley 1985). Since the interpolating formulas for the pion and the deuteron electromagnetic form factors are found to reproduce the available form-factor data in the region $|t| \lesssim 2$ GeV^2, the model predictions are expected to be valid in the same region of $|t|$ in which the predicted positions of dips and maxima are found to be confined.

Available experimental data on the differential cross-sections at $P_{\text{lab}} = 9{\cdot}0$ and 15·2 GeV/C agree with the model predictions reasonably well for $0{\cdot}4 \lesssim |t| \lesssim 1{\cdot}5$ GeV^2; but in the diffraction peak region, when $|t| < 0{\cdot}4$ GeV^2, the predicted cross-section seems to be 50–100% more than the experimental data. Since we have ignored the real part effects completely, the model prediction is expected to be the same as, or less than the available data in the diffraction peak region, depending upon whether the real part effects are negligible, or significant, for such energies. Such type of apparently contrasting behaviour can be understood if the experimental cross-section has a possible slope break around $-t \simeq 0{\cdot}1$ GeV^2, for which the model furnishes no exact explanation. Further, by noting that the Chou-Yang model is meant to apply at higher energies, we expect better agreement between the model predictions and the future experimental data for $P_{\text{lab}} \simeq 50$–100 GeV/C. We predict that a shoulder in the differential cross-section would continue to appear in the region $-t \simeq 0{\cdot}6$–$0{\cdot}8$ GeV^2 until such energies for which $\mu \simeq 20$ GeV^{-2} and $\sigma_T \simeq 80$ mb; but a conspicuous dip structure would appear only for $\mu \gtrsim 25$ GeV^{-2}, or $\sigma_T \gtrsim 90$ mb. As μ increases from 25 GeV^{-2} to 30 GeV^{-2} with increasing energy, the diffraction peak is predicted to shrink and the dip is predicted to move from $|t_d| \simeq 0{\cdot}55$ GeV^2 to $|t_d| \simeq 0{\cdot}4$ GeV^2. These features of the dip in respect of the energy and the position, at which it is expected to appear, are in sharp contrast to pp scattering where the first appearance of the dip takes place at $P_{\text{lab}} \simeq 50$–100 GeV/C and $|t_d| \lesssim 1{\cdot}5$ GeV^2. But the highest dip position in $\pi^{-}d$ scattering is larger than the corresponding one in pd scattering, for which, using the Chou-Yang model, we predicted $|t_d|^{\max} \simeq 0{\cdot}35$–$0{\cdot}4$ GeV^2 (Parida and Patel 1981). At present, it is possible to predict the appearance and position of a dip as a function of μ or σ_{tot}, but not as a function of energy, since measurements of σ_{tot} at sufficiently high energies are not available.

As discussed in § 1, detailed analyses of the pp and $\bar{p}p$ differential cross-section data at the ISR and CERN collider energies have revealed that the Chou-Yang model fails to account for the observed variation of the differential cross-section near and away from the dip region. Also it fails to account for the observed slope break at $|t| = 0{\cdot}1$ GeV^2 in pp scattering. Despite these and other failures, the gross features of the forward peak upto $|t| \simeq 1{\cdot}1$ GeV^2 in pp at the ISR energies and $|t| \simeq 0{\cdot}75$ GeV^2 in $\bar{p}p$ at the collider energies are described by the model reasonably well. Also, as noted by Chou and Yang

(1979), the positions of the dip, as a function of total cross-section or energy, are predicted and found to agree reasonably well with the observed values in *pp* at the ISR energies. For $P_{lab} \lesssim 400$ GeV/C, however, the agreement of the model predictions with the high energy data in the forward peak and dip regions have been found to be good. In the high-energy hadron nucleus scattering, expecting similar successes and failures, we note that the future experimental data on $\pi^- d$ scattering would agree with our predictions upto at least $P_{lab} \simeq 500$ GeV/C. For $P_{lab} \gg 500$ GeV/C, disagreements may appear in the region $|t| \gtrsim 0{\cdot}5$ GeV^2 when the dip and the secondary maximum are conspicuous.

Acknowledgement

The authors are thankful to the Computer Centre, Utkal University for help in computation.

References

Amaldi U and Schubert K R 1980 *Nucl. Phys.* **B166** 301
Bradamante F *et al* 1970 *Phys. Lett.* **B31** 87
Chan S C, Lo S T, Low H B and Phua K K 1978 *Phys. Rev.* **D17** 802
Chou T T and Yang C N 1967 *High energy physics and nuclear structure* (ed.) G Alexander (Amsterdam: North Holland) 348
Chou T T and Yang C N 1968a *Phys. Rev. Lett.* **20** 1213
Chou T T and Yang C N 1968b *Phys. Rev.* **170** 1591
Chou T T and Yang C N 1979 *Phys. Rev.* **D19** 3268
Chou T T and Yang C N 1983 *Phys. Lett.* **B128** 457
Clarke D J and Lo S Y 1974 *Phys. Rev.* **D10** 1519
Durand L and Lipes R 1968 *Phys. Rev. Lett.* **20** 637
Fearnley T 1985 CERN Report EP/85-137
Hayot F and Sukhatme U P 1974 *Phys. Rev.* **D17** 802
Kac M 1973 *Nucl. Phys.* **B62** 402
Parida M K 1979 *Phys. Rev.* **D19** 3320
Parida M K and Patel S 1981 *Phys. Rev.* **D24** 1865
Parida M K, Patel S and Agrawalla K N 1983 *Phys. Rev.* **D27** 1187