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Weak electron acoustic double layers in a multicomponent plasma

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Abstract. Formation of electron acoustic double layers in a magneto-plasma with two ion species is investigated. The existence of double layers propagating almost perpendicular to the magnetic field in a plasma with two distinct ion species and cold electron is discussed.

Keywords. Electron acoustic double layer; multicomponent plasma; single-ion species, two-ion species.

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1. Introduction

A large number of theoretical (Block 1972; Torven 1981), computer simulation (Hubbard and Joyce 1979; Kim and Crystal 1984) and experimental (Coakley and Hershkowtiz 1979; Temerin *et al* 1982) investigations have been carried out on the double layers. Recently Kim (1983) and Schamel (1983) showed that slow electron acoustic double layers (SEADL) and slow ion acoustic double layers (SIADL) can be constructed considering the effects of reflected electrons and reflected ions. However, Goswami and Bujarbarua (1985, 1986a) have shown that small amplitude ion acoustic double layers can be constructed without considering the effects of reflected particles. But in such a situation at least two types of free electrons (each described by a Maxwellian distribution) are necessary to construct such double layers. From the earlier theory of double layers (DL) it has been found that they are generally associated with the ion acoustic branch, while theoretical studies of electron acoustic solitary waves, propagating almost perpendicular to the magnetic field in a plasma with ion temperature very much larger than the electron temperature, have been performed extensively (Arefev 1970; Goedbloed *et al* 1973). Buti (1980) investigated an exact nonlinear electron acoustic waves in a multicomponent plasma and showed the effects of second ion components. With this idea in mind, the existence of small amplitude electron acoustic double layers or shocks in plasma has been investigated by Goswami *et al* (1986).

In this paper, we study the existence of small amplitude electron acoustic double layers or shocks in a multicomponent plasma. Using fluid equation for electrons and describing ion components by the Maxwell-Boltzmann relation, i.e. the free particle temperature and the reflected particle temperature of the two-ion species to be equal, we first describe the method of finding solutions with a potential ϕ that varies monotonically from a minimum value, zero to maximum value ψ (amplitude of the double layer). We then describe the classical potential $V(\phi)$ which is obtained by integrating the densities with respect to ϕ. The existence of a double layer solution demands that the classical potential $V(\phi)$ and the electric field $\partial V/\partial\phi$ vanish at two extremum of ϕ i.e. $\phi = 0$ and $\phi = \psi$. The variation of classical potential $V(\phi)$ for a double layer against its potential ϕ is shown in figure 1.

In § 2, we discuss the DL solution in the presence of a single Boltzmann ion. In § 3, we obtain the DL solutions in the presence of two positive ion species (each described by a Maxwellian-Boltzmann distribution). In § 4, we obtain the DL solution in the presence of both positive and negative ions (each described by a Maxwellian-Boltzmann distribution) and § 5 contains a discussion and conclusion of our results.

2. Single ion species

We consider a homogeneous plasma with electrons and ions in the presence of external magnetic field $\vec{B} = B_0\hat{z}$ in the $\hat{Z}$ direction. We assume that $T_i > T_e$, where T_i and T_e are the ion temperature and the electron temperature respectively. Here the electron acoustic waves are assumed to propagate almost perpendicular to the magnetic field and the direction of propagation lie on the y-z plane. The electrons are governed by the following set of fluid equations

$$\partial_t n + \partial_y(nv_y) + \partial_z(nv_z) = 0, \tag{1}$$

$$\partial_t v_x + (v_y\partial_y + v_z\partial_z)v_x = -v_y, \tag{2}$$

$$\partial_t v_y + (v_y\partial_y + v_z\partial_z)v_y = v_x + \partial_y\phi, \tag{3}$$

$$\partial_t v_z + (v_y\partial_y + v_z\partial_z)v_z = \partial_z\phi. \tag{4}$$

In equations (1) to (4), we have normalized the densities by the equilibrium density n_0, the velocities by the electron acoustic speed $C_e = (T_i/m_e)^{1/2}$, the potential ϕ by T_i/e, the space variables by $\rho = C_e/\Omega_e$, and the time by the inverse of electron gyro-frequency $\Omega_e = eB_0/m_e$ and the others are the standard notations.

We consider a new frame of reference which is moving with velocity M with respect to the rest frame in the direction of the wave. In this frame the space co-ordinate is given by, $\xi = k_y y + k_z z - Mt$ where $k_y^2 + k_z^2 = 1$ and k_y, k_z are the direction cosines.

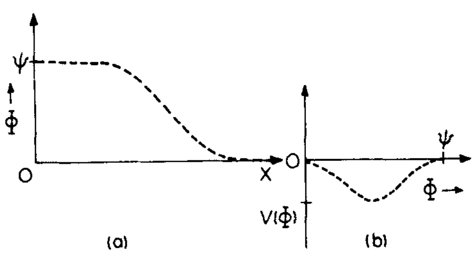

Figure 1. (a) A qualitative plot of the electrical potential $\phi(x)$ of a monotonic double layers as a function of x. (b) The corresponding classical potential $v(\phi)$ as a function of ϕ.

Therefore (1) to (4) can be written as

$$(k_y v_y + k_z v_z) = M\left(1 - \frac{1}{n}\right), \tag{5}$$

$$\frac{M}{n}\partial_\xi v_x = -v_y, \tag{6}$$

$$-\frac{M}{n}\partial_\xi v_y = v_x + k_y \partial_\xi \phi, \tag{7}$$

$$-\frac{M}{n}\partial_\xi v_z = k_z \partial_\xi \phi. \tag{8}$$

Integrating (8), we get

$$v_z = -\frac{k_z}{M} I(\phi), \tag{9}$$

where $$I(\phi) = \int_0^\phi n\,\mathrm{d}\phi. \tag{10}$$

Substituting v_y from (5) and using (8), we get

$$v_x = \frac{1}{k_y}\left[\left(-1 - \frac{M^2}{n^3}\frac{\partial n}{\partial \phi}\right)\frac{\partial \phi}{\partial \xi}\right]. \tag{11}$$

Substituting (11) and (5) into (6) and combining with (9), we get

$$\frac{1}{2}\partial_\phi\left(P\frac{\partial \phi}{\partial \xi}\right)^2 = P\left[\frac{k_z^2}{M^2} nI(\phi) + n - 1\right] = -\frac{\partial V}{\partial \phi}, \tag{12}$$

where $V(\phi)$ is the classical potential and

$$P = -1 - (M^2/n^3)\,\partial_\phi n. \tag{13}$$

Since for electron acoustic waves, the frequencies are much larger than the ion cyclotron frequency and the wavelengths are much smaller than the ion Larmour radius, the ions may be considered as unmagnetized and we assume that the ion density n_i obey the Boltzmann relation (Mohan and Buti 1980)

$$n_i = \exp(-\phi). \tag{14}$$

We consider small amplitude waves and assume a quasineutral condition which is valid when the electron Debye length is smaller than the electron gyro-radius. Therefore the ion density n_i may be substituted for n in equation (12).

For small ϕ, the ion density can be written as

$$n_i = 1 - \phi + \tfrac{1}{2}\phi^2 - \tfrac{1}{6}\phi^3 + \cdots. \tag{15}$$

To the lowest order, we can easily write from (13) and (15) i.e. $P \simeq -1 + M^2$. Inserting the value of n from (15) into (12), we get

$$P[B_1\phi + B_2\phi^2 + B_3\phi^3] \equiv -\partial_\phi V, \tag{16}$$

where
$$B_1 = \left(\frac{k_z^2}{M^2} - 1\right), \tag{17a}$$

$$B_2 = \left(\frac{1}{2} - \frac{3}{2}\frac{k_z^2}{M^2}\right), \tag{17b}$$

and
$$B_3 = \left(\frac{7}{6}\frac{k_z^2}{M^2} - \frac{1}{6}\right). \tag{17c}$$

Integrating (16) and imposing the boundary condition $V(0) = 0$, we get the classical potential as

$$-V(\phi) = P[\tfrac{1}{2}B_1\phi^2 + \tfrac{1}{3}B_2\phi^3 + \tfrac{1}{4}B_3\phi^4]. \tag{18}$$

The nonlinear dispersion relation is obtained by imposing the boundary condition $V(\phi) = 0$ at $\phi = \psi$ in (18) and we get

$$B_1 = -\tfrac{2}{3}B_2\psi - \tfrac{1}{2}B_3\psi^2. \tag{19}$$

To get the DL solution we introduce another boundary condition i.e. $\partial V/\partial\phi = 0$ at $\phi = \psi$ and we get from (16),

$$B_1 = -B_2\psi - B_3\psi^2. \tag{20}$$

Equating (19) and (20), we get

$$B_1 = \tfrac{1}{2}B_3\psi^2, \tag{21a}$$

$$B_2 = -\tfrac{3}{2}B_3\psi. \tag{21b}$$

From (21a), (21b) and (18), we get

$$-V(\phi) = \frac{B_3 P}{4}\phi^2(\phi - \psi)^2. \tag{22}$$

Using (22), the potential of the double layer can be written from (12) as (Goswami and Bujarbarua 1985)

$$\phi = \frac{\psi}{2}(1 - \tanh\kappa\xi) \quad \text{with} \quad \kappa = (B_3/8P)^{1/2}\psi. \tag{23}$$

Equation (23) represents a DL provided $B_3 < 0$ (since P is negative). The velocity of this type of DL can be calculated from (17) and (21) and is given by

$$M = k_z\left(1 - \frac{1}{3}\frac{\psi}{2}\right). \tag{24}$$

However it is found from (17c) that $B_3 > 0$. Hence the electron acoustic double layer with single species of ions and electrons does not exist.

3. Two positive ions

We now consider the case when two isothermal positive ions are present in the system. If the gyroperiods and Larmor radii of both the ion species are much larger than the wave period and the wavelength, the ions can be taken as unmagnetized and they have a Boltzmann type distribution (Buti 1980). Therefore the total ion density can be written as

$$n = \alpha_1 \exp(-a_1\phi) + \alpha_2 \exp(-a_2\phi), \tag{25}$$

where $\alpha_1 = n_1/n_0$, $\alpha_2 = n_2/n_0$, $a_1 = T_{if}/T_1$ and $a_2 = T_{if}/T_2$ and n_1, n_2 are the densities and T_1, T_2 are the temperatures of the positive ions. T_{if} is the effective ion temperature i.e. $T_{if} = T_1T_2/(\alpha_1T_2 + \alpha_2T_1)$.

In a small amplitude limit, the total ion density can be written as,

$$n = 1 - A_1\phi + \tfrac{1}{2}A_2\phi^2 - \tfrac{1}{6}A_3\phi^3 + \cdots, \tag{26}$$

where

$$A_1 = 1, \tag{27a}$$

$$A_2 = (\alpha_1 + \alpha_2 T_1^2/T_2^2)/(\alpha_1 + \alpha_2 T_1/T_2)^2, \tag{27b}$$

and

$$A_3 = (\alpha_1 + \alpha_2 T_1^3/T_2^3)/(\alpha_1 + \alpha_2 T_1/T_2)^3. \tag{27c}$$

Substituting the value of n from (26) into (12), we get (16) where B_1, B_2 and B_3 are as follows,

$$B_1 = \left(\frac{k_z^2}{M^2} - A_1\right), \tag{28a}$$

$$B_2 = \left(\frac{1}{2} - \frac{3}{2}\frac{k_z^2}{M^2}\right), \tag{28b}$$

$$B_3 = \left[\frac{k_z^2}{M^2}\left\{\frac{2}{3}A_2 + \frac{1}{2}\right\} - \frac{A_3}{6}\right]. \tag{28c}$$

The potential of this type of DL is given by (23) when the condition $B_3 < 0$ is satisfied, and the velocity of such a type of DL is given by

$$M = k_z\left\{1 - \frac{1}{6}\left[\frac{3}{2} - \frac{1}{2}\left(\frac{\alpha_1 + \alpha_2 T_1^2/T_2^2}{(\alpha_1 + \alpha_2 T_1/T_2)^2}\right)\right]\psi\right\}. \tag{29}$$

It can be shown that in the presence of two isothermal positive ions, the value of B_3 will be negative for certain values of A_2 and A_3. For example (a) $\alpha_1 = 0{\cdot}95$, $T_1/T_2 = 10$, $A_2 = 2{\cdot}83$, $A_3 = 16{\cdot}7$, $B_2 = -0{\cdot}085$ and $B_3 = -0{\cdot}396$, (b) $\alpha_1 = 0{\cdot}95$, $T_1/T_2 = 20$, $A_2 = 5{\cdot}51$, $A_3 = 54{\cdot}11$, $B_2 = 1{\cdot}255$, $B_3 = -4{\cdot}845$. Since for case (a) both B_2 and B_3 are negative and to satisfy (21b), $\psi(\phi)$ has to be negative in this case corresponding to rarefactive DL. For case (b) B_2 is positive and B_3 is negative and in this case we get the compressive DL. Hence we may conclude that for moderate values of T_1/T_2, the ratio of two positive ions, electron acoustic double layers can exist when the cold positive ion concentration i.e. α_2 is very low.

4. Presence of negative ions

Next we consider the case when both positive and negative ions are present in the system. In this case, the net positive ion density can be written as

$$n = \frac{1}{1-\alpha}\exp(-a_1\phi) - \frac{\alpha}{1-\alpha}\exp(a_2\phi), \tag{30}$$

where $\alpha = n_{j0}/n_{i0}$, $a_1 = T_{if}/T_i$ and $a_2 = T_{if}/T_j$ and the subscripts i and j denote the positive and negative ions respectively. Here T_{if}, the effective ion temperature is given by,

$$T_{if} = T_iT_j \bigg/ \left(\frac{1}{1-\alpha}T_j + \frac{\alpha}{1-\alpha}T_i\right).$$

In the small amplitude limit, the expansion of (30) is the same as (26), the quantities A_1, A_2 and A_3 are given by

$$A_1 = 1, \tag{31a}$$

$$A_2 = \left(\frac{1}{1-\alpha} - \frac{\alpha}{1-\alpha}\frac{T_i^2}{T_j^2}\right)\left(\frac{1}{1-\alpha} + \frac{\alpha}{1-\alpha}\frac{T_i^2}{T_j}\right)^{-2}, \tag{31b}$$

$$A_3 = \left(\frac{1}{1-\alpha} + \frac{\alpha}{1-\alpha}\frac{T_i^3}{T_j^3}\right)\left(\frac{1}{1-\alpha} + \frac{\alpha}{1-\alpha}\frac{T_i^2}{T_j}\right)^{-3}. \tag{31c}$$

Applying the same procedure as in the case of two positive ion species, we obtain (16), where B_1, B_2 and B_3 are given by (28a) to (28c). The potential of DL is given by (23) with the condition $B_3 < 0$ and the velocity of this type of double layer is given by

$$M = k_z\left[1 - \frac{1}{6}\left\{\frac{3}{2} - \frac{1}{2}\left(\frac{\left(\frac{1}{1-\alpha} - \frac{\alpha}{1-\alpha}\frac{T_i}{T_j}\right)}{\left(\frac{1}{1-\alpha} + \frac{\alpha}{1-\alpha}\frac{T_i}{T_j}\right)^2}\right)\right\}\psi\right]. \tag{32}$$

Let us consider the following examples (a) $\alpha = 0{\cdot}1$, $T_i/T_j = 4$, $A_2 = -0{\cdot}275$, $A_3 = 2{\cdot}17$, $B_2 = -1{\cdot}6375$ and $B_3 = -0{\cdot}0467$; (b) $\alpha = 0{\cdot}1$, $T_i/T_j = 10$, $A_2 = -2{\cdot}02$, $A_3 = 10{\cdot}2$, $B_2 = -2{\cdot}51$ and $B_3 = -2{\cdot}54$. From the above examples cases (a) and (b) corresponding to the moderate values of negative ion concentration α and the ratio of positive to the negative ion temperatures, T_i/T_j. It has been found that both B_2 and B_3 are negative. Thus we conclude that the rarefactive electron acoustic double layers can exist when the densities and the temperature of positive ions are greater than the negative ions.

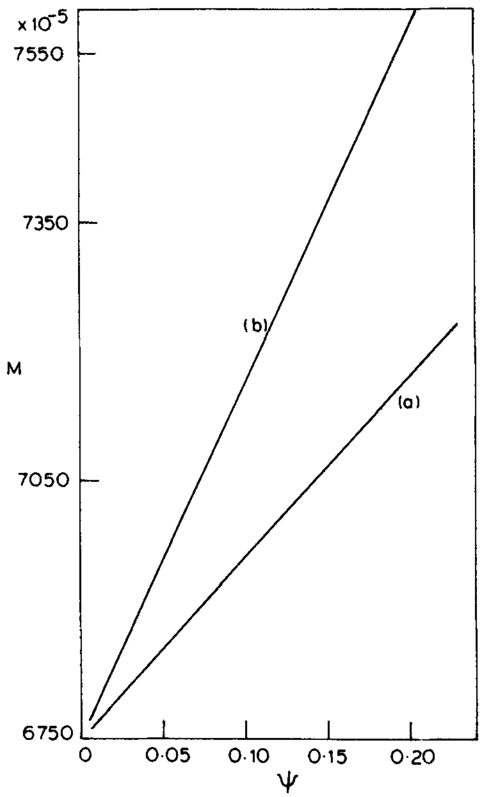

Figure 2. Variation of the double layer velocity against its amplitude. (a) $\alpha_1 = 0{\cdot}95$, $\alpha_2 = 0{\cdot}05$, $k_z = 0{\cdot}09$ and $T_1/T_2 = 10$. (b) $\alpha_1 = 0{\cdot}95$, $\alpha_2 = 0{\cdot}05$, $k_z = 0.09$ and $T_1/T_2 = 20$.

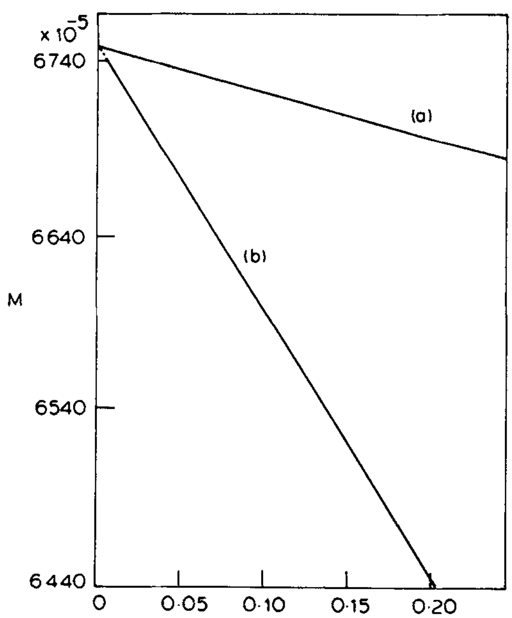

Figure 3. Variation of the double layer velocity against its amplitude. (a) $\alpha = 0{\cdot}1$, $k_z = 0{\cdot}09$ and $T_i/T_j = 4$. (b) $\alpha = 0{\cdot}1$, $k_z = 0{\cdot}09$ and $T_i/T_j = 10$.

5. Discussion

We have studied electron acoustic double layers in three different cases viz (a) single positive ions (b) two positive ions (c) positive and negative ions. It has been shown that for case (a) no DL solutions exist. This result agrees with the results on ion acoustic double layers (Goswami and Bujarbarua 1985, 1986) that no weak ion acoustic double layers solution exist in a plasma containing one free electron and one ion species. In the present paper, we have shown that electron acoustic DL or shock solution exists in a plasma consisting of two types of positive ions each described by a Boltzmann relation, that the concentration of colder ions species is smaller than the hotter one and that both compressive and rarefactive DL solutions are also possible in this case. We have also studied the effects of negative ions on the electron acoustic double layers and it has been shown that when the concentration and temperature of the negative ions are smaller than the positive ions, such DL solution exists.

The results of our investigation are shown graphically in figures 2 to 5. In figure 2, we have plotted the velocity of the double layer against its amplitude, for fixed values of positive ion densities and k_z and for two different values of T_1/T_2. Similarly in figure 3, we have plotted the velocity of the DLs against its amplitude for fixed values of positive

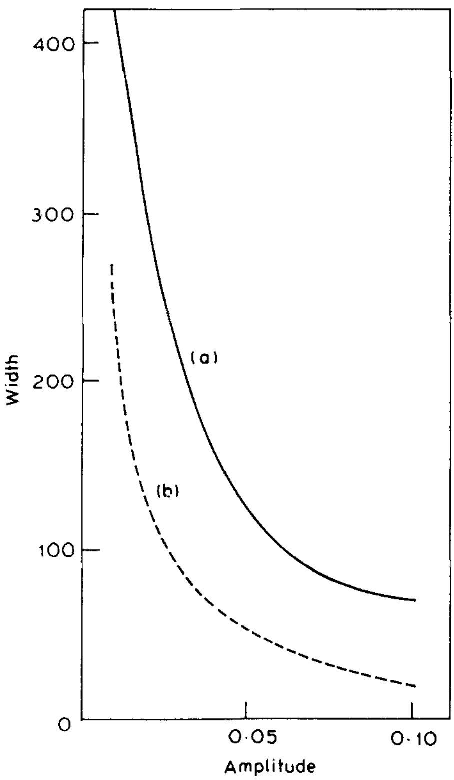

Figure 4. Width of the double layer plotted against its amplitude. (a) $\alpha_1 = 0{\cdot}95$, $\alpha_2 = 0{\cdot}05$, $k_z = 0{\cdot}09$, $M = 0{\cdot}09$ and $T_1/T_2 = 10$ and (b) $\alpha_1 = 0{\cdot}95$, $\alpha_2 = 0{\cdot}05$, $k_z = 0{\cdot}09$, $M = 0{\cdot}09$ and $T_1/T_2 = 20$.

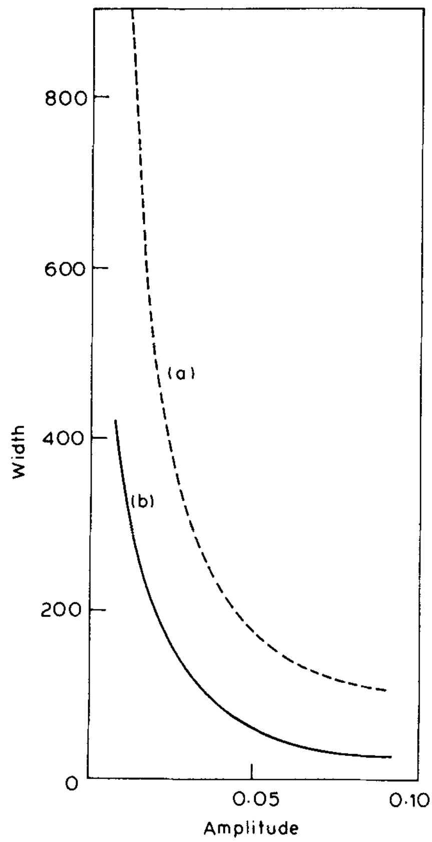

Figure 5. Width of double layer plotted against its amplitude. (a) $\alpha = 0{\cdot}1$, $k_z = 0{\cdot}09$, $M = 0{\cdot}09$ and $T_i/T_j = 4$ and (b) $\alpha = 0{\cdot}1$, $k_z = 0{\cdot}09$, $M = 0{\cdot}09$ and $T_i/T_j = 10$.

and negative ion densities and k_z and for two different values of T_i/T_j. It is seen that in the presence of two positive ions, the velocity of the double layer increases with increases of its amplitude (figure 2). But in the presence of both positive and negative ions, the velocity of the double layers decreases with increase of its amplitude (figure 3).

However, the width of the double layer decreases rapidly as the amplitude increases and then becomes nearly constant as the amplitude increases still further. The behaviour of the variation of the width with its amplitude for fixed values of temperature ratio is shown in figures 4 and 5.

Summarizing, we conclude that both rarefactive and compressive electron acoustic double layers or shocks exist in a multicomponent plasma depending upon the concentration of the different species of ions.

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