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Production of tachyons and antiparticles in extended manifolds of general relativity

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Abstract. The extended space-time manifold in a uniformly accelerating reference frame is considered both for positive and negative accelerations. An analogy between the light barrier and a black hole event horizon in the theory of relativity is drawn. It is shown that bradyon-tachyon-antibradyon transformations are possible in the proper reference frame by a constant acceleration, i.e. for the light barrier penetration.

Keywords. Tachyon; antiparticle; accelerating motion; black hole.

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1. Introduction

In most studies on special and general relativity (GR) there exist similar peculiarities, a light barrier and an event horizon, the occurrence of which is associated with the principle of short-range interaction which is common for both the theories, i.e. the finite fundamental speed of propagation of interaction. The expressions connecting the proper with the coordinate time and distance are the following for uniform motion and the Schwarzschild gravitational field, respectively (Misner *et al* 1973):

$$t = t_0(1 - V^2/c^2)^{-1/2}, \quad l = l_0(1 - V^2/C^2)^{1/2}, \tag{1}$$

$$t = t_0(1 - 2GM/rc^2)^{1/2}, \quad l = l_0(1 - 2GM/rc^2)^{-1/2}, \tag{2}$$

where t_0 and l_0 are the proper time and the proper distance. The first pair of the equations for $V = c$ and the second pair for $r = r_g = 2GM/c^2$ (G is the gravitational constant and c is the light speed) become singular. For $V > c$ and $r < r_g$ all the left-hand values become imaginary. If we pass over to the non-relativistic limit, $c \to \infty$, then the singularities will disappear together with the division of the space-time into subluminal and superluminal regions as well as into regions over the horizon and under it. Thus, non-Euclidean topology of space-time (pseudo-singular surfaces, the higher dimensionality, etc) in relativity is conditioned by the finiteness of the fundamental speed (Trofimenko 1984).

Superluminal objects are obtained in extended (special) relativity through the generalization of the Lorentz transformations (Recami and Mignani 1974; Pavšič and Recami 1977; Recami and Rodrigues 1982) forming the generalized Lorentz group

(Recami and Mignani 1974)

$$G = \{+\Lambda_<\} \cup \{-\Lambda_<\} \cup \{+i\Lambda_>\} \cup \{-i\Lambda_>\}. \quad (3)$$

Here the sets $\{\pm\Lambda_<\}$ are proper orthochronous and nonorthochronous transformations and $\{\pm i\Lambda_>\}$ are superluminal ones. The above studies extend the usual Lorentz group by including the CPT-inverse and the operation of the replacement of V by the dual V^{-1} with instantaneous multiplication by $i = (-1)^{1/2}$.

The above four sets of transformations make the transitions between an original frame and a frame moving with respect to it with the arbitrary velocity ($\lessgtr c$). Through two consequent superluminal transformations, one can come to inverted frames corresponding to the antiobjects. Thus, extended relativity divides all the variety of objects into four separate classes: bradyons, tachyons, antibradyons and antitachyons. In the superluminal reference frame tachyons are bradyons (the duality principle) and with respect to bradyons, which are in the superluminal reference frame, antiparticles will be tachyons, i.e. double orthochronous transformations lead to inversion. Thus in order to become an antiparticle a bradyon has to become a tachyon: antiparticles are twice tachyonic objects (Trofimenko 1984).

In this paper we consider the extended manifold construction procedure in the reference frame of a uniformly accelerating observer. The possibility of transition through motion in the proper reference frame with intersection of pseudosingularities from bradyons to tachyons and to antiobjects is shown. An object passing beyond the event horizon in the proper reference frame can pass to a superluminal region, i.e. it can become a tachyonic object, with another superluminal transition, namely transition beyond the second event horizon when it becomes an antiobject. Such transitions are possible also in the Kerr extended manifold.

2. Extended manifold of a uniformly accelerating observer

Let us consider the geometry of the reference frame for an observer moving with a constant acceleration $A(>0)$ or deceleration $A(<0)$ (Kinnersley and Walker 1970; Gibbons and Perry 1980):

$$\mathrm{d}s^2 = (1 + Ar\cos\theta)^{-2}[(1 - A^2r^2)\mathrm{d}t^2 - (1 - A^2r^2)^{-1}\mathrm{d}r^2 - r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2)], \quad (4)$$

where r is the distance from the observer, the origin of the coordinate system which is associated with the accelerating object. We use here the geometrized units where $c = G = 1$.

We consider here only the conformal structure of this spacetime, which is sufficient for the present work. This metric is very similar to the well-known de Sitter metric with constant curvature (see Tolman 1964). It takes the de Sitter form in the equatorial plane $\theta = \pi/2$ and in the renotation of the constant, $A^2 = \Lambda/3$, where Λ is the cosmological constant. In such a spacetime manifold (STM) the surfaces $r = \pm A^{-1}$ are the horizons. The de Sitter metric extension beyond the horizon was performed by Graves and Brill (1960), Geyer (1980) and Gautreau (1983). We shall apply this analysis to the metric of a uniformly accelerating observer and demonstrate the possibility of motion across the horizon surfaces $r = \pm A^{-1}$.

In order to remove the pseudosingularities at horizons in the metric (4) we adopt the new coordinates (Graves and Brill 1960)

$$u = 2C\exp(\gamma r^*)\cosh(\gamma t), \quad v = 2C\exp(\gamma r^*)\sinh(\gamma t) \tag{5}$$

with the ranges $-\infty < u < +\infty$, $-\infty < v < +\infty$, where C and γ are constants; then the metric gets the form

$$ds^2 = (1 + Ar\cos\theta)^{-2}[f^2(u,v)(dv^2 - du^2) - r^2(d\theta^2 + \sin^2\theta d\varphi^2)]. \tag{6}$$

Here we define

$$f^2(u,v) = \frac{1}{4}C^{-2}\gamma^{-2}(1 - A^2r^2)\exp(-2\gamma r^*), \tag{7}$$

and
$$r^* = \int(1 - A^2r^2)dr = \frac{1}{2}\log[(1 + Ar)/(1 - Ar)]. \tag{8}$$

We can easily obtain

$$f^2(u,v) = \frac{1}{4}C^{-2}\gamma^{-2}(1 + Ar)^{1-\gamma}(1 - Ar)^{1+\gamma}. \tag{9}$$

The metric of a uniformly accelerating observer (4) will not be singular at the horizons $r = \pm A^{-1}$ if the parameters are constrained within certain conditions. We choose $\gamma = \pm 1$ for $A \lessgtr 0$, respectively, and C is an arbitrary constant. Then

$$\begin{aligned} f^2 &= (1 + Ar)^2/4C^2 \quad \text{for} \quad A > 0 \ (\cos\theta > 0), \\ f^2 &= (1 - Ar)^2/4C^2 \quad \text{for} \quad A < 0 \ (\cos\theta < 0). \end{aligned} \tag{10}$$

Therefore, the new non-singular space-time coordinates are connected with (t, r)-ones by the following formulas:

$$\begin{aligned} u &= C[(1 \mp Ar)/(1 \pm Ar)]^{1/2}\cosh(\mp At) \quad \text{for} \quad A \gtrless 0, \\ v &= C[(1 \mp Ar)/(1 \pm Ar)]^{1/2}\sinh(\mp At) \quad \text{for} \quad A \gtrless 0. \end{aligned} \tag{11}$$

The inverse transformations are

$$t = \mp(1/A)\cosh^{-1}(u/v), \tag{12}$$

$$r = \pm(1/A)(C^2 - u^2 + v^2)(C^2 + u^2 - v^2)^{-1}. \tag{13}$$

With the help of the set of correlations the STM are represented as the Kruskal-like diagram in the (u, v)-coordinates. For both the $A > 0$ (acceleration) and $A < 0$ (deceleration) cases these space-times are drawn as the same diagrams (figure 1).

The compact representation of the conformal space-time structure can be done with the help of Penrose-like diagrams by transforming (u, v)-coordinates to (p, q)-ones according to the formulas

$$u = \tan p, \quad v = \tan q, \tag{14}$$

with the ranges $-\pi/2 \leqslant q \leqslant +\pi/2$ and $-\pi/2 \leqslant p \leqslant +\pi/2$ (Misner *et al* 1973). In figure 2 two cases of STM's under consideration are shown separately.

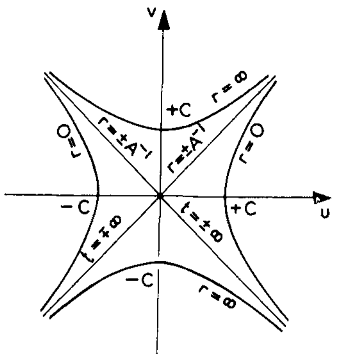

Figure 1. The Kruskal-like diagram for the extended STM of a uniformly accelerating reference frame.

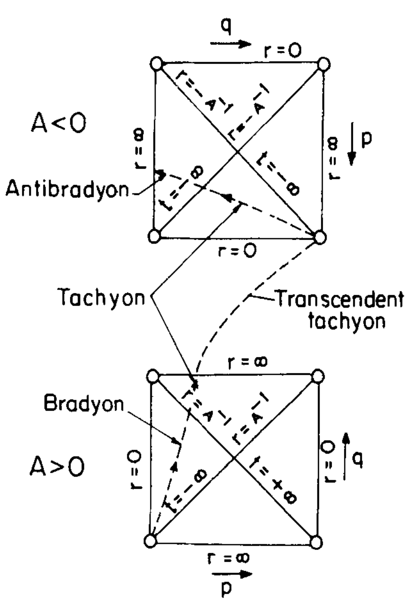

Figure 2. The representation of the extended STM's of uniformly accelerating and decelerating reference frames by means of Penrose conformal diagram. This is obtained from figure 1 through compact conformal transformations. The dashed line shows motion with transitions: bradyon → tachyon → antibradyon in the limited case via a transcendent tachyon.

One can see four regions on the diagram of the uniformly accelerating or decelerating STM which are divided by horizons. They are bounded by the origin $r = 0$ and the spatial infinity $r = \infty$, which are not singularities but emerge as physical boundaries of the space-time. Some similiarities can be seen between this representation of the uniformly accelerating STM and the analogous one for the Schwarzschild STM (see, for example, Misner *et al* 1973). The asymptotically flat Schwarzschild region corresponds to $r = 0$ here, the Schwarzschild singularity at $r = 0$ does to $r = \infty$ in our case and the Schwarzschild horizon at $r = 2M$ shows analogous properties with $r = A^{-1}$ surface.

3. Geodesic motion

Let us consider now the geodesic equations in the uniformly accelerating geometry (4) when $\theta = \pi/2$:

$$dr/d\tau = k(E^2 - 1 + r^2A^2 - L^2r^{-2} + L^2A^2)^{1/2}, \tag{15}$$

$$d\varphi/d\tau = Lr^{-2}, \tag{16}$$

$$dt/d\tau = E(1 - r^2A^2)^{-1}, \tag{17}$$

where τ is the proper time, E and L are the energy and the angular momentum of a moving particle per unit mass, respectively and $k = \pm 1$ depending on whether r increases or decreases in time. The geodesics behave differently depending on the value of $E^2 + L^2A^2$. In the case of $E^2 + L^2A^2 < 1$ there is the turn point at r_i defined by the equation $E^2 + L^2A^2 = r_i^2A^2 - L^2r_i^{-2}$. On achieving this point the geodesic set off to infinity of the space, and the speed of the particle increases continuously with respect to some point $r = \text{const}$. In the case of $E^2 + L^2A^2 > 1$ the particle on the geodesic is captured at $r = 0$. The case of $E^2 + L^2A^2 = 1$ is transitional where the timelike geodesic is given by the equation $r = 0$.

The geodesic equations (15) and (16) do not reveal any peculiar features on the horizons $r = \pm A^{-1}$, and the singularity in (17) is associated with the presence of the coordinate time t. We investigate the proper and coordinate time behaviour in the given STM by integrating (15) and (17) together:

$$t - t_c = kE \int_{r_c}^{r} (E^2 - 1 + x^2A^2 - L^2x^{-2} + L^2A^2)^{1/2} (1 - A^2x^2)^{-1} dx. \tag{18}$$

Here x is some radial variable of the integration, t_c and r_c are the initial coordinates of motion. It can be seen from (18) that the infinite value of the t-coordinate corresponds to the approximation to $r = \pm A^{-1}$, i.e. the flowing of the coordinate time is infinitely delayed. The proper time for the uniformly accelerating observer does not have such peculiarities; it is calculated by integrating the equation (15):

$$\tau - \tau_c = k \int_{r_c}^{r} (E^2 - 1 + x^2A^2 - L^2x^2 + L^2A^2)^{1/2} dx, \tag{19}$$

where τ_c is an initial value of the proper time. It can be seen that (19) remains finite at $r = \pm A^{-1}$, besides the coordinate and the proper time behaviour in the STM under consideration is very much analogous to that of the Schwarzschild geometry. The last fact indicates again that the passage to the region $r > A^{-1}$ from $r < A^{-1}$, i.e. from the subluminal region to the superluminal one, corresponds to the passage from the region over an event horizon to the one under it (Goldoni 1975, 1978; de Sabbata *et al* 1977; Narlikar and Dhurandhar 1978). Moreover, it can be concluded on the basis of these and other studies (Srivastava 1983; Gurin 1984, 1985; Gurin and Trofimenko 1985), that every horizon divides space-time into subluminal and superluminal regions forming an STM with non-trivial topology. In other words, taking into account a many-dimensional interpretation of superluminal phenomena and phenomena under black hole event horizon (Trofimenko 1986; Gurin 1984; Gurin and Trofimenko 1985;

Srivastava 1983; Chandola and Rajput 1984, 1985) the horizons and the light barrier divide ranges of real and imaginary coordinates.

Thus, in the proper reference frame the geometry covers both the regions $r < +A^{-1}$ or $r < -A^{-1}$ and $r > +A^{-1}$ or $r > -A^{-1}$.

Let us consider the qualitative picture of the accelerating motion. When an object approximates to the horizon moving with a constant acceleration, it will gradually disappear becoming asymptotic to the $r = +A^{-1}$ surface as regards the observer being at rest at some point. Such a situation is similar to some object "disappearing" when it falls on a black hole. But in the reference frame of an observer at rest the object will never pass across the surface $r = +A^{-1}$. In the proper reference frame of an accelerating body this surface is not peculiar at all, and passing across it the object goes to the superluminal region (see figure 2), i.e. it becomes a tachyon.

As can be easily seen the last will also be true for a constant deceleration crossing the $r = -A^{-1}$ surface from the superluminal region to the subluminal one. In this case a tachyon will be transformed into a bradyon with respect to the original reference frame (see figure 2).

To consider the two consequent processes of acceleration and deceleration we must take the results of extended relativity (Recami and Mignani 1974) on double superluminal transformations which lead to the complete inverse, i.e. to transition from an object to an antiobject. Hence, having accelerated across the horizon $r = +A^{-1}$ an original bradyon turns into a tachyon which can be decelerated across the horizon $r = -A^{-1}$ and turned into an antibradyon. Naturally, to go from an acceleration to a deceleration an action of force is required. Thus uniformly accelerating and decelerating STM's make it possible to produce tachyons and antiparticles in principle. It must be emphasized that such a conclusion is valid for proper reference frames of the objects and for a rest frame above the transitions are unobservable.

4. Discussion

In the extended relativity theory by Recami and Mignani (1974) superluminal speeds of inertial motion are considered on equal footing with subluminal ones, but the value of the light speed remains limited for both tachyons and bradyons. In other words, the light barrier penetration is impossible, that is bradyons will always remain as bradyons and tachyons will always be as tachyons with respect to an inertial reference frame. This thesis is unquestionably true for inertial motion and in the absence of gravitational fields.

Antiobjects also appear in extended relativity on equal footing and a transition between objects and antiobjects is the discrete operation. In general, bradyons, tachyons, and antiobjects are represented as separate classes of physical objects.

The present conclusions show that it is possible to transform continuously from one class of objects to another and this is realized due to the non-inertial motion. However, the light barrier penetration is possible only in a properly uniform accelerating or decelerating reference frame without violating the principles of relativity as compared to the relativity-violating analogous conclusions (Everett 1976; Froning 1983). Note that our results might seem a little unexpected, although the possibility of moving infinitely along with an acceleration was mentioned not once (see, for example, the

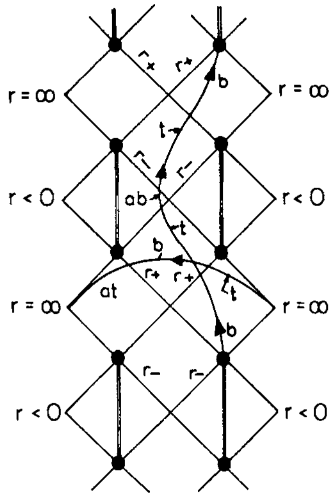

Figure 3. The Penrose diagram for the Kerr STM (Misner *et al* 1973) along the symmetry axis (the singularity is pictured conditionally). An analogous chain of transitions is shown: b—bradyon → t—tachyon → a–b—antibradyon → at—antitachyon → b—bradyon along a timelike geodesics (i.e. for an originate *b*); t—tachyon → b—bradyon → at—antitachyon along a spacelike geodesics (i.e. for an originate *t*).

book by Bondi 1964 and the paper by Morgan 1973) and they can be easily understood within the framework of GR. They are the consequence of the fundamental principle of equivalence and do not contradict the principles of special relativity because when analyzing the accelerating motion we leave its realm and pass over to GR, where, as we have shown, the light barrier penetration is quite possible.

Continuing the analogy between space-time structure with horizons for the uniformly accelerating and decelerating frames and gravitational fields we can propose the possibility of similar transformations from bradyons via tachyons to antibradyons in some black hole STM's, for example, Kerr STM (figure 3). This problem will be considered in greater detail by the authors in future. Note that the tachyon production from the simplest case of black hole, the Schwarzschild hole, was recently investigated by Srivastava (1983).

Let us perform some estimates concerning the possibility of light barrier penetration. Consider an object from the point of view of its own reference frame which travels with the constant acceleration A equalled, say, to the free fall acceleration in the earth field, g. Such an object can travel for an unlimited time without any restrictions (in its proper reference frame!). In such an accelerating frame local effects analogous to those occurring in the earth is gravitational field will be observed. In this case the distance to the accelerating horizon is $r = c^2 g^{-1} \approx 10^{18}$ cm $\approx$ a light year (Trofimenko 1984).

In conclusion, we would like to propose some astrophysical appearance of transitions between bradyons, tachyons and antiparticles considered here. Such processes can possibly develop as higher-energetic effects (gamma-bursts, quasars, superdense D-bodies by Ambartsumian, etc.). From the point of view of the STM theory they may be named by white hole explosions (the bibliography on this topic has been recently given by the present authors (Trofimenko and Gurin 1986).

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