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Equivalent potentials for a nonsymmetric non-local interaction

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Abstract. Scattering formalisms which incorporate antisymmetrization of the projectile with respect to identical particles in the target result in a nonsymmetric non-local interaction. Such an interaction constraints the relative wavefunctions to be orthogonal to redundant states forbidden by the Pauli principle. Concentrating on the nonsymmetric non-local kernel of Saito we try to visualize the mechanisms by which a potential can ensure the required orthogonality. We achieve this by replacing the Saito kernel by an effective symmetric non-local potential. The constructed symmetric potential is found to be phase-equivalent only but not off-shell equivalent to the original kernel. This difference in the off-shell behaviour is attributed to the dynamical origin simulating the redundant states. In close analogy with one of our recent works we also derive an energy-momentum dependent equivalent to the local potential. Our solution of the pseudo inverse problem is exact and provides a basis for writing the phase—and quasiphase—equations. We present numerical results in support of this.

Keywords. Equivalent potentials; radial wavefunction; non-local interaction; Saito potential; phase method.

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1. Introduction

The resonating group method (RGM) (Wildermuth *et al* 1966) represents a microscopic theory for the description of the interaction between nucleon clusters or composite nuclei. As an approximation to RGM, Saito (1969) introduced the orthogonality condition model (OCM). In OCM, non-symmetric non-local potentials occur which constraint the relative wave functions to be orthogonal to redundant states forbidden by the Pauli principle. The essential feature for maintaining orthogonality provides for extra nodes in the scattering wave functions (Krasnopol'skii and Kukulin 1975). An important aspect of Saito's theory is that the redundant states appear in the model potentials. For example, the s-wave Saito potential in the presence of one redundant state $v_1(r)$ is given by

$$V(r, r') = v_1(r) v_1(r') \frac{d^2}{dr'^2}. \tag{1}$$

Here $v_1(r)$ is represented by a normalized function (Okai *et al* 1972)

$$v_1(r) = (4a^3)^{1/2} r \exp(-ar), \tag{2}$$

with $a = 2\ \text{fm}^{-1}$. It has been observed (Englefield *et al* 1974) that the potential in

equation (1) can simulate the effect of the repulsive core of the α-α interaction as well as that of the n-p interaction in the $^1S_0$ channel.

In the present paper we consider the problem of symmetrization as well as of localization of the non-symmetric non-local potential in equation (1). The symmetrization procedure helps us visualize the mechanisms by which a potential can ensure the required orthogonality in terms of extra nodes in the scattering wave functions. The local potential constructed by us is momentum-dependent. This dependence may be expected to assume some significance when considering electromagnetic interactions (Yamaguchi 1954).

The Schrödinger equation for the Saito potential and its adjoint admits of simple analytical solution (Ghosh *et al* 1983). In § 2 we present some of these results and see that the Saito model freezes the redundant states or continuum bound states (CBS) at zero energy. We also note that although the Saito potential and its adjoint are phase-equivalent, they exhibit altogether different off-shell properties. We symmetrize the Saito potential in § 3. The symmetrization procedure converts the zero energy CBS to another redundant state called the spurious state (Coz *et al* 1970). The spurious state, however, occurs at a very high energy. Interestingly, both symmetric and non-symmetric potentials exhibit the same on-shell behaviour but they differ very much in their off-shell properties. This verifies that off-shell properties of a potential are sensitive to the nature of the redundant state it can support. We localize the Saito potential and its adjoint in § 4. In the case of a local potential the only circumstance which can cause an extra node is the presence of a bound state. In the course of our study we shall see that the constructed local potentials do not involve the effect of any extra node whatsoever. Thus the localization procedure carried out here does not spoil the repulsive nature of the Saito interaction. In § 5 we examine the quality of our local potentials by using a combined variable phase off-shell scattering theory.

2. Radial wave functions

At a positive energy $k^2 > 0$ the s-wave solutions of the radial Schrödinger equation for the potential in equation (1) belonging to the regular, irregular and outgoing wave boundary conditions are (Ghosh *et al* 1983).

$$\phi_s(k,r) = \frac{\sin kr}{k} - \frac{8a^4k^2}{D_s(k)(a^2+k^2)^4}\Bigg[(a^2+k^2)r\exp(-ar) + 2a\left\{\exp(-ar) - \cos kr + \frac{a^2-k^2}{2ak}\sin kr\right\}\Bigg], \tag{3}$$

$$f_s(k,r) = \exp(ikr) - \frac{4a^3k^2(a+ik)^2}{\Delta_s(k)(a^2+k^2)^4}[(a^2+k^2)r+2a]\exp(-ar), \tag{4}$$

and

$$\psi_s^+(k,r) = \sin kr + \frac{8a^4k^3}{D_s^+(k)(a^2+k^2)^4}[2a(\exp(ikr) - \exp(-ar)) - (a^2+k^2)r\exp(-ar)]. \tag{5}$$

In the above we have used the subscripts to specify quantities directly related to

scattering by the Saito potential. The same convention is also followed for the corresponding adjoint potential. We shall, however, use no subscript for quantities related to equivalent potentials to be constructed in subsequent section. Here

$$D_s(k) = 1 - a^2(a^2 - k^2)/(a^2 + k^2)^2, \tag{6}$$

represents the Fredholm determinant associated with $\phi_s(k, r)$. Also $D_s(k) = \Delta_s(k)$, the Fredholm determinant (Bagchi *et al* 1978). The Fredholm determinant associated with $\psi_s^{(+)}(k, r)$ is given by

$$D_s^{(+)}(k) = 1 - (a^2 + k^2)^{-4}(a^8 + 9a^6k^2 - 9a^4k^4 - a^2k^6 + 16\,i\,a^5k^3). \tag{7}$$

The Jost function is determined by the behaviour of the irregular solution near the origin and in this case we have

$$f_s(k) = f_s(k, 0) = 1 - \frac{8a^4k^2}{\Delta_s(k)(a^2 + k^2)^4}(a + ik)^2. \tag{8}$$

The phase of the Jost function is the negative of the scattering phase shift (Newton 1966). Thus the expression in (8) provides a basis for calculating the scattering phase shifts induced by the Saito potential.

In general the simultaneous zeros of $D^{(+)}(k)$ and $D(k)$ for $k \neq 0$ are called continuum bound states (Gourdin and Martin 1957). The quantities $D_s(k)$ and $D_s^{(+)}(k)$ given in (6) and (7) can be simultaneously zero only for $k = 0$. Heurestically, this is often expressed by saying that the Saito model freezes all redundant states at zero energy (Bagchi *et al* 1980). The term spurious state is reserved for a zero of $D(k)$ for $D^{(+)}(k) \neq 0$ (Coz *et al* 1970; Arnold and Mac Keller 1971). Clearly this cannot occur in our case and the Saito potential does not support a spurious state.

For the adjoint potential to that in equation (1) we denote the regular, irregular and outgoing wave solutions by $\tilde{\phi}_s(k, r)$, $\tilde{f}_s(k, r)$ and $\tilde{\psi}_s^{(+)}(k, r)$ respectively. The results for these solutions are given by

$$\tilde{\phi}_s(k, r) = \frac{\sin kr}{k} - \frac{8a^5}{(a^2 + k^2)^3 \tilde{D}(k)}\left[\frac{a(a^2 + 3k^2)}{k(a^2 + k^2)}\sin kr - \frac{2k^2}{(a^2 + k^2)}\cos kr + \left\{\frac{2k^2}{(a^2 + k^2)} - ar\right\}\exp(-ar)\right], \tag{9}$$

$$\tilde{f}_s(k, r) = \exp(ikr) + \frac{4a^4(a + ik)^2}{(a^2 + k^2)^3 \tilde{\Delta}_s(k)}\left[ar\exp(-ar) - \frac{2k^2}{(a^2 + k^2)}\exp(-ar)\right], \tag{10}$$

and

$$\tilde{\psi}_s^{(+)}(k, r) = \sin kr + \frac{8a^5k}{(a^2 + k^2)^4 \tilde{D}_s^{(+)}(k)}[a(a^2 + k^2)\,r\exp(-ar) - 2k^2(\exp(-ar) - \exp(ikr))], \tag{11}$$

with

$$\tilde{D}_s(k) = (8a^4 + 3a^2k^2 + k^4)/(a^2 + k^2)^2, \tag{12}$$

$$\tilde{\Delta}_s(k) = D_s(k) = \Delta_s(k), \tag{13}$$

and

$$\tilde{D}_s^{(+)}(k) = D_s^{(+)}(k). \tag{14}$$

From (10) we get the Jost function for the adjoint potential in the form

$$\tilde{f}_s(k) = 1 - \frac{8a^4k^2}{(a^2+k^2)^4\,\tilde{\Delta}_s(k)}\,(a+ik)^2. \tag{15}$$

Equations (8), (13) and (15) show that the Saito potential and its adjoint are phase-equivalent (Bagchi *et al* 1978). These two, however, exhibit entirely different off-shell properties. For example, the half-shell t-matrix for the Saito potential is given by (Fuda and Whiting 1973; Ghosh *et al* 1983)

$$t_s(k, q; k^2) = \frac{32a^5q^2}{\pi f_s(k)\,\Delta_s(k)\,(a^2+k^2)^2\,(a^2+q^2)^2}, \tag{16}$$

while that for the corresponding adjoint potential is

$$\tilde{t}_s(k, q; k^2) = -\frac{32\,a^5k^2}{\pi f_s(k)\,\Delta_s(k)\,(a^2+k^2)^2\,(a^2+q^2)^2}, \tag{17}$$

where q is an off-shell momentum. Clearly $t_s(k, q; k^2) \neq \tilde{t}_s(k, q; k^2)$.

3. Symmetrization of the Saito potential

The radial Schrödinger equation for the potential in equation (1) is given by

$$\left(\frac{d^2}{dr^2} + k^2\right)\psi_s(k, r) = 4a^3\,r\exp(-ar)\int_0^\infty r'\exp(-ar')\frac{d^2}{dr'^2}\,\psi_s(k, r')\,dr'. \tag{18}$$

From (18) one has $\langle v_1 | \psi_s \rangle = 0$. To summarize the Saito kernel we proceed as follows.

Consider the Yamaguchi potential written as (Yamaguchi 1954)

$$V(r, r') = \lambda \exp[-\beta(r+r')]. \tag{19}$$

Here λ and β are energy-independent strength and range parameters. The regular solution for this potential is given by (Ghosh *et al* 1983)

$$\phi(k, r) = \frac{\sin kr}{k} + \frac{\lambda}{D(k)\,(\beta^2+k^2)^2}\left[\exp(-\beta r) + \frac{\beta}{k}\sin kr - \cos kr\right], \tag{20}$$

with the Fredholm determinant

$$D(k) = 1 - \lambda/2\beta\,(\beta^2+k^2). \tag{21}$$

The Fredholm determinant associated with the physical solution of Yamaguchi potential is

$$D^{(+)}(k) = D(k) + \frac{\lambda(\beta+ik)}{(\beta^2+k^2)^2}. \tag{22}$$

From (21) and (22) it is clear that no values of λ and β will make $D^{(+)}(k)$ zero, and therefore, a continuum bound state cannot be associated with the Yamaguchi form factor. On the other hand, $D(k)$ can be zero for a wide range of values of λ and β. Mulligan *et al* (1976) illustrated that the Yamaguchi form factor supports a spurious state at $E_{lab} = 400$ MeV for $\lambda = 21{\cdot}219\ \text{fm}^{-3}$ and $\beta = 1{\cdot}5\ \text{fm}^{-1}$. Thus if we demand that $\langle v_1|\phi\rangle = 0$, it will amount to simulating the effect (which is essentially an extra node or an inner oscillation) of continuum bound state of the Saito potential through a spurious state associated with the Yamaguchi form factor and both symmetric and non-symmetric potentials will be expected to be phase-equivalent. From $\langle v_1|\phi\rangle = 0$ we immediately get

$$\lambda(k^2) = -2a\beta(\beta^2+k^2)(a+\beta)^2/[\beta(a^2+k^2)-a(a^2-\beta^2)]. \tag{23}$$

Thus for a given β the Yamaguchi potential can have the same inner oscillation as that of the Saito potential provided the coupling constant becomes energy-dependent. The result in (23) was obtained by Bagchi *et al* (1980) by using the Fredholm's third theorem for a scattering solution to exist when $D(k)$ or $D^{(+)}(k)$ is zero.

We have numerically found that for $\beta = 1\ \text{fm}^{-1}$ the Yamaguchi potential with the energy-dependent coupling constant given in (23) is phase-equivalent to the Saito potential. Although the potential strength changes sign at $k^2 = 2\ \text{fm}^{-2}$, this does not affect the continuity of the phase shift. Bagchi *et al* (1980) claim this as a necessary requirement for the scattering wave function to exhibit proper continuity and nodal behaviour.

The half off-shell t-matrix for the symmetric potential under consideration is given by

$$t_{sym}(k, q; k^2) = \frac{2\lambda(k^2)q}{\pi q f(k)(\beta^2+k^2)(\beta^2+q^2)D_{sym}(k)}, \tag{24}$$

with

$$f(k) = 1 + \frac{\lambda(\beta+ik)}{(\beta^2+k^2)D_{sym}(k)}. \tag{25}$$

The expression for $D_{sym}(k)$ is obtained from (21) by replacing λ by $\lambda(k^2)$. It can be seen that numerical results for $t_{sym}(k, q; k^2)$ do not agree with those for $t_s(k, q; k^2)$ or $\tilde{t}_s(k, q; k^2)$. This implies that the off-shell behaviour of the constructed symmetric potential is different from that of the Saito potentials.

4. Equivalent local potentials

We have introduced the radal Schrödinger equation for the Saito potential in (18). For adjoint scattering (Bagchi *et al* 1978) the Schrödinger equation corresponding to (18) is

$$\left(\frac{d^2}{dr^2}+k^2\right)\tilde{\psi}_s(k, r) = 4a^3\frac{d^2}{dr^2}[r\exp(-ar)] \times \int_0^\infty r'\exp(-ar')\,\tilde{\psi}_s(k, r')\,dr'. \tag{26}$$

For the non-local potentials in (18) and (26) we have obtained the equivalent local potentials in the form

$$V_{\text{eq}}^{(i)}(k,r) = g_1^{(i)}(k,r) + g_2^{(i)}(k,r)\frac{\mathrm{d}}{\mathrm{d}r}, \qquad i = 1, 2, \tag{27}$$

by following a prescription given by us earlier (Talukdar *et al* 1985). The subscript $i = 1$ corresponds to (18) while $i = 2$ corresponds to (26). The expressions for $g_1^{(i)}(k,r)$ and $g_2^{(i)}(k,r)$ are given as

$$g_1^{(i)}(k,r) = \{\beta_1^{(i)}(k,r)\gamma_1^{(i)}(k,r)\, k \sin kr - \beta_2^{(i)}(k,r)\gamma_2^{(i)}(k,r) k \cos kr\} \times \frac{1}{(\beta_1^{(i)2} + \beta_2^{(i)2})}, \tag{28a}$$

and

$$g_2^{(i)}(k,r) = \{\beta_1^{(i)}(k,r)\gamma_1^{(i)}(k,r)\cos kr + \beta_2^{(i)}(k,r)\gamma_2^{(i)}(k,r)\sin kr\} \times \frac{1}{(\beta_1^{(i)2} + \beta_2^{(i)2})}, \tag{28b}$$

with

$$\beta_1^{(1)}(k,r) = \frac{(3a^2+k^2)}{8a^4}\left[1 - \frac{8a^4}{(3a^2+k^2)}\left\{\frac{r\exp(-ar)(-a\cos kr + k\sin kr)}{(a^2+k^2)} - \exp(-ar)\frac{\{(a^2-k^2)\cos kr - 2ak\sin kr\}}{(a^2+k^2)^2} - \frac{a^2-k^2}{(a^2+k^2)^2}\right\}\right], \tag{29a}$$

$$\beta_2^{(1)}(k,r) = \frac{r\exp(-ar)(-a\sin kr - k\cos kr)}{(a^2+k^2)} - \frac{\exp(-ar)\{(a^2-k^2)\sin kr + 2ak\cos kr\}}{(a^2+k^2)^2} + \frac{2ak}{(a^2+k^2)^2}, \tag{29b}$$

$$\beta_1^{(2)}(k,r) = (8a^4 + 3a^2k^2 + k^4)\left[1 + \frac{8a^4}{(8a^4+3a^2k^2+k^4)}\left\{a^2\left(\frac{r\exp(-ar)(-a\cos kr + k\sin kr)}{(a^2+k^2)} - \frac{\exp(-ar)\{(a^2-k^2)\cos kr - 2ak\sin kr\}}{(a^2+k^2)^2} + \frac{a^2-k^2}{(a^2+k^2)^2}\right) - 2a\left(\frac{\exp(-ar)(-a\cos kr + k\sin kr)}{(a^2+k^2)} + \frac{a}{(a^2+k^2)}\right)\right\}\right], \tag{29c}$$

$$\beta_2^{(2)}(k,r) = (8a^4 + 3a^2k^2 + k^4)\left[\frac{8a^4}{(8a^4+3a^2k^2+k^4)}\{a^2(r\exp(-ar)\right.$$

$$\times \frac{(-a\sin kr - k\cos kr)}{(a^2+k^2)} - \frac{\exp(-ar)\{a^2-k^2)\sin kr + 2ak\cos kr\}}{(a^2+k^2)^2} + \frac{2ak}{(a^2+k^2)^2}\Bigg) - 2a\left(\frac{\exp(-ar)(-a\sin kr - k\cos kr)}{(a^2+k^2)} + \frac{k}{(a^2+k^2)}\right)\Bigg\}\Bigg], \tag{29d}$$

$$\gamma_1^{(1)}(k,r) = -r\exp(-ar), \tag{30a}$$

$$\gamma_2^{(1)}(k,r) = r\exp(-ar), \tag{30b}$$

$$\gamma_1^{(2)}(k,r) = 8a^4[a^2 r\exp(-ar) - 2a\exp(-ar)], \tag{30c}$$

and

$$\gamma_2^{(2)}(k,r) = 8a^4[a^2 r\exp(-ar) - 2a\exp(-ar)]. \tag{30d}$$

In writing the expressions for $g_1^{(i)}(k,r)$ and $g_2^{(i)}(k,r)$ we have followed the prescription of Kermode and Melhem (1983) to ensure that the equivalent potentials are not singular for any value of k and r.

5. Phase method

To study the on- and off-shell properties of our local potentials we now make use of a generalized phase method (Talukdar *et al* 1985). In this method the equations for the phase function $\delta(k,r)$ and quasiphase function $\Delta(k,q,r)$ for the potential in (27) are given by (McKeller *et al* 1965; Talukdar *et al* 1985)

$$\delta^{(i)\prime}(k,r) = -k^{-1}[g_1^{(i)}(k,r)\sin(kr+\delta(k,r)) + kg_2^{(i)}(k,r)\cos(kr+\delta(k,r))] \times \sin(kr+\delta(k,r)), \tag{31}$$

and

$$\Delta^{(i)\prime}(k,q,r) = -[q^{-1}\sin qr + k^{-1}\Delta(k,q,r)\cos(kr+\delta(k,r))] \times \{g_1^{(i)}(k,r)\sin(kr+\delta(k,r)) + kg_2^{(i)}(k,r)\cos(kr+\delta(k,r))]. \tag{32}$$

Here prime denotes differentiation with respect to r. Initial conditions on (31) and (32) are $\delta^{(i)}(k,0) = \Delta^{(i)}(k,q,0) = 0$. The scattering phase shift $\delta^{(i)}(k) = \lim_{r\to\infty} \delta^{(i)}(k,r)$ and quasiphase (Talukdar *et al* 1983) $\Delta^{i}(kq) = \lim_{r\to\infty} \Delta^{(i)}(k,q,r)$. The quasiphase $\Delta^{(i)}(k,q)$ is related to the off-shell extension function (Noyes 1965) $F^{(i)}(k,q)$ by

$$\Delta^{(i)}(k,q) = F^{(i)}(k,q)\sin\delta^{(i)}(k). \tag{33}$$

Interestingly, $\lim_{q\to k} \Delta^{(i)}(k,q) = \sin\delta^{(i)}(k)$ and thus $F^{(i)}(k,k) = 1$.

The potentials in (18) and (26) which simulate the repulsive core of the α-α and n-p interactions has only one free parameter. Equivalent local potentials $V_{eq}^{(i)}(k,r)$ corresponding to these in (18) and (26) are given in (27) with $i = 1$ and 2. In figure 1 we plot $V_{eq}^{(i)}(k,r)$ as a function of r at $E_{lab} = 10$ MeV with the energy momentum conversion factor 1/41·468 $MeV^{-1}\,fm^{-2}$. The functions $g_1^{(i)}(k,r)$ are highly attractive while the corresponding $g_2^{(i)}(k,r)$ are attractive at short distances and repulsive elsewhere.

The phase-shifts induced by the Saito potentials are negative. Also on a very general ground Saito (1968) and Bagchi and Mulligan (1979) have shown that phase-shifts for the potential in equation (1) and its adjoint are equal. It is, therefore, interesting to examine if the potentials in (27) predict similar results. To that end we have calculated the phase-shifts at $E_{lab} = 10$ MeV from (18) and (26) by using a formalism developed earlier (Talukdar *et al* 1979) and found that $\delta(k) = -0{\cdot}5490$ rad. For the equivalent local potentials we have solved (31) by the Runge-Kutta method with an appropriate stability check and obtained $\delta^{(i)}(k) = 2{\cdot}5926$ rad for both $V_{eq}^{(i)}(k,r)$ in agreement with the conjecture of Saito. Clearly,

$$\delta(k) = \delta^{(i)}(k) - \pi. \tag{34}$$

The result in (34) tells us that by localizing the Saito potential we have removed the effect of CBS.

To consider the action of the different regions of the potentials in (27) in producing the scattering phase shift $\delta^{(i)}(k)$ we plot in figure 2 the phase function $\delta^{(i)}(k,r)$ as a

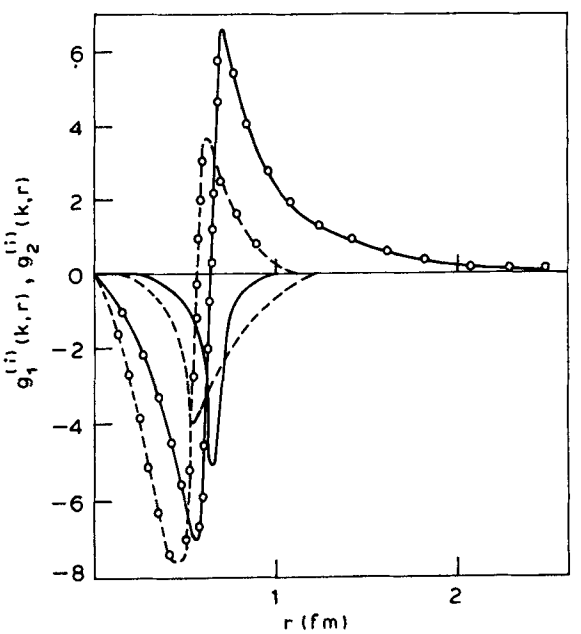

Figure 1. $g_1^{(i)}(k,r)$ and $g_2^{(i)}(k,r)$ as a function of r for the equivalent local potentials in (6). The solid curve and solid curve with circle represent the variation of $g_1^{(1)}(k,r)$ and $g_2^{(1)}(k,r)$ with r while the dashed curve and dashed curve with circle represent similar variation of $g_1^{(2)}(k,r)$ and $g_2^{(2)}(k,r)$.

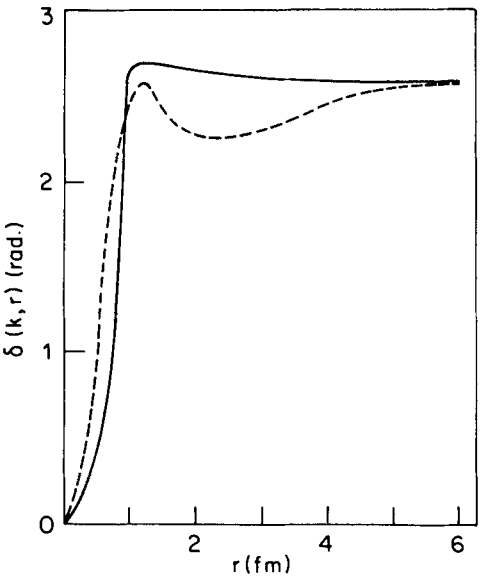

Figure 2. Phase function $\delta^{(i)}(k, r)$ for equivalent local potentials in (6) as a function of r. The solid curve represents the variation of $\delta^{(1)}(k, r)$ with r while the dashed curve represents similar variation for $\delta^{(2)}(k, r)$.

function of r. For both potentials $\delta^{(i)}(k, r)$'s increase very rapidly for small r. This is a peculiarity of non-local potentials. The asymptotic value is, however, reached through different paths. Thus, Saito and its adjoint kernel act differently in producing the phase shift.

From the above it is clear that the potentials in (27) are phase-equivalent to those in (18) and (26). This is no guarantee that these are also off-shell equivalent. To compare the off-shell behaviour of the original Saito potentials with that of $V^{(i)}_{eq}(k, r)$ we have proceeded as follows.

(i) We calculated the extersion functions $F^{(i)}(k, q)$ for the non-local kernels by a prescription given by one of us (Talukdar *et al* 1979).

(ii) For the equivalent local potentials we have solved the coupled differential equations (31) and (32) and made use of (33).

In table 1 we display our numbers for $\Delta^{(i)}(k, q)$ and $F^{(i)}(k, q)$ for $V^{(i)}_{eq}(k, r)$ as a function of q. These numbers are in exact agreement with those calculated by the method in (i). Thus $V^{(i)}_{eq}(k, r)$ are not only phase-equivalent to the Saito potentials but also off-shell equivalent. Looking closely into our numbers we see that the results for $\Delta^{(1)}(k, q)$ [or $F^{(1)}(k, q)$] and $\Delta^{(2)}(k, q)$ [or $F^{(2)}(k, q)$] do not exhibit similar behaviour as a function of q. For example, $\Delta^{(1)}(k, q)$ [or $F^{(1)}(k, q)$] first increases with q and takes up a maximum value at $q = 6k$ and then decreases. As expected, at the on-shell point $q = k$, $\Delta^{(i)}(k, k) = |\sin \delta(k)|$ for both i's and $F^{(i)}(k, k) = 1$. From our numbers it is clear that the off-shell properties of the Saito potential and its adjoint are different. This is in agreement with our observation with regard to equations (16) and (17).

Table 1. Quasiphase $\Delta^{(i)}(k, q)$ and off-shell extension function $F^{(i)}(k, q)$ as a function of q at $E_{lab} = 10$ MeV.

	Saito potential		Its adjoint	
q	$\Delta^{(1)}(k, q)$	$F^{(1)}(k, q)$	$\Delta^{(2)}(k, q)$	$F^{(2)}(k, q)$
$k/8$	0·5531	1·0599	0·0199	0·0228
$2k/8$	0·5515	1·0569	0·0378	0·0724
$3k/8$	0·5490	1·0521	0·0875	0·1677
$4k/8$	0·5454	1·0452	0·1396	0·2675
$5k/8$	0·5408	1·0364	0·2146	0·4113
$6k/8$	0·5353	1·0259	0·3044	0·5834
$7k/8$	0·5289	1·0136	0·4083	0·7825
k	0·5218	1·0000	0·5218	1·0000
$2k$	0·4409	0·8450	1·7682	3·3886
$4k$	0·2502	0·4795	4·0387	7·7399
$6k$	0·1274	0·2442	4·5918	8·7999
$8k$	0·0646	0·1238	4·1368	7·9279
$10k$	0·0344	0·0659	3·4419	6·5962
$12k$	0·0195	0·0374	2·8006	5·3672
$14k$	0·0116	0·0222	2·2723	4·3547

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