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Near perpendicular propagation of ion cyclotron modes in a deuterium-hydrogen-oxygen fusion plasma

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Abstract. A dispersion relation for the near perpendicular propagation of the electromagnetic ion cyclotron wave has been derived in a fusion plasma that has deuterium as a majority species, hydrogen as a minority species and fully ionized oxygen as an impurity constituent; all being modelled by loss cone distribution functions. The wave has a frequency ω around the deuterium ion gyrofrequency-Ω_D and a wavelength much longer than its Larmor radius γ_{LD} $(k_\perp \gamma_{LD} < 1)$; the plasma itself being characterized by large ion plasma frequencies $(\omega_{PD}^2 \gg \Omega_D^2)$. Two modes, a low frequency (LF) and a high frequency (HF) mode of opposite electrical energy can propagate in the plasma; the instabilities that arise are thus due to an interaction of modes of opposite energies. We find that while hydrogen tends to destabilize the plasma, the impurity oxygen ions have the reverse effect. Also the plasma is most stable when the ratios of the perpendicular components of oxygen-to-deuterium and hydrogen-to-deuterium temperatures are kept low. Detailed studies of the wave propagation characteristics and energy reveal the close resemblance of a loss cone plasma containing oxygen to a stable Maxwellian plasma in regard to wave stability, propagation and energy.

Keywords. Ion cyclotron wave; fusion plasma; dielectric tensor; approximation scheme; tensor elements; dispersion relation; wave group velocity.

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1. Introduction

In early research in fusion plasma it was realized that the resonances between the natural motion of an ion and an electromagnetic (EM) wave at the same frequency provided an excellent, alternative approach to plasma heating (Stix and Palladino 1958). This method of plasma heating is of great relevance today as fusion physicists find it difficult to achieve the high temperatures necessary by the conventional method of ohmic heating of plasmas. Of the many options available, EM wave in the ion cyclotron range of frequencies (ICRF) has distinguished itself as one of the most successful methods available for the bulk heating of plasmas (Equipe 1981; Perkins 1984).

Fusion plasmas often contain mixtures of deuterium and hydrogen (in a relatively low concentration). Such plasmas, which are in the keV range, also contain metallic impurities such as iron, nickel and chromium. Multiply-ionized oxygen impurity ions can be found in the peripheral low temperature regions (Janzen 1981).

EM wave propagation in multi-ion species plasmas is of great interest since heavy ions, even in small concentrations and low temperatures, greatly modify the propagation and amplification of these waves. In particular, it has been shown that new resonances and cut-offs occur, the wave group velocity is greatly altered and that strong coupling may occur in the vicinity of the cross-over frequency (Smith and Brice 1964; Leer *et al* 1978). Such studies, for waves around the ICRF, have been carried out for 2, 3 and n-ion component plasmas by Jessup and McCarthy (1978) and Janzen (1980, 1981); the investigations on the hybrid resonances between the two ion components have been used to successfully explain wave heating results.

On the experimental side, as mentioned above, heating of plasmas by waves around the ICRF has been very successful. Thus in a plasma containing deuterium as the majority species and hydrogen as the minority species considerable increases in both ion and electron temperatures have been achieved (Equipe 1982; Owens *et al* 1983). Similar increases in ion and electron temperatures have also been achieved in the PDX and PLT machines, these plasmas containing either hydrogen or helium as the minority species (Davies *et al* 1983).

Our representative fusion plasma therefore has deuterium (denoted by D^+) as the majority species, hydrogen (H^+) as the minority species and fully ionized oxygen (O^{8+}) as the impurity constituent. The electrons provide only a neutralizing background; all the four constituents of the plasma have been modelled by loss cone distributions to simplify the algebra.

This paper purports to examine whether this successful method of plasma heating by waves around the ICRF could give rise to the reverse process in the form of instabilities. We have therefore derived a dispersion relation for the near perpendicular propagation of the EM ion cyclotron wave in the above representative plasma. The ion cyclotron wave has a frequency around the fundamental harmonic of the D^+ ion gyrofrequency Ω_D and a wavelength much larger than its ion Larmor radius $\gamma_{LD}(k_\perp \gamma_{LD} < 1)$; the plasma itself being characterized by large ion plasma frequencies ($\omega_{PD}^2 \gg \Omega_D^2$). A study of the dispersion relation, which can be used for both the anisotropic Maxwellian and the loss cone plasma, shows that the plasma can support two modes: a higher frequency mode (HF mode) which starts at $Z_D \approx 1 (Z_D = \omega/\Omega_D$; ω being the wave angular frequency) and a lower frequency mode (LF mode) which starts with a small frequency and finally attains a value of $Z_D \approx 1$. We find the LF and HF modes to be of positive and negative electrical energies respectively; the instabilities that arise in the plasma due to a coalescing of the modes can thus be interpreted physically as being due to an interaction between modes of opposite electrical energy (Hasegawa 1975). The stability of these modes has also been studied as a function of three temperature ratios namely, $T_{\perp,eD} = (T_{\perp,e}/T_{\perp,D})$, $T_{\perp,HD} = (T_{\perp,H}/T_{\perp,D})$ and $T_{\perp,OD} = (T_{\perp,O}/T_{\perp,D})$ where $T_{\perp,D}$, $T_{\perp,H}$, $T_{\perp,O}$ and $T_{\perp,e}$ are the temperatures of D^+, H^+, O^{8+} and electrons perpendicular to the ambient magnetic field. We find the plasma to be most stable when $T_{\perp,eD}$ is large and $T_{\perp,HD}$ and $T_{\perp,OD}$ are low. Also while fully ionized oxygen has a stabilizing influence on the plasma, the lighter hydrogen ions tend to enhance the instability of the modes. Detailed studies of the wave propagation velocity, polarization and energy have also been made.

2. The dielectric tensor

We start with the wave equation

$$\frac{c^2}{\omega^2}[\mathbf{k} \times (\mathbf{k} \times \mathbf{E})] + K \cdot \mathbf{E} = 0, \tag{1}$$

where c is the velocity of light and $\mathbf{k}$ the wave vector. $\mathbf{k}$ has components

$$\mathbf{k} \equiv (k_x = k_\perp; \quad k_y = 0 \quad \text{and} \quad k_z = k_\parallel).$$

$\boldsymbol{K}$, the hot plasma dielectric tensor, can be derived from the coupled kinetic Vlasov-Maxwell equations (Montgomery and Tidman 1964) and the expressions for the individual components $K_{ij}\,(i, j = x, y, z)$ are well known.

As mentioned earlier, we are interested in ion cyclotron wave propagation in a plasma in which all the four components namely deuterium (D), hydrogen (H), oxygen (O) and electrons (e) are modelled by loss cone distribution functions. The equilibrium particle distribution function f_0 is therefore, in general, given by

$$f_0 = [j!\,\pi^{3/2} W^{2j+2} U]^{-1} \left(\frac{v_\perp^2}{W^2}\right)^j \exp\left[-\frac{v_\parallel^2}{U^2} - \frac{v_\perp^2}{W^2}\right] \tag{2}$$

where

$$U^2 = 2T_\parallel / M \quad \text{and} \quad W^2 = 2T_\perp / M(j+1).$$

In (2) j is the loss cone index while T and M denote the temperature and mass respectively.

Substituting (2) into the expressions for the elements K_{ij} (Landau and Cuperman 1971) and carrying out the $\mathrm{d}v_\parallel$ and $\mathrm{d}v_\perp$ integrations we get the following expressions for the elements of the dielectric tensor $\boldsymbol{K}$:

$$\begin{array}{l} K_{xx}-1 \\[2ex] K_{xy} \\[2ex] K_{yy}-1 \end{array} = \sum_s \frac{\omega_p^{2'}}{z} C' \sum_{n=-\infty}^{\infty} \begin{array}{c} \dfrac{n^2}{\alpha^2} \\[2ex] \dfrac{j'n}{\alpha} \\[2ex] \ \end{array} \left\{ \begin{array}{lcr} I_p^j & & I_p^{j-1} \\[2ex] I_{\alpha p}^j & \left[\dfrac{1-E}{n-z} - \dfrac{AE}{z}\right] - j\left[\dfrac{1-E}{n-z} - \dfrac{E}{z}\right] & I_{\alpha p}^{j-1} \\[2ex] I_{\alpha\beta p}^j & & I_{\alpha\beta p}^{j-1} \end{array} \right\}$$

$$\begin{array}{l} K_{zx} \\[2ex] \\[2ex] K_{zy} \end{array} = \sum_s \frac{\omega_p^{2'}}{z} C' \sum_{n=-\infty}^{\infty} \begin{array}{c} \dfrac{n}{\theta\alpha^2} \\[2ex] \ \\[2ex] \dfrac{j'}{\theta\alpha} \end{array} \left\{ \begin{array}{lcr} I_p^j & & I_p^{j-1} \\[2ex] & \left[E + \dfrac{A}{z}(n-z)E\right] - j\left[E + \dfrac{1}{z}(n-z)E\right] & \\[2ex] I_{\alpha p}^j & & I_{\alpha p}^{j-1} \end{array} \right\}$$

and

$$\begin{aligned} K_{zz} - 1 = \sum_s \frac{\omega_p^{2'}}{z} C' \sum_{n=-\infty}^{\infty} \frac{1}{\theta^2\alpha^2} \Bigg\{ & I_p^j \left[\frac{nA}{z} + \frac{W^2}{U^2}\right](z-n)E \\ & - j\left[\frac{n}{z}(z-n)E\right] I_p^{j-1} \Bigg\}. \end{aligned} \tag{3}$$

In (3) the summation over s indicates a summation over the species and the definitions of the other parameters, with the subscripts suppressed are:

$$\Omega = \pm\frac{eB_0}{MC}, \quad \omega_p^2 = \frac{4\pi ne^2}{M}, \quad \omega_p^{2'} = \omega_p^2/\Omega^2, \quad j' = \sqrt{-1},$$

$$z = \frac{\omega}{\Omega}, \quad \alpha = \frac{k_\perp}{\Omega}, \quad \theta = \frac{k_\parallel}{k_\perp}, \quad A = 1 - \frac{W^2}{U^2} \quad \text{and}$$

$$C' = 4[j!\,W^{2j+2}]^{-1}, \tag{4a}$$

$$I_p^j = pL^j I(\alpha, \alpha), \quad I_p^{j-1} = L^{j-1} I(\alpha, \alpha), \tag{4b}$$

$$I_{\alpha p}^j = pL^j\left(\frac{\mathrm{d}}{\mathrm{d}\alpha}\frac{I(\alpha,\alpha)}{2}\right), \quad I_{\alpha p}^{j-1} = L^{j-1}\left(\frac{\mathrm{d}}{\mathrm{d}\alpha}\frac{I(\alpha,\alpha)}{2}\right), \tag{4c}$$

$$I_{\alpha\beta p}^j = pL^j\left(\frac{\mathrm{d}^2 I(\alpha,\beta)}{\mathrm{d}\alpha\,\mathrm{d}\beta}\right), \quad I_{\alpha\beta p}^{j-1} = L^{j-1}\left(\frac{\mathrm{d}^2 I(\alpha,\beta)}{\mathrm{d}\alpha\,\mathrm{d}\beta}\right), \tag{4d}$$

where

$$p \equiv \frac{1}{W^2}, \quad L^j = (-1)^j\frac{\mathrm{d}^j}{\mathrm{d}p^j} \quad \text{and} \quad L^{j-1} = (-1)^{j-1}\frac{\mathrm{d}^{j-1}}{\mathrm{d}p^{j-1}}. \tag{4e}$$

$I = I(\alpha, \beta)$ arises from the $\mathrm{d}v_\perp$ integrations and in its most general form is given by

$$I(\alpha, \beta) = \frac{W^2}{2}\exp\left(-\frac{(\alpha^2+\beta^2)W^2}{4}\right)I_n(\alpha\beta W^2/2). \tag{5}$$

In (4b) we have $\alpha = \beta$ in $I(\alpha, \beta)$, in (4c) we differentiate $I(\alpha, \alpha)$ with respect to α and in (4d) we differentiate $I(\alpha,\beta)$ with respect to α and β and finally set $\alpha = \beta$. These are then differentiated j or $(j-1)$ times with respect to p. The arguments of the Bessel function $I_n = I_n(l'_\perp)$, where

$$l'_\perp = \alpha^2 W^2 = \frac{2}{(j+1)}l_\perp, \tag{6}$$

with

$$l_\perp = k_\perp^2 T_\perp/\Omega^2 M. \tag{7}$$

The E-function, which arises from the $\mathrm{d}v_\parallel$ integration, is defined as (Landau and Cuperman 1971)

$$E(s) = -\tfrac{1}{2}Z'(s/\sqrt{2}), \quad s = (z-n)/\theta\sqrt{l_\parallel}$$

with

$$l_\parallel = k_\perp^2 T_\parallel/\Omega^2 M. \tag{8}$$

Z' is the derivative of the plasma dispersion function of Fried and Conte.

In addition to the above and the definitions of $T_{\perp,\mathrm{eD}}$, etc we also define

$$N_{\mathrm{HD}} = N_{\mathrm{H}}/N_{\mathrm{D}}, \quad N_{\mathrm{OD}} = N_{\mathrm{O}}/N_{\mathrm{D}} \quad \text{and} \quad N_{\mathrm{eD}} = N_{\mathrm{e}}/N_{\mathrm{D}}, \tag{9}$$

where N_{D}, N_{H}, N_{O} and N_{e} are the densities of D^+, H^+, O^{8+} and electrons respectively.

We also use the following simplifying relations

$$M_{\mathrm{D}} = 2M_{\mathrm{H}} \quad \text{and} \quad M_{\mathrm{O}} = 8M_{\mathrm{D}},$$

where M_{D}, M_{H} and M_{O} are the masses of deuterium, hydrogen and oxygen ions respectively. Thus

$$Z_{\mathrm{H}} = \omega/\Omega_{\mathrm{H}} = \tfrac{1}{2}Z_{\mathrm{D}}, \tag{10a}$$

$$Z_{\mathrm{O}} = \omega/\Omega_{\mathrm{O}} = Z_{\mathrm{D}}, \tag{10b}$$

$$Z_{\mathrm{e}} = \omega/\Omega_{\mathrm{e}} = -(M_{\mathrm{e}}/M_{\mathrm{D}})Z_{\mathrm{D}}, \tag{10c}$$

and

$$\omega_{p\mathrm{H}}^{2'} = \frac{N_{\mathrm{HD}}}{2}\omega_{p\mathrm{D}}^{2'}, \tag{10d}$$

$$\omega_{p\mathrm{O}}^{2'} = 8N_{\mathrm{OD}}\omega_{p\mathrm{D}}^{2'}, \tag{10e}$$

$$\omega_{pe}^{2'} = N_{\mathrm{eD}}\frac{M_{\mathrm{e}}}{M_{\mathrm{D}}}\omega_{p\mathrm{D}}^{2'}. \tag{10f}$$

3. The approximation scheme

We are interested in the near perpendicular propagation ($k_{\parallel} \ll k_{\perp}$) of the *EM* ion cyclotron wave around the fundamental harmonic of the ion gyrofrequency Ω_{D} of the majority species deuterium. The wave, with a wavelength larger than the D^+ Larmor radius γ_{LD} ($k_{\perp}\gamma_{\mathrm{LD}} < 1$), can have a small range of frequencies both above and below Ω_{D}; we denote this deviation from the exact first harmonic by means of a small parameter ε. Again, it is assumed that this plasma, which is characterized by large ion plasma frequencies, is approximately temperature isotropic not only with respect to the temperatures parallel and perpendicular to the magnetic field ($T_{\parallel}$ and $T_{\perp}$ respectively) but also with respect to the different species themselves. These assumptions regarding the ion cyclotron wave and the plasma in which it propagates permit us the following ordering scheme in terms of a small parameter ε:

$$\nu_{\mathrm{D}} = 1 - Z_{\mathrm{D}}^2 \sim \varepsilon; \quad l'_{\perp,\mathrm{D}}, l_{\parallel,\mathrm{D}}, \theta \quad \text{and} \quad 1/\omega_{p\mathrm{D}}^{2'} \sim \varepsilon;$$

$$T_{\mathrm{eD}}, T_{\mathrm{HD}}, T_{\mathrm{OD}} \quad \text{and} \quad T_{\perp}/T_{\parallel} \sim 1 \quad \text{and} \quad M_{\mathrm{e}}/M_{\mathrm{D}} \sim \varepsilon^2. \tag{11a}$$

where

$$T_{\mathrm{eD}} = T_{\mathrm{e}}/T_{\mathrm{D}}, \text{ etc.}$$

From the definition of ν_{D} and using (10a), we get

$$\nu_{\mathrm{H}} = 1 - Z_{\mathrm{H}}^2 = 1 - \frac{Z_{\mathrm{D}}^2}{4} = \tfrac{1}{4}(3 + \nu_{\mathrm{D}}).$$

Thus whenever ν_{H} occurs in the denominator it can be expanded binomially as

$$\frac{1}{\nu_H} \simeq \tfrac{4}{3} - \tfrac{4}{9}\nu_{\mathrm{D}} \ldots \sim (1 - \varepsilon) \tag{11b}$$

and from the definition of $l'_{\perp}$, namely (6), we have

$$l'_{\perp,\mathrm{H}} = \tfrac{1}{2} T_{\perp,\mathrm{HD}} l'_{\perp,\mathrm{D}} \sim \varepsilon. \tag{11c}$$

These quantities in the case of O^{8+}, become

$$\nu_{\mathrm{O}} = 1 - Z_{\mathrm{O}}^{2} = \nu_{\mathrm{D}} \sim \varepsilon \tag{11d}$$

and

$$l'_{\perp,\mathrm{O}} = \tfrac{1}{8} T_{\perp,\mathrm{OD}} l'_{\perp,\mathrm{D}}. \tag{11e}$$

Though from (11e) we find $l'_{\perp,\mathrm{O}}$ to be only $\frac{1}{8}$ of $l'_{\perp,\mathrm{D}}$ (for $T_{\perp,\mathrm{OD}} = 1{\cdot}0$) the term was retained as such in the expressions for the tensor elements for the following reasons: firstly, since $l'_{\perp,\mathrm{O}}$ always occurred in the numerator and secondly (but more importantly) because it allows us to consider temperature ratios $T_{\perp,\mathrm{OD}} > 1$. In fact a value as high as 8 can be considered when $l'_{\perp,\mathrm{O}} = l'_{\perp,\mathrm{D}}$.

4. The tensor elements and dispersion relation

The expressions for the various tensor elements in (3) retaining terms of order $1/\varepsilon$, 1 and ε and using the ordering scheme of (11a) were derived earlier (Chandu Venugopal, 1983 hereinafter referred to as I). This was for a two-component plasma consisting of protons and electrons; for a compact description of these and the present tensor elements we introduce the following notation. The tensor elements will be written down as a sum of simple brackets containing numerals and carrying subscripts—the numerals indicate the number of terms in the above descending order (that is, the number of terms of order $1/\varepsilon$, 1 and ε) while the subscripts denote the contribution from the particular constituent of the plasma. With this notation, the tensor elements in I can be written as:

$$K_{xx}/\omega_{pi}^{2'} = (1, 2, 6)_i; \quad K_{yy}(Z_i/\omega_{pi}^{2'}) = (1, 3, 6)_i + (0, 0, 1)_e$$

$$\frac{K_{xy}}{j'}\frac{Z_i}{\omega_{pi}^{2'}} = (1, 2, 5)_i + (0, 1, 0)_e; \quad K_{xz}/\omega_{pi}^{2'} = (0, 1, 3)_i,$$

and

$$\frac{K_{zy}}{j'}\frac{Z_i}{\omega_{pi}^{2'}} = (0, 1, 3)_i + (0, 1, 2)_e,$$

where the subscript i refers to ions and $j' = \sqrt{-1}$. Only one term, of order $1/\varepsilon^2$, was retained in K_{zz}. The following equality relations were also found to hold among these tensor elements, namely

$$K_{xy} = j' K_{yy} = j' K_{xx}$$

and

$$K_{yz} \approx -j' K_{xz}. \tag{12}$$

We have assumed a loss cone distribution for all the four constituents of the present plasma and hence their initial contributions to the tensor elements are all similar; however, a number of terms drop out on conversion to the deuterium parameters (using

(10a) to (10f) and (11b) to (11e)). We shall now briefly describe the tensor elements. The deuterium ion contributions to the tensor elements are similar to the ionic contributions in I (noting, however, the change of ionic species). As mentioned earlier the electrons form only a neutralizing background and hence their contributions to the tensor elements can be got by multiplying the electron contributions in I by N_{eD} (we have normalized all quantities with respect to the deuterium parameters and $N_{eD} = 1$ only for a two-component plasma). Since $v_O = v_D$ and $l'_{\perp,O} \sim l'_{\perp,D}$ the contributions of O^{8+} to the tensor elements are very similar to the contributions of the D^+ ions. And finally, the H^+ ions contribute only to K_{xx}, K_{xy} and K_{yy} and that too only terms of order 1 and ε due to the binomial expansion of terms involving $1/v_H$. Thus the tensor elements in a four-component plasma have the form

$$
\begin{aligned}
&K_{xx}/\omega_{pD}^{2'} = (1, 2, 6)_D + (0, 1, 2)_H + (1, 2, 5)_O,\\
&K_{yy}(Z_D^2/\omega_{pD}^{2'}) = (1, 3, 6)_D + (0, 1, 2)_H + (1, 3, 5)_O + (0, 0, 1)_e,\\
&\frac{K_{xy}}{j'}\frac{Z_D}{\omega_{pD}^{2'}} = (1, 2, 5)_D + (0, 1, 2)_H + (1, 2, 5)_O + (0, 1, 0)_e,\\
&K_{xz}/\omega_{pD}^{2'} = (0, 1, 3)_D + (0, 1, 3)_O,\\
&\frac{K_{zy}}{j'}\frac{Z_D}{\omega_{pD}^{2'}} = (0, 1, 3)_D + (0, 1, 3)_O + (0, 1, 2)_e. \qquad (13)
\end{aligned}
$$

Again only one term of order $1/\varepsilon^2$, was retained for K_{zz}; the equality relations of (12) also hold good. Once more the numerals indicate the number of terms of order $1/\varepsilon$, 1 and ε.

It was shown in I that the dispersion formula for the near perpendicular propagation of ion cyclotron waves is given by

$$\frac{K_{xx}}{\omega_{pD}^{2'}}\left[-\frac{l_{\perp,D}}{\beta_{\perp,D}} + K_{yy}\frac{Z_D^2}{\omega_{pD}^{2'}}\right] + \left[\cdot K_{xy}\frac{Z_D}{\omega_{pD}^{2'}}\right]^2 = 0, \qquad (14)$$

when the dielectric tensor elements are of the above order and the equality relations of (12) hold good. $l_{\perp,D}/\beta_{\perp,D}$, in (14), is ~ 1 and $\beta_{\perp,D}$ is defined as

$$\beta_{\perp,D} = (4\pi N_D T_{\perp,D})/B_0^2.$$

Thus substituting the explicit expressions for the tensor elements K_{xx}, K_{xy} and K_{yy} (whose general form is given in (13)) in (14), simplifying and retaining terms ~ 1 we get the dispersion relation as

$$Av^2 + Bv - C - D = 0, \qquad (15)$$

where

$$A = \frac{2}{3}N_{HD}\left[\frac{4}{3}(1 + 8N_{OD}) + 2N_{HD} - 4N_{eD} + \frac{l_\perp}{\beta_\perp}\right] + N_{eD}^2$$

$$B = -(1 + 8N_{OD})\left(1 + \frac{2}{3}N_{HD} + 8N_{OD} - \frac{l_\perp}{\beta_\perp}\right) - \sigma$$

with

$$\sigma = \Bigg[(1+8N_{\mathrm{OD}})\left(\tfrac{1}{3}-2N_{\mathrm{eD}}T_{\perp,\mathrm{eD}}-\tfrac{16}{15}N_{\mathrm{HD}}T_{\perp,\mathrm{HD}}+\tfrac{1}{3}N_{\mathrm{OD}}T_{\perp,\mathrm{OD}}\right)$$

$$+(1+N_{\mathrm{OD}}T_{\perp,\mathrm{OD}})(1+\tfrac{8}{3}N_{\mathrm{HD}}+8N_{\mathrm{OD}}-4N_{\mathrm{eD}})$$

$$+(1+N_{\mathrm{OD}}T_{\perp,\mathrm{OD}})\frac{l_\perp}{\beta_\perp}\Bigg]+\frac{2}{\omega_{p\mathrm{D}}^{2'}}(1+8N_{\mathrm{OD}})$$

$$C = l_\perp^2\Bigg[\frac{3}{4}(1+8N_{\mathrm{OD}})\frac{(j+2)}{(j+1)}\left(1+\frac{N_{\mathrm{OD}}T_{\perp,\mathrm{OD}}^2}{8}\right)$$

$$-(1+N_{\mathrm{OD}}T_{\perp,\mathrm{OD}})^2\Bigg] \quad \text{and}$$

$$D = 4\frac{\theta^2 l_\parallel}{v}\left(1+N_{\mathrm{OD}}\frac{T_{\parallel,\mathrm{O}}}{T_{\parallel,\mathrm{D}}}\right)\left(1+\frac{2}{3}N_{\mathrm{HD}}+8N_{\mathrm{OD}}-\frac{l_\perp}{\beta_\perp}\right).$$

In (15) all un-subscripted parameters refer to the majority species deuterium. And as a check on our results we note that for $N_{\mathrm{HD}} = N_{\mathrm{OD}} = 0$ ($N_{\mathrm{eD}} = 1{\cdot}0$), (15) reduces to the dispersion relation in I.

Inspecting (15) we find that in the assumed ordering more terms may be dropped: for the first three terms to be of order ε^2 we need to set

$$\left(1+\frac{2}{3}N_{\mathrm{HD}}+8N_{\mathrm{OD}}-\frac{l_\perp}{\beta_\perp}\right) \quad \text{to be} \sim \varepsilon.$$

Unfortunately now the D term is of order ε^3 and thus does not contribute to the dispersion relation as the other terms are all of order ε^2. Thus the dispersion relation for the near perpendicular propagation of ion cyclotron waves is of the form $Av^2+Bv-C=0$.

5. Wave group velocity, polarization and energy

Differentiating (15) with respect to $k_\perp \gamma_{\mathrm{LD}}$ we get a dimensionless quantity $\partial Z_{\mathrm{D}}/\partial k_\perp \gamma_{\mathrm{LD}}$ which is, however, proportional to the group velocity V_{gr} of the ion cyclotron waves. We find this expression for V_{gr} to be sensitively dependent on the loss cone index j and on the number densities and temperatures of the constituents of the plasma.

The wave equation (1) can be expanded as a system of homogeneous, simultaneous equations in the perturbed electric fields E_x, E_y and E_z. These can then be solved and expressions obtained for E_z/E_x and E_y/E_x in terms of the dielectric tensor elements K_{ij}. Substituting for the expressions for the tensor elements K_{ij} from (13), one can obtain, after a long and tedious simplification, expressions for E_z/E_x and $j'E_y/E_x$.

It was found that the expression for $j'E_y/E_x \to \infty$ as $Z_{\mathrm{D}} \to 0$ (linear polarization); it is equal to 1 when $Z_{\mathrm{D}} = 1$ (circular polarization) and is greater than 1 when $Z_{\mathrm{D}} > 1$ (elliptic polarization).

The expressions for E_z/E_x and E_y/E_x can also be used to study the wave electrical energy, which we define to be

$$E^2/E_x^2 = [1+(E_y^2/E_x^2)+(E_z^2/E_x^2)]. \tag{16}$$

6. Results

We consider typical fusion conditions. $n_D = 10^{14}$ particles cm^{-3}, $T_{\perp,D} = T_{\parallel,D} = 20$ keV and $B_0 = 10$ kG. With these parameters $\beta_{\perp,D} = 0{\cdot}4023$.

Figure 1 depicts the dispersion relation (15) as plots of Z_D versus $k_\perp \gamma_{LD}$ for $\beta'_\perp = 2\beta_{\perp,D}$, $j = 10$ and all temperature ratios $= 1{\cdot}0$. The individual plots 1(a) to 1(d) correspond to plasmas of the following compositions: D^+, $D^+ + 10\%H^+$, $D^+ + 5\%$ O^{8+} and $D^+ + 10\%H^+ + 5\%O^{8+}$ respectively. Plot 1(a) also depicts (15) for a purely Maxwellian plasma ($j = 0$ and indicated by dotted lines). We find that in all the cases two modes can propagate in the plasma: a LF mode which starts with a frequency $\omega < \Omega_D$ and finally reaches $\omega \approx \Omega_D$ and an HF mode which starts at $\omega \approx \Omega_D$ and reaches $\omega > \Omega_D$. A common characteristic of all the plots is the mode conversion that exists between these waves. The existence of mode conversion between two waves propagating in an ICRF-heated plasma was anticipated very early (Jacquinot *et al* 1977); mode conversion has since then been experimentally observed in ICRF-heated D^+ and H^+ plasmas on a number of occasions (Ida *et al* 1984). Such mode conversion mechanisms are of interest in RF-heated thermonuclear plasmas as they are expected to play a key role in the understanding of wave damping mechanisms (Colestock and Kashuba 1983).

We now consider the individual plots of figure 1. Plot 1(a) depicts the variation of Z_D versus $k_\perp \gamma_{LD}$ for two values of j ($= 0$ and 10 and depicted as dotted and solid lines respectively). We find the two modes to be well-separated for $j = 0$. They, however,

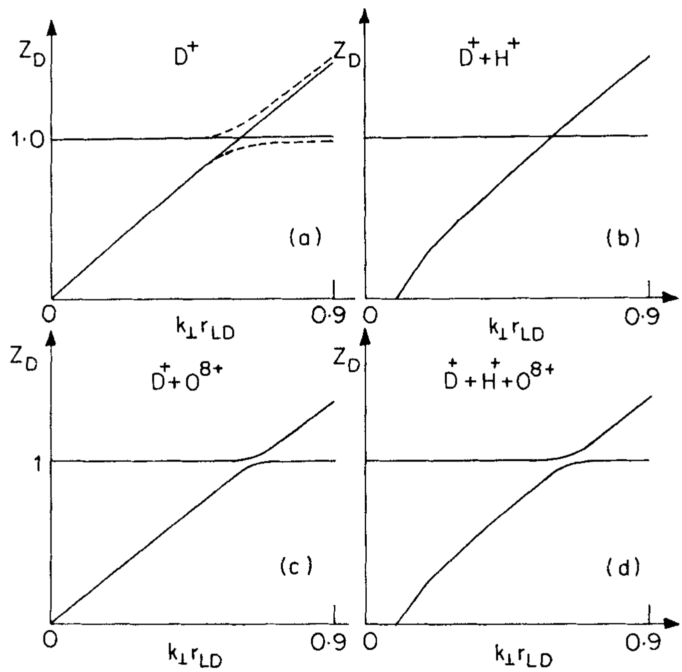

Figure 1. Plots of Z_D vs $k_\perp \gamma_{LD}$ for $\beta'_\perp = 2\beta_{\perp,D}$ and all temperature ratios equal to 1. The solid lines correspond to $j = 10$, the dotted lines of plot 1(a) correspond to $j = 0$. The plasma is stable when the dispersion curves do not intersect.

come closer for $j = 10$ suggesting that they may coalesce for some higher value of j. (A coalescing of the two modes indicates an instability as it results in a pair of complex conjugate roots for the dispersion relation). In plot 1(b), which is for a plasma containing D^+ and 10% H^+, the two modes coalesce at $k_\perp \gamma_{LD} = 0{\cdot}6$ thus indicating an instability. Thus, with all other parameters being the same, the introduction of a small amount of H^+ makes the plasma unstable. Plot 1(c) is for a plasma containing D^+ and 5%O^{8+}. We find the two modes to be well separated; this dispersion diagram closely resembles the dotted line plot in 1(a) which was for a stable Maxwellian plasma ($j = 0$). The plot of 1(d) is for a plasma of $D^+ + 10\%H^+ + 5\%O^{8+}$. Once again the two modes are well separated; the instability in plot 1(b) is thus removed by the addition of a small amount of O^{8+}. We can thus conclude that the impurity O^{8+} has a stabilizing influence on the plasma. Based on our discussions in § 5 we find that the polarizations of the modes change: the LF mode changes its polarization from linear to circular and the HF mode from circular to elliptic.

Figure 2 reveals the plots of V_{gr} versus $k_\perp \gamma_{LD}$ for $\beta'_\perp = \beta_{\perp,D}$, $j = 10$ and all temperature ratios equal to 1; the compositions of plots 2(a) to 2(d) being identical to that in plots 1(a) to 1(d). Plot 2(a) thus reveals the variation of V_{gr} vs $k_\perp \gamma_{LD}$ for two values of j, namely 0 and 10 in a plasma containing D^+ alone. When $j = 0$ (Maxwellian plasma) the group velocities of both the modes are essentially positive over the entire wavelength range studied. When $j = 10$, however, both modes have wavelength ranges over which the group velocities are negative; the LF and HF modes have peak values of negative V_{gr} at $k_\perp \gamma_{LD} = 0{\cdot}6$ and $0{\cdot}5$ respectively. Thus in a plasma prone to an instability the group velocities of the modes can have negative values. Plot 2(b) is for a

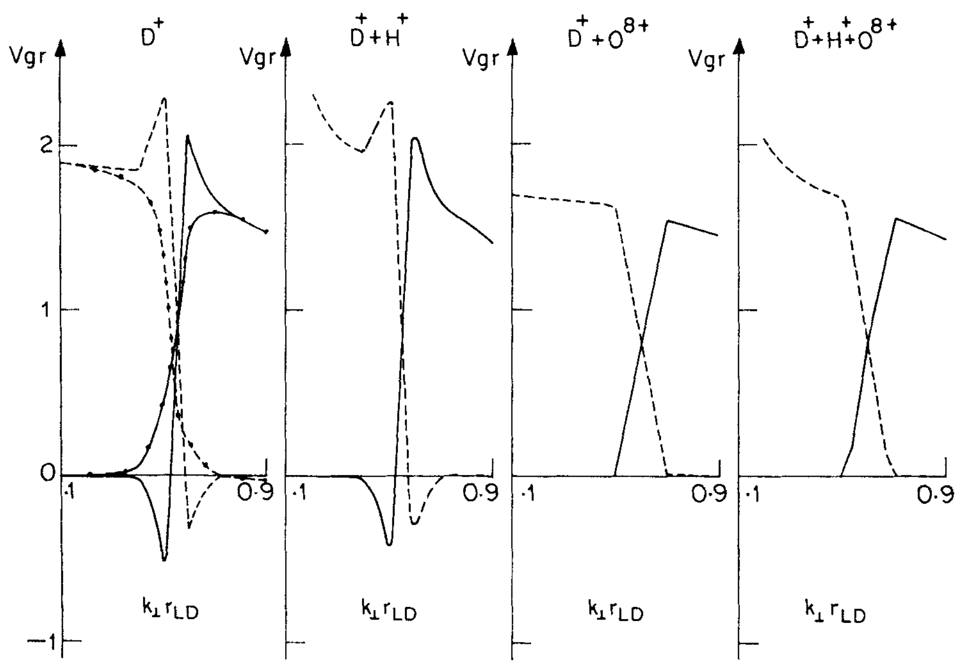

Figure 2. Plots of V_{gr} vs $k_\perp \gamma_{LD}$ for $\beta'_\perp = \beta_{\perp,D}$ and all temperature ratios equal to 1. The solid and dotted lines correspond to the group velocities of the HF and LF modes respectively for $j = 10$; the same for $j = 0$ is indicated by dots in the figures of plot 2(a).

plasma containing D^+ and $10\%H^+$ with $j = 10$. As already mentioned the addition of H^+ tends to make the plasma unstable and thus the negative group velocities of the modes persists for this case also. Plots 2(c) and 2(d) are for plasmas containing O^{8+}. In both cases the group velocities of the modes are positive. From plots 1(a), 1(c) and 1(d) we know that the plasma is stable when $j = 0$ or when it contains O^{8+} ions. Thus we may conclude from plots 2(a), 2(c) and 2(d) that the group velocities of the modes are positive in a stable plasma.

Figure 3 depicts the variation of V_{gr} with $k_\perp \gamma_{LD}$ for two values of $T_{\perp,OD}$, namely 1·0 and 8·0 for a plasma containing D^+ and $5\%O^{8+}$ with $j = 10$ and $\beta'_\perp = \beta_{\perp,D}$. Plot 3(a) is the same as 2(c); the group velocities of both the modes are positive as the plasma is stable. A solution of the dispersion relation (15) shows that the plasma is unstable for $T_{\perp,OD} \geqslant 4{\cdot}0$. In concurrence with our earlier conclusion we find that the group velocities of the modes are negative in this unstable plasma containing O^{8+}. Also the group velocities of the modes are a maximum at the coalescing point $k_\perp \gamma_{LD} = 0{\cdot}6$.

A plot similar to figure 3 was made for a plasma containing D^+ and $10\%H^+$ for two values of $T_{\perp,HD}$($= 0{\cdot}1$ and 2·0), the other parameters of the plasma being $\beta'_\perp = \beta_{\perp,D}$, $j = 10$ and $T_{\perp,eD} = 1{\cdot}0$. The plots were similar to 2(b) which was for $T_{\perp,HD} = 1{\cdot}0$. However, we find that with increasing $T_{\perp,HD}$ the maximum positive value of V_{gr} increases for the LF mode and decreases for the HF mode. The reverse, however, holds true on the negative side: V_{gr} increases for the HF mode and decreases for the LF mode.

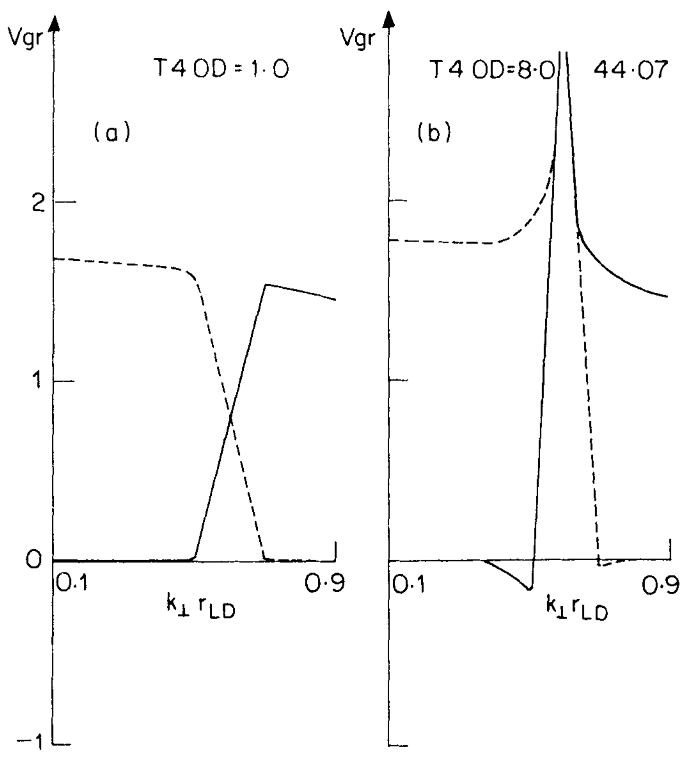

Figure 3. Plots of V_{gr} vs $k_\perp \gamma_{LD}$ in a plasma of $D^+ + 5\%O^{8+}$ for two values of $T_{\perp,OD}$($= 1{\cdot}0$ and 8·0) with $j = 10$, $T_{\perp,eD} = 1{\cdot}0$ and $\beta'_\perp = \beta_{\perp,D}$. The solid and dotted lines correspond to the HF and LF modes respectively.

We plot, in figure 4, the wave electrical energy E^2/E_x^2 versus $k_\perp \gamma_{LD}$ for a plasma characterized by $\beta'_\perp = \beta_{\perp,D}$, $j = 10$ and all temperature ratios equal to 1: the plasmas of the individual plots having the same compositions as the plasmas in plots of figures 1 and 2. We find that, in general, the LF mode to be a positive energy wave while the HF mode has a negative energy over a major portion of the wavelength region studied. Thus when an instability occurs due to a coalescing of the modes, it may be interpreted as being due to an interaction between waves of opposite electrical energy (Hasegawa 1975). We now consider the individual plots. Plot 4(a) compares the wave electrical energy for two values of j namely 0 and 10. For $j = 0$ the two modes are well separated in energy; in particular, the LF wave has a positive energy throughout the wavelength region studied. However, in a plasma prone to an instability (plot 4(a) when $j = 10$ and 4(b)), the LF mode has a wavelength region of very small negative electrical energy. In cases where the two modes coalesce (for example when $\beta'_\perp > 2\beta_{\perp,D}$ or when $T_{\perp,OD} \geqslant 4{\cdot}0$), the two modes have the same energy at the coalescing point. In stable plasmas containing O^{8+} (plots 4(c) and 4(d)) the LF mode, as for the case of a Maxwellian plasma, is a positive energy wave over the wavelength region studied. Also as can be seen from these plots the two modes are well separated in energy thus precluding any instability.

7. Conclusions

We have, in this paper, studied the near perpendicular propagation of ion cyclotron waves in a fusion plasma containing deuterium as the majority species, hydrogen as the

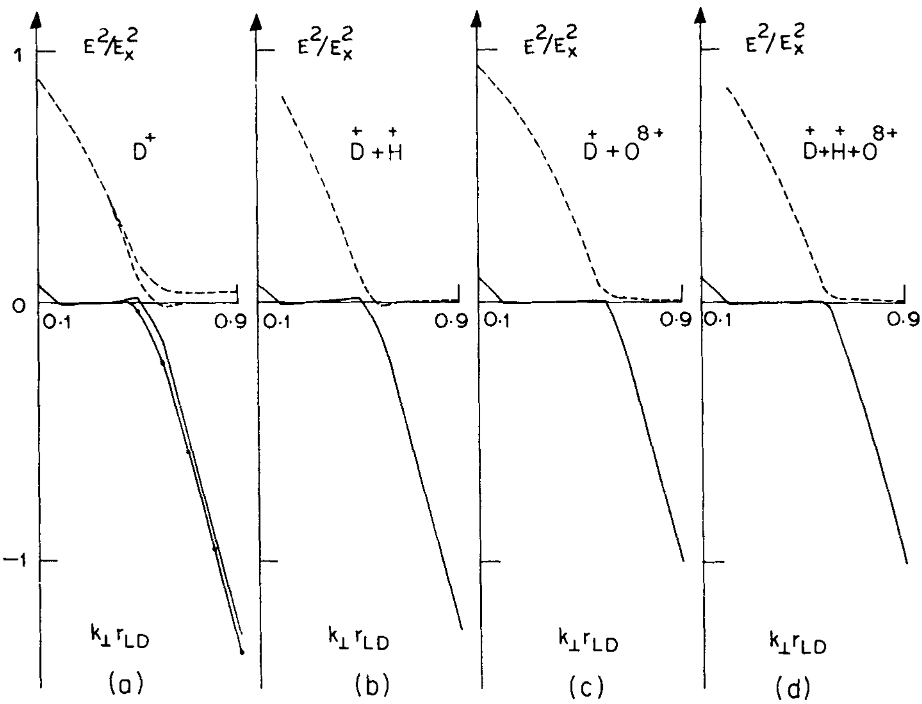

Figure 4. Plots of E^2/E_x^2 vs $k_\perp \gamma_{LD}$ for $\beta'_\perp = \beta_{\perp,D}$ and all temperature ratios equal to 1. The solid and dotted lines correspond to the HF and LF modes respectively for $j = 10$; the same for $j = 0$ in plot 4(a) is indicated by dots on the curves.

minority species and fully ionized oxygen as the impurity constituent. We find the plasma to be stable when $\beta_{\perp,\mathrm{D}}$ is low and when the perpendicular component of oxygen-to-deuterium and hydrogen-to-deuterium temperature ratios are also kept low. The resemblance of a loss cone plasma containing a small amount of impurity O^{8+} ions with a Maxwellian plasma as regards wave stability, propagation and energy has also been brought out.

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