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Slope parameter and scaling of differential cross-section of Λ-p scattering

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Abstract. The claim of Mohapatra and Maharana that $tb(s)$ is a better scaling variable than $t(\ln s)^2$ is put to test in the case of Λ-p scattering, after parametrizing $b(s)$ as $C_1 + C_2(\ln s)^\alpha$. It was observed that in this case the data also prefer an α value which is close to those obtained by Mohapatra and Maharana for other scattering processes.

Keywords. Λ-p scattering; slope parameter; scaling; unitarity bound.

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1. Introduction

During the last decade, the scaling phenomenon has been qualitatively well established. Auberson *et al* (1971) were the first to investigate the scaling of scattering amplitude, $F(s, t)$. While studying the analytic properties of the Pomeranchuk-theorem-violating amplitudes in the high energy limit, they demonstrated that for a sequence $(s_n) \to \infty$ $\lim f(s_n, \tau(\ln s_n)^{-2})$ exists and is a nontrivial function of τ where $\tau = 1(\ln s)^2$, and $f(s, t) = F(s, t)/F(s, 0)$. Cornille and Simao (1971) extended the results of Auberson *et al* (1971) for other forward high energy behaviour of the scattering amplitude and arrived at similar conclusions regarding the scaling of $f(s, t)$. Auberson and Roy (1977) deduced the bounds on slope and curvature of the diffraction peak and further observed that $\tau = t(\ln s)^2$ can be chosen as a scaling variable. Thus in the recent past, there has been a general unanimity over the conclusion that $\tau = t(\ln s)^2$ is a good scaling variable and $b(s)$ is likely to grow as $(\ln s)^2$.

However, in a recent paper Mohapatra and Maharana (1983) demonstrated that the data of pp, $\bar{p}p$, $K^{\pm}p$, $\pi^{\pm}p$ scatterings perhaps do not favour such a conclusion. Through the analysis of data they claim that (i) if $b(s)$ is parametrized as $C_1 + C_2(\ln s)^\alpha$, then the data prefer an α value close to 1·25 instead of 2, and (ii) $tb(s)$ is a better scaling variable than $t(\ln s)^2$.

In this paper we have tried to test their claim by applying their method of analysis to Λp scattering. We are conscious of the fact that (a) data on Λp scattering are scanty because of the short lifetime (de Swart *et al* 1970) of Λ beam; (b) the available data are

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in the incident momentum range of 0·13 GeV/c to 20 GeV/c only (Hauptman 1974); and (c) $(d\sigma/dt)(s, t)$ values are available only upto 6 GeV/c (Hauptman 1974). Though this momentum is not high enough to conclusively show the scaling of $(d\sigma/dt)(s, t)$ of Λp scattering, we assume that it is high enough to test the validity of the claim of Mohapatra and Maharana (1983).

The plan of the paper is as follows: In § 2 a brief summary of previous work is given as we are attempting to test the conclusions of the earlier work. Section 3 contains our analysis of data and conclusion.

2. Relevant previous work

Defining

$$f(s, t) = \frac{d\sigma}{dt}(s, t) \Big/ \frac{d\sigma}{dt}(s, 0) \tag{1}$$

and the slope and curvature, respectively, as

$$b(s) = \frac{d}{dt} f(s, t)_{t=0}, \tag{2}$$

$$c(s) = \frac{d^2}{dt^2} \ln f(s, t)_{t=0}. \tag{3}$$

Maharana (1978), using

$$w(s) = \ln \frac{s^2}{(d\sigma/dt)(s, 0)} \tag{4}$$

proved that if

$$b(s) \simeq [w(s)]^{\alpha} \tag{5}$$

for s large and all zeros, t_i, of $f(s, t)$ lie in a domain $\mathrm{Im}\, t_i < \varepsilon |t_i|^2$, ε being some s-dependent and arbitrarily small positive number and if

$$\tfrac{5}{4} \leqslant \alpha \leqslant 2, \tag{6}$$

then $\tau = tb(s)$ is a scaling variable.

This bound (equation (6)) is important for two reasons: (i) The upper bound for α is reminiscent of Froissart's bound and agrees well with the observation of Auberson and Roy (1977). (ii) The lower bound of α in (6) is of interest in the sense that if the data on various scattering processes favour this then one has to possibly take a second look at the earlier results on scaling and the Froissart's bound.

3. Analysis of data and conclusion

Keeping in view the findings of Mohapatra and Maharana (1983) we parametrized $b(s)$ as,

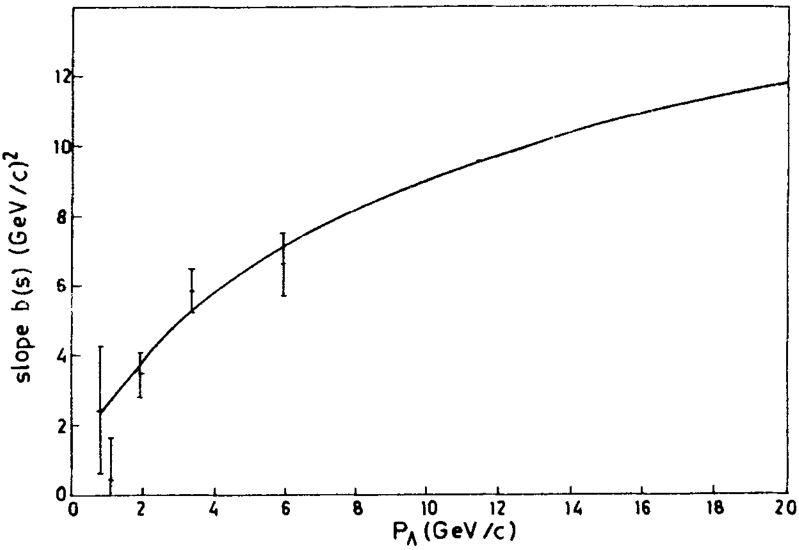

Figure 1. Fit for the slope parameter $b(s)$ and the extrapolated value of it for P_Λ value upto 20 GeV/c.

$$b(s) = C_1 + C_2 (\ln s)^\alpha \tag{7}$$

and made a least $-\chi^2$ fit to the data on $b(s)$ figure 1. For our best fit, the values of the parameters are

$$C_1 = 0{\cdot}64638, \quad C_2 = 0{\cdot}69443, \quad \text{and} \quad \alpha = 1{\cdot}35 \begin{array}{l} +0{\cdot}035 \\ -0{\cdot}047 \end{array}.$$

Assuming that our theoretical curve for $b(s)$ faithfully represents the actual $b(s)$ for Λp scattering, we generated the data on $b(s)$ by extrapolating our theoretical curve to $P_\Lambda = 20$ GeV/c for future use. As, in this analysis, we have converged our interests on the value of α only, we have not searched for the allowed errors in C_1 and C_2. The α value clearly shows that α does not saturate to the unitarity bound, rather the data also favour a value close to the lower bound of α as observed by Mohapatra and Maharana (1983). Since the value of α is consistent with the condition (equation (6)) it satisfies the requirements of scaling. Hence $\tau = tb(s) = t(C_1 + C_2 (\ln s)^\alpha)$ should be the correct scaling variable instead of $\tau = t(\ln s)^2$. However, to test this claim of Mohapatra and Maharana (1983), we are handicapped in two ways. (i) Though data on $b(s)$ are now available upto a P_Λ value of 20 GeV/c, data on $(\mathrm{d}\sigma/\mathrm{d}t)(s, t)$ are available upto a P_Λ value of 6 GeV/c. (ii) There are no published numbers for the experimental values of $(\mathrm{d}\sigma/\mathrm{d}t)(s, 0)$. However, data (Hauptman 1974) are available on $|\rho|^2$ (upto 6 GeV/c) and σ_T, where ρ is the ratio of the real to imaginary part of the scattering amplitude. We would like to use them to our advantage and note that the scattering amplitude $f(q, \theta)$ is in general given by,

$$f(q, \theta) = a_1(q, \theta) + i a_2(q, \theta), \tag{8}$$

so that $\quad \mathrm{d}\sigma/\mathrm{d}\Omega = a_1^2(q, \theta) + a_2^2(q, \theta).$

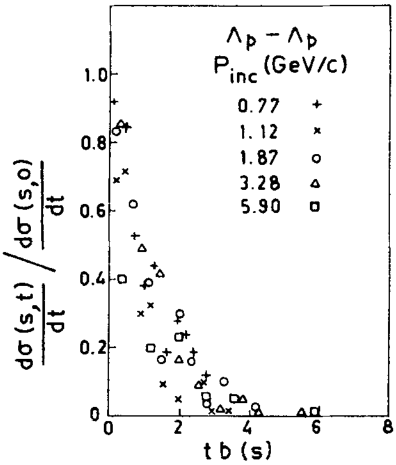

Figure 2. Plot of $\{[\mathrm{d}\sigma(s,t)]/\mathrm{d}t\}/\{[\mathrm{d}\sigma(s,0)]/\mathrm{d}t\}$ vs $tb(s)$.

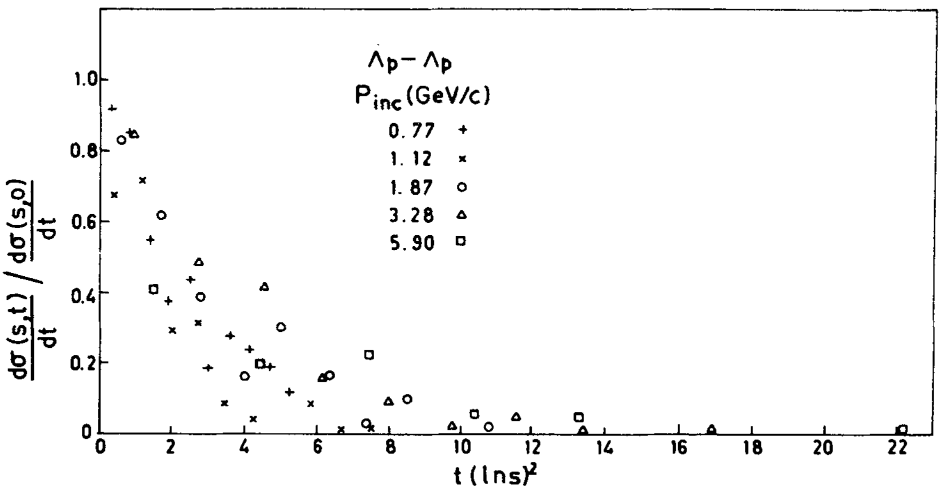

Figure 3. Plot of $\{[\mathrm{d}\sigma(s,t)]/\mathrm{d}t\}/\{[\mathrm{d}\sigma(s,0)]/\mathrm{d}t\}$ vs $t(\ln s)^2$.

Using
$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}=\frac{\mathrm{d}\sigma}{\mathrm{d}t}\frac{\mathrm{d}t}{\mathrm{d}\Omega}=A\exp(bt)\frac{q^2}{\pi}. \tag{9}$$

We obtain,
$$a_1^2(q,0)+a_2^2(q,0)=A\frac{q^2}{\pi}. \tag{10}$$

From the optical theorem we get,
$$\sigma_T=\frac{4\pi}{q}\,\mathrm{Im}\,f(q,0)=\frac{4\pi}{q}a_2(q,0). \tag{11}$$

From (10) and (11) we obtain

$$a_1^2(q, 0) = q^2\left(\frac{A}{\pi} - \frac{\sigma_T^2}{16\pi^2}\right). \tag{12}$$

Then we have

$$|\rho(q)|^2 = a_1^2/a_2^2 = (16\pi A/\sigma_T^2) - 1 \tag{13}$$

where of course $A = (\mathrm{d}\sigma/\mathrm{d}t)(s, 0)$. Thus using the data (Hauptman 1974) on $|\rho(q)|^2$ and σ_T we computed the values of A from (13) and used these numbers as our data for $(\mathrm{d}\sigma/\mathrm{d}t)(s, 0)$.

In figures 2 and 3

$$\frac{\mathrm{d}\sigma}{\mathrm{d}t}(s, t)\bigg/\frac{\mathrm{d}\sigma}{\mathrm{d}t}(s, 0)$$

versus $\tau = tb(s) = t(C_1 + C_2(\ln s)^\alpha)$ and $\tau = t(\ln s)^2$ is plotted upto a momentum range of 6 GeV/c. Since the scales in both the graphs are the same, a comparison of the spread in the data points (in the two figures) clearly indicates that the observations of Mohapatra and Maharana (1983) are correct. Also as $s \to \infty$

$$\tau \simeq C_2\, t(\ln s)^\alpha \tag{14}$$

and one is likely to think that $t(\ln s)^\alpha$ may be a good scaling variable. But this can only be tested when high energy data are available on Λp scattering. This present analysis revalidates the results of Mohapatra and Maharana (1983) that $t(C_1 + C_2(\ln s)^\alpha)$ and not $t(\ln s)^2$ could be the real scaling variable for all processes, although its universality can only be tested at energies for which $(\ln s)^\alpha$ will be comparable to C_1/C_2, and α does not necessarily saturate to the unitarity bound.

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