Pramāṇa – J. Phys., Vol. 28, No. 1, January 1987, pp. 23–34. © Printed in India.

# Phenomenological structure functions and Gribov-Lipatov relation

D K CHOUDHURY and A K MISRA[†]

Department of Physics, [†]Department of Chemistry, University of Gauhati, Gauhati 781 014, India

MS received 27 September 1984; revised 29 September 1986

**Abstract.** We make an analysis of the Gribov-Lipatov relation using the phenomenological forms of the structure function $F_2^{ep}$. Our analysis indicate breakdown of the relation at PETRA energies. Plausible reasons of the breakdown of Gribov-Lipatov relation are discussed together with its phenomenological form.

**Keywords.** Gribov-Lipatov relation; structure function; scale breaking.

**PACS Nos** 13·65; 13·60

## 1. Introduction

The present paper deals with an analysis of Gribov-Lipatov relation (Gribov and Lipatov 1971; 1972) at high energies using the phenomenological forms of structure functions proposed by various authors (Perkins *et al* 1977; Karlinger and Sullivan 1978; Roy *et al* 1978) as alternatives to QCD structure (Buras and Gaemers 1978).

Phenomenological analysis of Gribov-Lipatov relation was done earlier by Gilman (1974) and Brandelik *et al* (1979) using the empirical structure functions of Miller *et al* (1972) and Riordan *et al* (1974) respectively. While Gilman (1974) observed approximate validity of the Gribov-Lipatov relation, Brandelik *et al* (1979) pointed out its violation. There was also an attempt to study scale-breaking patterns (Choudhury 1976; Choudhury and Vanryckghem 1978) assuming the validity of Gribov-Lipatov relation.

The parametrization of the phenomenological structure functions used by some of the authors (Perkins *et al* 1977; Karlinger and Sullivan 1978; Roy *et al* 1978) is essentially determined by low $Q^2$ data ($Q^2 \sim 15$–$20\,\mathrm{GeV}^2$). We will therefore test their validity in the high $Q^2$ regime of EMC (Aubert *et al* 1981; Dress 1981) and CDHS (Wotschalk 1981) before using them in our analysis at high $Q^2$ PETRA data ($Q^2 \sim 900\,\mathrm{GeV}^2$).

The inference of our analysis might be dependent on the form of low $Q^2$ structure functions used (Perkins *et al* 1977; Karlinger and Sullivan 1978; Roy *et al* 1978). Hence we will also use the parametrization from high $Q^2$ data of CDHS (Abramowicz *et al* 1983), EMC (Aubert *et al* 1981) and NA 10 (Betev *et al* 1985) to study the status of the Gribov-Lipatov relation. Low $Q^2$ region is generally assumed to have significant higher twist effect (Abbott *et al* 1980; Aubert *et al* 1981; Godbole and Roy 1982). We will comment on such a possibility in our analysis. Finally the plausible reasons of the breakdown of Gribov-Lipatov relation will be discussed together with its phenomenological form.

## 2. Formalism

The Gribov-Lipatov relation (Gribov and Lipatov 1971; 1972) is defined as

$$F_2^{ep}(x,Q^2)=\frac{x}{\beta\sigma(e^+e^-\to\mu^+\mu^-)}\frac{d\sigma}{dx}(e^+e^-\to\bar{P}+X), \tag{1}$$

where $\beta$ is the ratio of the momentum and energy of $\bar{P}$ and other variables have usual meanings.

In order to test the relation (1), one needs to know the theoretical structures of $F_2^{ep}(X,Q^2)$ and experimental information about $(d\sigma/dx)$ $(e^+e^-\to\bar{P}+X)$. In the scaling model, $F_2^{ep}$ is a function of quark distribution $V_u$, $V_d$, $\xi$ and $\xi_c$ defined as

$$F_2^{ep}(x,Q^2)=x\left[\tfrac{4}{9}V_u(x)+\tfrac{1}{9}V_d(x)+\tfrac{4}{3}\xi(x)+\tfrac{8}{9}\xi_c(x)\right]. \tag{2}$$

Here $V_u$ and $V_d$ denote the valence contributions, $\xi$ and $\xi_c$ represent SU(3) symmetric and $c\bar{c}$ component of the sea respectively. These have been parametrized as (Berger and Phillips 1974):

$$\begin{aligned}V_u&=0{\cdot}594x^{-1/2}(1-x^2)^3+0{\cdot}461x^{-1/2}(1-x^2)^5+0{\cdot}621x^{-1/2}(1-x^2)^7,\\ V_d&=0{\cdot}072x^{-1/2}(1-x^2)^3+0{\cdot}206x^{-1/2}(1-x^2)^5+0{\cdot}621x^{-1/2}(1-x^2)^7,\\ \xi&=0{\cdot}145x^{-1}(1-x)^9,\end{aligned} \tag{3}$$

using the SLAC data (Riordan *et al* 1974) on $F_2^{ep}$ and $F_2^{en}$. By all evidence, the charm component $\xi_c$ is strongly suppressed (Sivers 1976) at low $Q^2$ and is taken to be zero in conformity with common practice (Buras and Gaemers 1978). Equation (3) will be taken as the low $Q^2$ input in the scale-breaking models discussed below.

Let us now record the phenomenological scale-breaking models as $F_2^{ep}$ to be studied later.

### 2.1 *PSS models*

Perkins *et al* (1977) have shown that scale-breaking in electron (muon) scattering on both proton and isoscalar target can be described in terms of a single form factor:

$$F_2(x,Q^2)=F_2(x,Q_0^2)\,(Q^2/Q_0^2)^{0{\cdot}25-x}, \tag{4}$$

where $Q_0^2$ is 3 GeV$^2$.

With the same form factor, they were also able to describe the $\nu P$ and $\nu N$ data. Since these four processes correspond to different combinations of the valence and the sea quark distribution, the model amounts to assuming identical forms for $V_u$, $V_d$, $\xi$ and $\xi_c$:

$$\begin{aligned}V_u(x,Q^2)&=V_u(x,Q_0^2)\,(Q^2/Q_0^2)^{0{\cdot}25-x},\\ V_d(x,Q^2)&=V_d(x,Q_0^2)\,(Q^2/Q_0^2)^{0{\cdot}25-x},\\ \xi(x,Q^2)&=\xi(x,Q_0^2)\,(Q^2/Q_0^2)^{0{\cdot}25-x},\\ \xi_c(x,Q^2)&=\xi_c(x,Q_0^2)\,(Q^2/Q_0^2)^{0{\cdot}25-x}.\end{aligned} \tag{5}$$

We shall use the parametrization of (3) for the low $Q^2$ input on the right hand side of (4). The variation of the effective $Q^2$ with $x$ in the parametrization, (3) will be taken care of by re-expressing the above equation as (Roy *et al* 1978)

(i) *PSS(I)*:

$$V_{u,d}(x, Q^2) = V_{u,d}(x, Q_1^2(x)) \left(\frac{Q}{Q_1^2(x)}\right)^{0\cdot25-x}, \tag{6}$$

where $Q_1^2(x)$ is the average $Q^2$ for this parametrization for a given $x$ given by (Roy *et al* 1978)

$$Q_1^2(x) \sim 13\cdot3x. \tag{7}$$

To check the sensitivity of the prediction of the Gribov-Lipatov relation on the latitude allowed by the ep data, we shall also use an alternative parametrization proposed by Roy *et al* (1978).

(ii) *PSS(II)*:

$$V_{u,d}(x, Q^2) = V_{u,d}(x, Q_1^2(x)) \left(\frac{Q^2}{Q_1^2(x)}\right)^{0\cdot075-0\cdot5x} \tag{8}$$

which is designed to mimic the QCD model (Buras and Gaemers 1978).

## 2.2 *KS models*

Karlinger and Sullivan (1978) considered two different parametrization of scale breaking in $eN$ and $\mu N$ scattering, one involving logarithmic type as suggested by asymptotic freedom and the other a power type, suggested by conventional renormalizable field theory. These are given by

(i) *KS(I)*

$$F_2^{\log}(x, Q^2) = \left(1 + b(x)\frac{\ln (Q^2/Q_0^2)}{\ln (Q_0^2/\Lambda^2)}\right) F_2(x, Q_0^2), \tag{9}$$

with $Q_0^2 = 3\,\mathrm{GeV}^2$, $\Lambda = 0\cdot4\,\mathrm{GeV}^2$, $b(x) = b_0[1-(x/x_0)^\alpha]$, $x_0^{-1} = 4\cdot489$, $b_0 = 0\cdot804$ and $\alpha = 0\cdot6343$. It is to be noted that (9) is to be used recursively for high $Q^2$ range.

(ii) *KS(II)*

$$F_2^{\text{power}}(X, Q^2) = (Q^2/Q_0^2)^{f(x)} F_2(X, Q_0^2) \tag{10}$$

with $Q_0^2 = 3\,\mathrm{GeV}^2$, $f(x) = a + bx + \varepsilon/x$, $a = 0\cdot21324$, $b = -0\cdot97955$ and $\varepsilon = 5\cdot9 \times 10^{-4}$.

## 2.3 *Effect of longitudinal cross-section*

We shall now consider the effect of non-zero longitudinal ($\sigma_L$) to transverse cross-section ($\sigma_T$) ratio

$$R = \frac{F_2}{xF_1} - 1. \tag{11}$$

This is incorporated simply by multiplying the left side of (1) by

$$\frac{1}{2}\left(\frac{3}{R+1}-1\right). \tag{12}$$

The modification of our results due to (12) will be shown for typical values $R = 0{\cdot}0$, 0·31 and 0·76.

## 3. Results

### 3.1 *Comparison of the phenomenological forms of the structure function in high $Q^2$ region*

In figure 1(a–d) we show the comparison with EMC data for a few representative $x$ values $x = 0{\cdot}03$, 0·25, 0·45 and 0·65. In figures (2) and (3) the corresponding comparisons are made with the CDHS data. In figure 1, we observe that PSS(II) fits the overall data better than the other three forms of the structure function viz PSS(I), KS(I) and KS(II). A similar inference about PSS(II) can also be drawn even from figure 2 with high $x$ CDHS data of $F_2^{\nu N}$. Figure 3 representing all the four models (KS I, KS II, PSS I and PSS II) for $xF_3$ does not however show such a definite trend. Hence PSS(II) may be preferred over other models of $F_2^{ep}$ and $F_2^{\nu N}$ both in EMC and CDHS data but cannot be preferred over them in $xF_3$ CDHS data. We however note that it is $F^{ep}$ that appear in our defining equation (1). We therefore select PSS(II) out of the four models to test it.

PSS(II) is still a phenomenological parametrization at low $Q^2$. There are high $Q^2$ parametrizations directly from EMC (Aubert *et al* 1981), CDHS (Abramowicz *et al* 1983) and NA 10 (Betev *et al* 1985) data. We will also test (1) with these high $Q^2$ parametrizations as well besides with PSS(II).

### 3.2 *Analysis of Gribov-Lipatov relation*

We now compare the predictions of the formalism with data for

$$\frac{x}{\beta\sigma(e^+e^-\to\mu^+\mu^-)}\frac{\mathrm{d}\sigma}{\mathrm{d}x}(e^+e^-\to\bar{P}+X).$$

To that end, we take data at the centre of mass energy $\sqrt{S} = 3{\cdot}74$, 4·72, 5·2 and 34·0 GeV from earlier results (Gilman 1974; Brandelik *et al* 1979; Mess and Wiik 1982; Wolf 1980; Jade Collaboration 1981).

In figure 4(a–d) we show the predictions of the models of $F_2^{ep}$ defined as PSS(II). In the same figure we also show the predictions of the parametrizations given by EMC, CDHS and NA 10 graphs. Although the large $x$ data at $\sqrt{S} = 3{\cdot}74$ GeV reported by Gilman (1974) give good agreement with the models under consideration, the rest of the data indicates break down of (1).

In fact the theoretical curves in general fall below the experimental values of

$$\frac{x}{\beta\sigma(e^+e^-\to\mu^+\mu^-)}\frac{\mathrm{d}\sigma}{\mathrm{d}x}(e^+e^-\to\bar{P}+X).$$

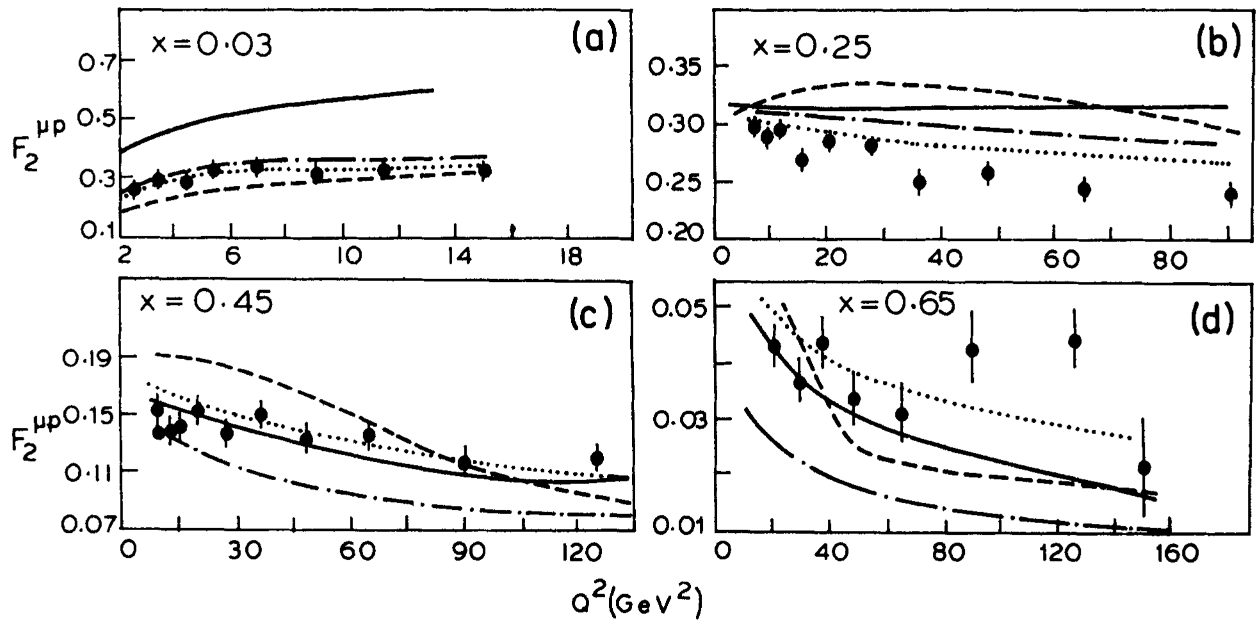


**Figure 1(a–d).** Comparison of EMC data $F_2^{\mu p}$ with phenomenological structure functions PSS(I) (continuous curve) PSS(II) (dotted curve), KS(I) (dashed curve) and KS(II) (dashed dot dashed curve) for $x = 0{\cdot}03$, $0{\cdot}25$, $0{\cdot}45$ and $0{\cdot}65$ respectively.

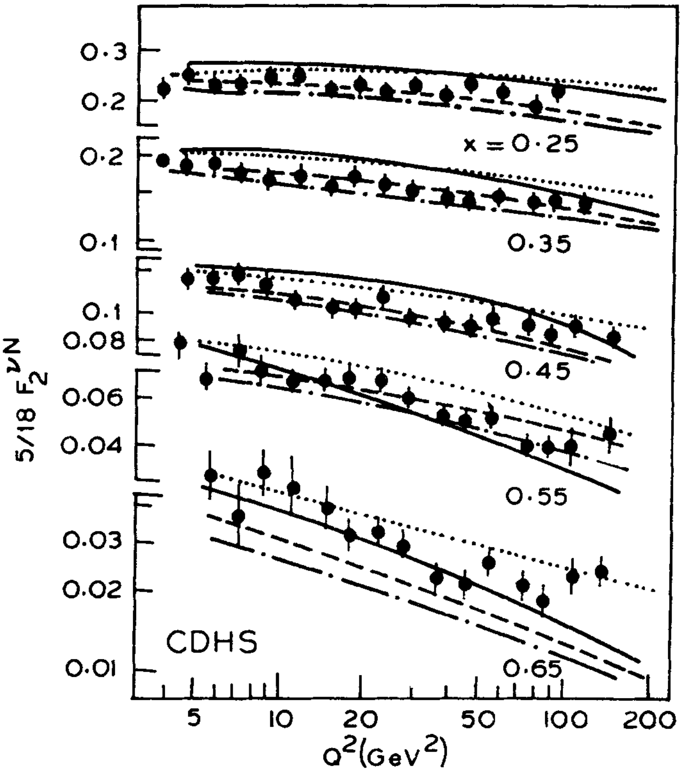


**Figure 2.** Comparison of CDHS data of (5/18) $F_2^{\nu N}$ with PSS (I), PSS (II), KS (I), KS (II) for $x = 0{\cdot}25$, 0·35, 0·45, 0·55 and 0·65. Curves have the same meaning as of figure 1.

Such an effect conveniently can be studied by defining the quantity

$$\bar{R}(x, Q^2) = \frac{F_2^{ep}(x, Q^2)}{\dfrac{x}{\beta\sigma(e^+e^- \to \mu^+\mu^-)} \dfrac{d\sigma}{dx}(e^+e^- \to \bar{P} + X)_{\text{expt}}}. \tag{13}$$

In figure 5(a–d) we evaluate typical values of $\bar{R}(x, Q^2)$ for $\sqrt{S} = 3{\cdot}65$, 4·42, 5·2 and 34·0 GeV respectively using an approximate QCD structure function (Buras and Gaemers 1978) resulting in $\bar{R} < 1$.

Let us now discuss the variation of our predictions with $R = (\sigma_L/\sigma_T)$.

In figure 6(a, b, c) we show the variation of

$$\frac{x}{\beta\sigma(e^+e^- \to \mu^+\mu^-)} \frac{d\sigma}{dx}(e^+e^- \to \bar{P} + X)$$

with $R$ using PSS II as a typical model of $F_2^{ep}$ and taking $R = 0{\cdot}0$, 0·31 and 0·76 at three typical energies $\sqrt{S} = 12$, 30 and 34 GeV. Figures 6(a, b, c) demonstrate that increase of $R$ makes the disagreement between the data and Gribov-Lipatov relation (1) more prominent.

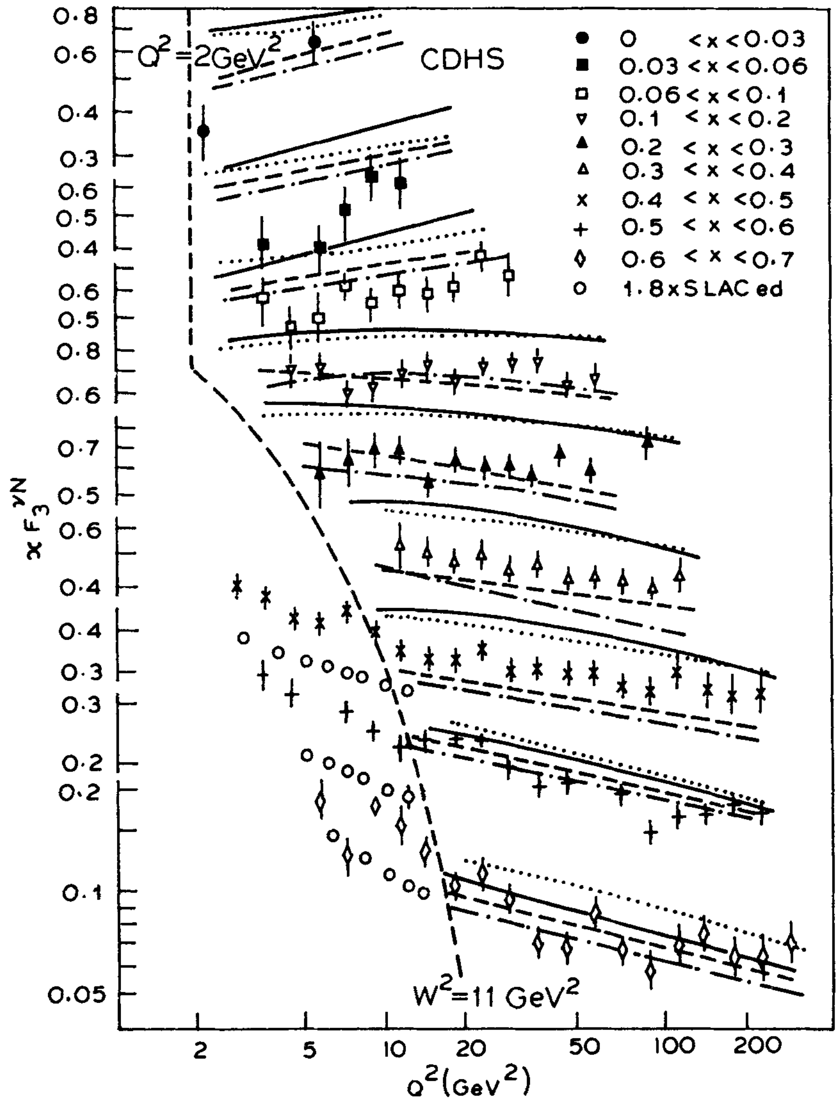


**Figure 3.** Comparison of $xF_3^{\nu N}$ from CDHS data with PSS(I), PSS(II), KS(I) and KS(II) for various ranges of $x$. Curves have the same meaning as of figure 1.

### 3.3 *Higher twist (HT) contribution*

In order to consider the effect of higher twist (HT) in the low end of $e^+e^-$ energy we assume the following form of the structure function (Abbott *et al* 1980; Abuert *et al* 1981; Bollini *et al* 1981; Godbole and Roy 1982)

$$F_2(x, Q^2) = F_2(x, Q^2)\left(1 + \frac{\mu^2}{(1-x)Q^2}\right). \tag{14}$$

The effect of (14) is shown in figure 5(a–d).

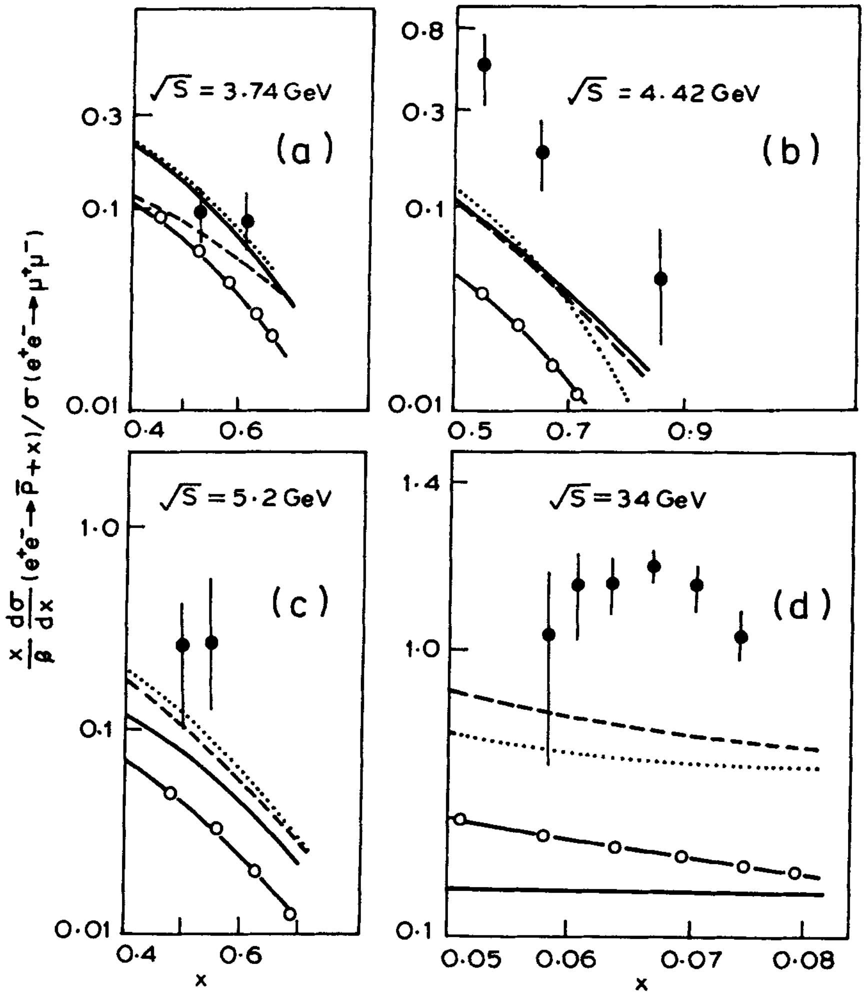


**Figure 4 (a–d).** Theoretical predictions at 34 GeV using PSS (II) (dotted curve) as well as parametrization of EMC (dashed curve) CDHS (continuous curve) and NA 10 (dashed crossed dashed curve) data.

A study of these figures shows that the higher twist effect enhances the value of $\bar{R}$ slightly, but not significant enough to alter the feature of breakdown of (1) as we have already obtained.

### 3.4 *Implication of violation of Gribov-Lipatov relation and its plausible significance*

Let us discuss the plausible theoretical significance of breakdown of Gribov-Lipatov relation (1).

The perturbative QCD gives this relation exactly as a result of leading logarithmic approximation for the case of single parton distribution (Konishi *et al* 1978; Dokshitzer 1977). However it is found (Kubota 1980; Floratas *et al* 1981) that even for single parton distribution, the next to leading logarithmic corrections in perturbative QCD

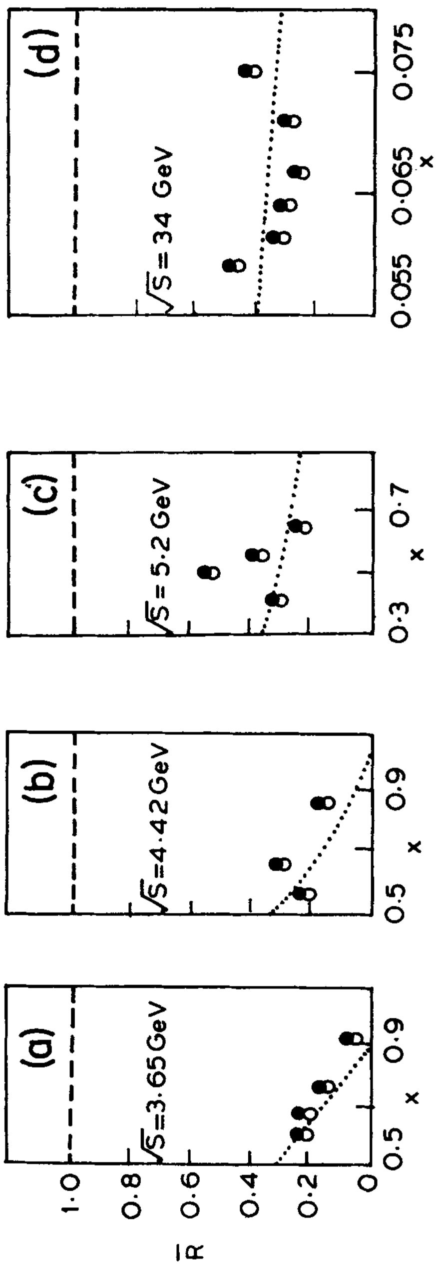


**Figure 5(a–d).** Values of $\bar{R}(x, Q^2)$ vs $x$ for $\sqrt{S}$ = 3·65, 4·42, 5·2 and 34 GeV (open circle). Effects of higher twist are shown by closed circles. The dotted curve corresponds to the prediction of Kato *et al* (1983).

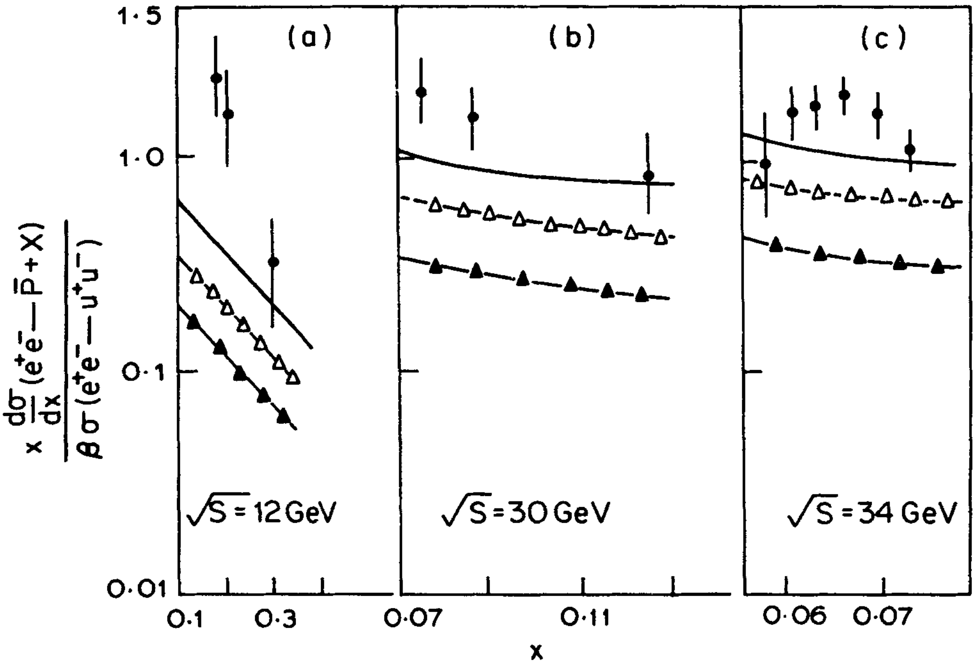


**Figure 6(a–c).** Variation with $R = 0{\cdot}0$ (continuous curve), 0·31 (dotted crossed dotted curve) and 0·76 (dashed cross dashed curve) for typical $\sqrt{S} = 12$, 30 and 34 GeV with PSS(II).

violates this relation. Similarly in the case of two-parton distribution which is required for hadronization, relation (1) might break down (Konishi *et al* 1978; Dokshitzer 1977).

Within the perturbative QCD itself there are other arguments as well on the implication of breakdown of Gribov-Lipatov relation. On the basis of pre-confinement hypothesis of Amati and Veneziano (1978), Kawabe (1981) studied the hadronization of quark jets and finds violation of Gribov-Lipatov relation. This is then interpreted as the evidence of the realization of the preconfinement picture. We note that in the preconfinement picture, the perturbative QCD itself prepares the colour singlet clusters with finite masses before their eventual fragmentation into hadrons. Thus within perturbative QCD, the violation of (1) indicates the presence of next to leading logarithmic correction and/or the validity of the preconfinement hypothesis. As $\bar{R} < 1$ invariably, one might also infer that $\bar{P}$ is produced in the sequential processes $e^+e^- \to h^+ + x$, $h^+ = \bar{\Lambda} \to \bar{P}$, $h^+ = \bar{\Sigma} \to \bar{P}$ etc besides the direct $(e^+e^- \to \gamma \to \bar{P} + x)$. Furthermore, massive violation of (1) at the hadronization level is perhaps the reason why $\bar{R}$ is so much away from unity ($\bar{R} \leqslant 0{\cdot}6$).

Let us now comment on the possible phenomenological expression of (13) as a measure of breakdown of (1). Such an expression of $\bar{R}$ can be obtained once the forms of the structure function $F_2^{ep}$ and $(\mathrm{d}\sigma/\mathrm{d}x)$ $(e^+e^- \to \bar{p} + x)$ are known. Taking PSS (II) as the typical structure function and the renormalized distribution of Kato *et al* (1983) from Monte Carlo simulation, we plot $\bar{R}(x, Q^2)$ vs $x$ for $\sqrt{S} = 3{\cdot}65$, 4·42, 5·2 and 34 GeV in figure 5(a–d) representing the gross feature.

## 4. Conclusion

To conclude, the Gribov-Lipatov relation breaks down within the present energy regime assuming the validity of the forms of the structure functions discussed in § 2. Such a breakdown was earlier pointed out by Brandelik *et al* (1979) using the empirical structure function of Riordan *et al* (1974). In the present work, we have confirmed its breakdown for higher energies using the recent models of structure function. Plausible reasons of its breakdown are discussed within QCD. Using a distribution given by Kato *et al* (1983) its gross feature is also represented phenomenologically.

## Acknowledgements

Both of us are grateful to Professor G Wolf of DESY for assistance with recent data of $e^+e^-$ physics. One of us (DKC) gratefully acknowledges financial support from the Department of Science, Technology and Environment of Government of Assam.

## References

Abbott L F, Atwood W B and Barnett R M 1980 *Phys. Rev.* **D22** 582
Abramowicz H *et al* 1983 *Z. Phys.* **C17** 283
Amati D and Veneziano G 1979 *Phys. Lett.* **B83** 87
Aubert J J *et al* 1981 *Phys. Lett.* **B105** 315
Berger V and Phillips R J N 1974 *Nucl. Phys.* **B73** 269
Betev B *et al* 1985 *Z. Phys.* **C28** 15
Bollini D *et al* 1981 *Phys. Lett.* **B104** 403
Brandelik *et al* 1979 *Nucl. Phys.* **D148** 189
Buras A J and Gaemers K J F 1978 *Nucl. Phys.* **B132** 249
Choudhury D K 1976 *Nuovo Cimento* **A31** 669
Choudhury D K and Vanryckeghem L G F 1978 *Phys. Rev.* **D17** 1766
Dokshitser Yu L 1977 *Sov. Phys. JETP* **46** 641
Dress J 1981 *Proc. Int. Symp. on Lepton and Photon Interaction at High Energies*, Bonn (ed.) W Pfeil
Floratas E G, Lacoze R and Kounnas 1981 *Phys. Lett.* **B98** 89
Gilman 1974 *Proc. Int. Conf. (XVII) on High Energy Physics, London* (ed.) J P Smith (Rutherford Laboratory, Chilton, Did cot, Barkeshire, England)
Godbole R M and Roy D P 1982 *Z. Phys.* **C15** 39
Gribov V N and Lipatov L N 1971 *Phys. Lett.* **B37** 78
Gribov V N and Lipatov L N 1972 *Yad. Fiz.* **15** 781
Gribov V N and Lipatov L N 1972 *Sov. J. Nucl. Phys.* **15** 438
Jade Collaboration 1981 DESY 81/028 (unpublished)
Karlinger I and Sullivan J D 1978 *Phys. Rev.* **D18** 3202
Kato K, Munehisa T, Okada H and Shimizu Y 1983 The perturbative QCD and the $e^+e^-$ single particle Inclusive Reaction, University of Tokyo preprint (UT-387)
Kawabe T 1981 *Prog. Theor. Phys.* **65** 1973
Konishi K, Ukaya A and Veneziano 1978 *Phys. Lett.* **B78** 24
Kubota T 1980 *Nucl. Phys.* **B165** 277
Mess K H and Wiik B H 1982 DESY 82/011 (unpublished)
Miller D H *et al* 1972 *Phys. Rev.* **D5** 528
Perkins D H, Schreiner P and Scott W C 1977 *Phys. Lett.* **B67** 347
Riordan E M, Bodek A and Coward D H 1974 *Phys. Lett.* **B52** 249
Roy D P, Paranjepe S, Pandita P N and Choudhury D K 1978 *Pramana – J. Phys.* **11** 619

Sivers D 1976 *Nucl. Phys.* **B106** 95
Wolf G 1980 DESY 80/85 (unpublished)
Wotschalk J 1981 *Proc. Int. Symp. on Lepton and Photon Interaction at High Energies* Bonn (ed.) W Pfeil