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Tree-level breaking of SU(2) × U(1) in general SUGRA theories

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Abstract. It is shown that SU(2) × U(1) can be broken at the tree level, without the occurrence of global potential minima that break $U(1)_{e.m}$, in supergravity models that are more general than those proposed by Nilles, Srednicki and Wyler. The study comprises an analysis of models with a general soft supersymmetry-breaking structure as well as those of the Hall-Lykken-Weinberg type.

Keywords. SU(2) × U(1) theory; supergravity; spontaneous symmetry breakdown.

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1. Introduction

It is now realized# that the breaking of supersymmetry (SUSY) in local SUSY theory, via the super-Higgs effect, generates an effective theory capable of incorporating the spontaneous breakdown of electroweak symmetry. If matter is coupled 'minimally' (Ellis 1983) to supergravity (SUGRA) and if the Higgs sector consists only of SU(2)-doublet fields, then the said breakdown does not occur at the tree level of the effective theory below the Planck mass M_P. Radiative corrections can trigger the breaking (Alvarez Gaume *et al* 1983; Ellis *et al* 1983) if some of the Yukawa couplings are large: this typically requires a top quark with mass ~ 45–50 GeV##.

The electroweak symmetry can be broken at the tree level itself if an SU(2) × U(1)-singlet superfield is present (Nilles *et al* 1983a). The singlet is known, however, (a) to destabilize the gauge hierarchy (Nilles *et al* 1983b; Labanas 1983) and (b) to induce global minima of the potential (Frere *et al* 1983) which spontaneously break the conservation of electric charge. Both these problems have been discussed in a special class of models proposed by Nilles *et al* (1983a), hereafter referred to as NSW. We point out in this paper that the above mentioned problem (b) does not exist in models more general than those of NSW. Coupled with the fact that there exists a way (Ferrara *et al* 1983) to solve problem (a), our observation makes tree-level breaking of SU(2) × U(1) a

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For a review and original references, Ellis (1983).

This is not an inescapable conclusion: SU(2) × U(1) breaking can also be driven with a light top quark in some models (Claudson *et al* 1983).

viable alternative for the construction of realistic models.

In § 2 we review the problem of electric-charge-breaking minima and discuss its solution with a potential which has a general soft SUSY-breaking structure. Potentials of the Hall-Lykken-Weinberg (HLW) type are considered in § 3, and it is argued that charge-conserving global minima probably exist in this case too. Appendix A gives details of minimization of an HLW potential.

2. Charge-preserving minima in general SUGRA theories

It is customary in a SUGRA theory to divide the spin-0 fields into two subsets: $S_A \sim (Z_i, Y_a)$. The 'hidden' fields Z_i acquire vacuum expectation values (VEV's) of $O(M_P)$, while Y_a are the ordinary fields. The scalar potential V at an energy scale M $(M = M_P/\sqrt{8\pi})$ is given by (Ellis 1983).

$$V = \exp(d/M^2)\left(d^{-1}_{AB}F^{\dagger}_{A}F_B - 3\frac{|W|^2}{M^2}\right) + \tfrac{1}{2}D^{\alpha}D^{\alpha}. \tag{1}$$

Here d is an arbitrary function of S_A and $S^{\dagger}_A$, and the super-potential W depends on S_A. Moreover,

$$d_{AB} = \frac{\partial^2 d}{\partial S_A \partial S^{\dagger}_B},$$

$$F_A = \frac{\partial W}{\partial S_A} + M^{-2}\frac{\partial d}{\partial S_A}W,$$

and

$$D^{\alpha} = \frac{\partial d}{\partial S_A} t^{\alpha}_{AB} S_B,$$

where (t^{α}) are generators of the internal-symmetry group.

The low energy $(\sim M_W)$ limit of V can be derived by eliminating hidden fields as well as those having $O(M_{\rm GUT})$ masses. Three cases can be distinguished:

(a) Here $d = S^{\dagger}_A S_A$, which brings the kinetic energy for scalars into the canonical form, and W is taken as the sum of separate terms involving hidden and ordinary fields respectively:

$$W(S_A) = f(Z_i) + \tilde{g}(Y_a). \tag{2}$$

If one neglects the presence of the GUT sector, the following effective potential is obtained at a scale $\sim M_W$ (Nilles *et al* 1983a)

$$V_{\rm eff} = |g_a|^2 + m^2_{3/2}Y^{\dagger}_a Y_a + m_{3/2}\left[(A-3)g(Y_a) + Y_a\frac{\partial g}{\partial Y_a} + \text{c.c.}\right], \tag{3}$$

where $g = \tilde{g}\exp(\langle Z^{\dagger}_i Z_i\rangle/M^2)$, $g_a = \partial g/\partial Y_a$ and A is a numerical constant. The mass parameter $m_{3/2}$ coincides with the gravitino mass.

(b) If, in addition to the above, the superpotential $g(Y_a)$ is assumed to depend on GUT fields, then after integrating out the latter one obtains the HLW potential (Hall *et al* 1983).

$$V_{\text{eff}} = \left|\frac{\partial g_{\text{eff}}}{\partial Y_m}\right|^2 + [m_{3/2}^2 g_3 + 2m_{3/2}\, g_2 + (4m_{3/2} - m'_{3/2})g_1 + \text{c.c.}]$$
$$+ m_{3/2}^2 |Y_m|^2 + \tfrac{1}{2} D^\alpha D^\alpha \tag{4}$$

$g_{\text{eff}} = g_3 + g_2 + g_1 + g_0$. In the above equation g_i is a homogeneous polynomial of degree i in the light fields Y_m, and $m'_{3/2}$ is a parameter of the same order of magnitude as $m_{3/2}$.

(c) Here W and d are taken to have the following most general allowed structure (Soni and Weldon 1983).

$$W = M^2 W_2(\xi_i) + M W_1(\xi_i) + W_0(Y_a, \xi_i), \tag{5a}$$

$$d = M^2 d_2(\xi_i, \xi_i^\dagger) + M d_1(\xi_i, \xi_i^\dagger) + d_0(\xi_i, \xi_i^\dagger, Y_a, Y_a^\dagger) \tag{5b}$$

with $$\left\langle \frac{\partial^2 d}{\partial Y_a \partial Y_b^\dagger} \right\rangle \equiv \frac{\partial^2 d_0(\langle \xi_i \rangle, \langle \xi_i^\dagger \rangle, Y_a, Y_a^\dagger)}{\partial Y_a \partial Y_b^\dagger} = \delta_{ab}, \qquad \xi_i \equiv Z_i/M.$$

The resulting V_{eff} describes a theory with SUSY broken softly by terms of the most general possible structure consistent with the absence of quadratic divergences.

$$V_{\text{eff}} = \left|\frac{\partial g}{\partial Y_a}\right|^2 + m_{3/2}^2\, S_{ab} Y_a^\dagger Y_b + m_{3/2}\,(h(Y_a) + \text{c.c.}). \tag{6}$$

Here g and h are *a priori* independent cubic polynomials in the fields Y_a, depending on the form of W_0 and d_0, while S_{ab} is a numerical matrix. In the special case $d = Z^\dagger Z^\dagger$, S_{ab} reduces to the identity matrix.

The simplest V_{eff} of (3) can break SU(2) × U(1) at the tree level. Consider two Higgs doublets H, H' with hypercharges $+1/2$, $-1/2$, and a singlet Y, with the following g:

$$g = \lambda H H' Y + \tfrac{1}{3}\sigma Y^3. \tag{7}$$

We are led, through (3), to the following V_{eff}:

$$V_{\text{eff}} = |\lambda|^2 (|H|^2 + |H'|^2)|Y|^2 + |\lambda H H' + \sigma Y^2|^2 + m_{3/2}^2 (|H|^2 + |H'|^2 + |Y|^2)$$
$$+ m_{3/2} A(\lambda H H' Y + \tfrac{1}{3}\sigma Y^3 + \text{c.c}). \tag{8}$$

It is possible to minimize this V_{eff} exactly (Joshipura *et al* 1986), although only approximate minima are given in the literature (Ellis 1983; Nilles *et al* 1983a; Frere *et al* 1983). The global minimum occurs at

$$\langle H^0 \rangle = \langle H^{0\prime} \rangle = \lambda^{-1} m_{3/2} \left[\frac{A^2 \sigma'}{(1+2\sigma')^2} - 1\right]^{1/2}$$

$$\langle Y \rangle = \lambda^{-1} m_{3/2}\, \frac{A}{1+2\sigma'}, \tag{9}$$

where $H^0(H^{0\prime})$ is the neutral component of $H(H')$ and $\sigma' = \sigma/\lambda$ provided

$$|A| > \frac{2\sigma' + 1}{(\sigma')^{1/2}}.$$

Thus SU(2) × U(1) is broken at the tree level. The potential at the minimum is given by (Joshipura *et al* 1986)

$$V_{\min} = \lambda^{-2} m_{3/2}^4 \left[-\frac{2\sigma'}{3} \frac{A^4}{(1+2\sigma')^3} - \frac{A^2}{1+2\sigma'} - 1 \right]. \tag{10}$$

It was noticed by Frere *et al* (1983) that (9) no longer corresponds to a global minimum of the potential if quarks and leptons are included. Let us consider the following g in the presence of leptons:

$$g = \lambda HH'Y + \lambda_e LH'E^+ \tag{11}$$

with L, E^+ respectively corresponding to $SU(2)_L$ doublet and singlet fields. Substitution in (3) shows that the V_{eff} for this theory has a charge-breaking minimum corresponding to non-zero VEV's for E^+ and L^-. The value of the potential at this minimum is scaled by λ_e^{-2} (cf. (10)). Now λ_e is the Yukawa coupling which controls the mass of the electron and hence is expected to be small ($\sim 10^{-6}$). Thus unless λ is also chosen to be at least as small as λ_e the charge-breaking minimum lies lower than the charge-preserving one. In the former case, $U(1)_{\text{e.m.}}$ can be preserved at the cost of the existence of a charged fermion lighter than the electron, as implied by the Yukawa interaction corresponding to $\lambda HH'Y$.

In view of the above serious problem, we address ourselves to the following question: can one obtain an acceptable pattern of SU(2) × U(1) breaking if one goes beyond the simple models described by (3)? The answer, as we show below, is yes. As long as one uses the V_{eff} of (3), the problem persists, since the cubic SUSY-breaking terms are fixed once g is fixed. This restriction is a consequence of the simple form assumed for W (equation (2)). By starting with a more general expression for W as given in (5), one can obtain a V_{eff} (equation (6)) in which the cubic terms are not simply related to g. We take h in (6) to be independent of g and demonstrate that a choice of h exists for which the charge-preserving minimum becomes absolute.

Let us consider the following h in a model with leptons and Higgs fields:

$$h = \lambda' HH'Y + \lambda_e' LH'E^+. \tag{12}$$

Symmetry-breaking terms with this structure are not allowed in NSW or HLW potentials. They can be obtained, however, if W_0 in (5) is chosen to depend nontrivially on hidden fields. With g and h as given by (11) and (12), substitution in (6) yields

$$\begin{aligned} V_{\text{eff}} = {} & |\lambda|^2 |H'|^2 |Y|^2 + |\lambda H^0 Y + \lambda_e L^- E^+|^2 + |\lambda H^\dagger Y + \lambda_e L^0 E^+|^2 \\ & + |\lambda HH'|^2 + |\lambda_e LH'|^2 + |\lambda_e E^+|^2 H'^\dagger H' \\ & + m_{3/2}^2 (H^{\dagger\prime} H' + H^\dagger H + L^\dagger L + |E^+|^2 + |Y|^2) \\ & + m_{3/2} (\lambda' HH'Y + \lambda_e' LH'E^+ + \text{c.c.}) + \tfrac{1}{2} D^\alpha D^\alpha. \end{aligned} \tag{13}$$

We have dropped the Y^3 terms from g for simplicity. They are not expected to change the qualitative conclusions but need to be retained in general to get rid of an unwanted global U(1) symmetry.

It can be seen from (13) that all charged fields are forced to acquire vanishing vacuum expectation values if $|A'| \equiv |\lambda'_e/\lambda_e| < 3$. The charge-preserving solution can still exist if $|A| \equiv |\lambda'/\lambda| \geq 3$. V at this minimum is given by (Frere *et al* 1983).

$$V_{\min}(1) = -m_{3/2}^4/\lambda^2\,(\Delta^2(\Delta^2-1)); \quad \Delta = \tfrac{1}{4}[|A| + (|A|^2-8)^{1/2}]. \tag{14}$$

In the NSW case (corresponding to W_0 being independent of ξ_i) $|A'| = |A|$ and the charge-breaking solution must exist. In general by allowing nontrivial dependence on ξ_i in W_0 one can make $A' < 3$ even when $A > 3$ and avoid the charge-breaking minimum altogether.

Even when $A' > 3$ there exists a reasonable range of parameters for which the charge-breaking minimum lies higher. For $A' > 3$, $\langle L^- \rangle = \langle E^+ \rangle = \langle H^{0\prime} \rangle = (m_{3/2}/\lambda_e)\Delta'$ with all other VEV's vanishing corresponds to a minimum with

$$V_{\min}(2) = -\frac{m_{3/2}^4}{\lambda_e^2}\Delta'^2(\Delta'^2-1); \quad \Delta' = \tfrac{1}{4}[|A'| + (|A'|^2-8)^{1/2}]. \tag{15}$$

A suitable choice of A and A' can make $V_{\min}(2)$ higher than $V_{\min}(1)$. Consider for example $|A|, |A'| \gg 3$, then $V_{\min}(2) > V_{\min}(1)$ provided

$$\frac{(\lambda_e|A|^2)^2}{(\lambda|A'|^2)^2} > 1,$$

i.e. $$|\lambda'/\lambda'_e| > |\lambda/\lambda_e|.$$

With $\lambda/\lambda_e \sim 10^4$, this needs $\lambda'/\lambda'_e \approx 10^6$. This can be achieved by fine-tuning the coefficients in $W_0(Y_a, \xi_i)$. Such a fine tuning is technically natural.

3. Charge-preserving minima in HLW theories

By starting with a complicated W in (5), it is possible to avoid the spontaneous breaking of $U(1)_{\text{e.m.}}$. Nevertheless, a theory with the superpotential separable as in (2) is simpler and it would be nice if this were free of the problem. This may be the case if one considers the HLW potential containing nontrivial effects of superheavy fields in the low-energy sector.

Consider again the light sector consisting of H, H' and Y, and choose the following g_{eff}:

$$g_{\text{eff}} = \lambda HH'Y + mHH'. \tag{16}$$

This leads to the following V_{eff}:

$$\begin{aligned} V_{\text{eff}} = &\left|\lambda\left(Y+\frac{m}{\lambda}\right)\right|^2 (H'^\dagger H' + H^\dagger H) + |\lambda HH'|^2 \\ &+ m_{3/2}^2\,(H'^\dagger H' + H^\dagger H + |Y|^2) \\ &+ (m'_{3/2}\lambda HH'Y + 2m_{3/2}mHH' + \text{c.c.}) + \tfrac{1}{2}D^\alpha D^\alpha. \end{aligned} \tag{17}$$

The value of $V_{\rm eff}$ of (17) is not the most general which could have been constructed. However, this form of $V_{\rm eff}$ is expected to hold in a large class of models. For example, the quadratic and linear terms in Y in $g_{\rm eff}$ do not arise in the minimal SU(5) model (Ellis 1983) or in its extension (Ferrara *et al* 1983) which makes the gauge hierarchy stable at one loop. The Y^3 term will also be absent if it is absent in the original superpotential.

As shown in Appendix A, the SU(2) × U(1)-breaking minimum of $V_{\rm eff}$ can and must exist if $A(\equiv m'_{3/2}/m_{3/2})$ does not lie in the following range:

$$2\hat{m}-1-2|\hat{m}-1| < A < 1+2\hat{m}+2|\hat{m}-1|. \tag{18}$$

This inequality reduces to the well-known $|A| < 3$ when $\hat{m} \equiv m/m_{3/2} = 0$.

Even in the presence of leptons, the above charge-preserving minimum may continue to be the global minimum. This is plausible for the following reason: the inclusion of leptons adds a term $\lambda_e LH'E^+$ to $g_{\rm eff}$. However, the presence of the mass term for HH' in $g_{\rm eff}$ spoils the inherent symmetric role of L and H existing in the NSW example. This symmetry was crucially responsible for leading to a global $\mathrm{U}(1)_{\rm e.m.}$-breaking minimum. It is easy to see that the point at which only L^-, E^+ and H'^0 acquire nonzero VEV's, as in the earlier example, is not even an extremum of $V_{\rm eff}$. The complexity of the potential does not allow us to rule out the existence of other charge-breaking minima. It is, however, conceivable that a reasonable range of parameters A and m may exist for which charge-breaking minima do not exist but the $\mathrm{U}(1)_{\rm e.m.}$-preserving minimum does.

4. Conclusions

We have investigated the breaking of SU(2) × U(1) at the tree level in a wider class of SUGRA models than that studied by NSW. It appears that, for some choice of parameters in the theories, the absolute minimum of the potential corresponds to the desired breaking of SU(2) × U(1) down to $\mathrm{U}(1)_{\rm e.m.}$. The existence of charge-breaking absolute minima in NSW models has led to the belief that, in a realistic SUGRA theory, tree-level electroweak breaking is impossible. Our work shows that this need not be the case if we go beyond the NSW structure. Taken together with the fact that there exist ways (Ferrara *et al* 1983) to avoid the gauge hierarchy problem in the presence of light singlets, tree-level electroweak breaking appears to be a viable possibility.

In the last couple of years, many papers dealing with the question of the electroweak symmetry breaking have appeared (Ibanez *et al* 1985). Most authors except Derendinger and Savoy (1984) rule out symmetry breaking through singlets on the basis of the results of Frere *et al.* Moreover, none of these papers have considered the consequences of allowing a general (Soni and Weldon 1983) soft breaking of supersymmetry. Considering the open status of supergravity model building, our results are thus still of relevance in that they enlarge considerably the class of phenomenologically reasonable models. It remains for us to extend the model of § 2 to include a full set of fermions and to work out in detail the consequences for low energy ($\sim 10^2$ GeV) physics.

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Appendix A

We consider here the minimization of the potential of equation (17). For the nonzero VEV's we write

$$\langle H_1^{0\prime} \rangle = \lambda^{-1} m_{3/2} a e^{i\alpha}, \quad \langle H_2^0 \rangle = \lambda^{-1} m_{3/2} v e^{i\omega},$$
$$\langle Y \rangle = \lambda^{-1} m_{3/2} y e^{i\eta} \tag{A1}$$

where a, v and y are chosen positive. The phases α, ω and η can be chosen as zero at the minimum without affecting the final conclusions.

With the above choices and definitions, V_{eff} takes a value V_0 given by

$$V_0 = \lambda^{-2} m_{3/2}^4 \hat{V}_0,$$
$$\hat{V}_0 = (y+\hat{m})^2(a^2+v^2) + v^2a^2 + (a^2+v^2+y^2)$$
$$-2av(AY+2\hat{m}) + \tfrac{1}{2} D^\alpha D^\alpha \lambda^2/m_{3/2}^4, \tag{A2}$$

where $\hat{m} = m/m_{3/2}$. It is easy to see that at an extremum

$$a = v.$$

This equality ensures that the D-terms vanish at an extremum of V_{eff}. The extremum conditions now take the form

$$v[(y+\hat{m})^2 + v^2 + 1 - (Ay+2\hat{m})] = 0, \tag{A3a}$$
$$v^2(2y - A + 2\hat{m}) + y = 0. \tag{A3b}$$

We are interested in solutions that break $SU(2) \times U(1)$, so we discard the solution $v = y = 0$, and define a new constant $A' = A - 2m$. In terms of A', the extremum conditions are

$$v^2 + y^2 - A'y + (\hat{m} - 1)^2 = 0, \tag{A4a}$$
$$2v^2 y - A'v^2 + y = 0. \tag{A4b}$$

The value of V_{eff} at the extremum is given by

$$V_0 = -v^4 + y^2. \tag{A5}$$

Equations (A4) can be reduced to a single cubic equation. If we write

$$x = (y/A') - \tfrac{1}{2}$$

we can eliminate v from (A4), obtaining

$$0 = f(x) \equiv x^3 + \left[\frac{(\hat{m}-1)^2}{A'^2} - \frac{1}{2A'^2} - \frac{1}{2}\right]x - \frac{1}{4A'^2}. \tag{A6}$$

By (A4b), $v^2 = -(2x)^{-1}(x+\frac{1}{2})$. Thus, to get a real v, we must look for solutions with $-\frac{1}{2} < x < 0$.

The symmetry-breaking extremum will lie lower than the symmetric solution $v = y = 0$ if and only if the right side of (A5) is negative, i.e. $y^2 < v^4$. This yields

$$2|A'x_0| < 1, \tag{A7}$$

where x_0 is a solution of (A6). Equation (A7) has a solution in the range $-\frac{1}{2} < x_0 < 0$ provided $|A'| > 1$ and $f(-1/2|A'|) > 0$. Substitution in the definition of $f(x)$ yields the inequality

$$(|A'|-1)^2 > 4(\hat{m}-1)^2,$$

i.e. $$|A-2\hat{m}| - 1 > 2(\hat{m}-1). \tag{A8}$$

This is equivalent to the restriction that A does not lie between $|2\hat{m}-1| - 2|\hat{m}-1|$ and $2\hat{m}+1+2|\hat{m}-1|$. Approximate values of v and y can be obtained by neglecting the cubic term in (A6). These are

$$y \approx \frac{A'}{2}\left[\frac{A'^2 - 2(\hat{m}-1)^2}{A'^2+1-2(\hat{m}-1)^2}\right],$$

$$v \approx \tfrac{1}{2}[A'^2 - 2(\hat{m}-1)^2].$$

In the large $|A'|$ limit, both y^2 and v^2 are proportional to A'^2.

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