Pramāṇa – J. Phys., Vol. 27, No. 6, December 1986, pp. 803–811. © Printed in India.

Design of diamagnetic coil with non-encircling compensation coil system for measuring the plasma diamagnetism in toroidal devices

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MS received 12 June 1986; revised 24 September 1986

Abstract. The paper describes a new scheme for suppressing the unwanted and dominant coupling of the diamagnetic coil with the toroidal field coils of a tokamak device.

Keywords. Diamagnetic coil; plasma; compensation

PACS Nos 06·30; 52·55; 52·70

1. Introduction

The plasma behaves like a diamagnetic medium in the magnetic field. According to magnetohydrodynamic theory the diamagnetic property of the plasma is due to the circular motion of ions and electrons transverse to the magnetic field. As a consequence, the plasma excludes a small amount of magnetic flux. From zero order pressure balance equation it can be easily shown (Mukhovatov and Shafranov 1971) that the magnetic flux excluded by a tokamak plasma in equilibrium is given by

$$\delta\phi = \mu_0^2 I_\phi (1 - \beta_\rho)/8\pi B_\phi, \tag{1}$$

where

$$\beta_\rho = \frac{16\pi^2}{\mu_0^2 I_\phi^2}\int_0^a n_e k(T_{e\perp} + n_i/n_e)T_{i\perp}\, \mathrm{d}r \tag{2}$$

is the poloidal beta, I_ϕ the plasma current, B_ϕ the toroidal magnetic field, a the plasma radius, T_e (T_i) the perpendicular component of electron (ion) temperature, and $\delta\phi$ the change in toroidal flux due to the plasma; all units are in MKS. This small change in magnetic flux is a measurable quantity and its measurement provides an estimate of either total transverse plasma energy or the poloidal beta.

An attractive feature of this diagnostic method is that it provides a continuous measure of the total transverse plasma energy throughout a single discharge in contrast to more sophisticated pulsed diagnostic techniques, such as laser scattering, which require many shots to generate a complete temporal profile. Consequently, it can be used in study of transient, and sometimes irreproducible phenomena such as the disruptive instability.

Two measuring techniques have been used successfully. In the first technique, the variation of the current in toroidal coils is recorded (Razumova 1966; Mirnov 1968; Thomas 1983). The disadvantage of this technique is that it is not applicable to all tokamaks. For example, those tokamaks with SCR bridges at the output of the toroidal field power supplies might have so much high frequency harmonic content that the noise level would be too high.

In the second method, the measurement of flux variation can be performed by a diamagnetic loop. In its simplest form, it consists of one or more turns of wire encircling the plasma column. It measures the rate of change of magnetic flux within the plasma as it is heated or cooled while confined by a magnetic field (Brzhechko *et al* 1966; Hooke *et al* 1965; Hutchinson 1976; Rothman *et al* 1966; UO 1965). However, the measurement is quite difficult since it is associated with large uncertainties and therefore has not been considered reliable.

The difficulties (Sand and Haegi 1975) stem from the fact that the flux variation is small ($\delta\phi/\phi \simeq 1\%$ for tokamaks), and so it can be measured only when coupling of the diamagnetic coil with the surrounding currents is suppressed. Another difficulty arises from the fact that diamagnetic coil suffers mechanical vibration which is generated in tokamaks (vessel and coils) during the shot. This results in large spurious signals, which must be avoided.

In order that only change in the toroidal flux is measured, it is important to suppress the dominant coupling with the toroidal field coils. This can be done by *in situ* compensation (Waelbroek 1962) of the diamagnetic signal from the diamagnetic coil by another coil whose active area encircles, but does not contain the plasma column. The two coils are chosen to have the same mutual inductances to the toroidal field coils. Since the plasma threads diamagnetic coil but not the compensating coil, the difference between the total fluxes coupling to them should give the diamagnetic flux. This compensation scheme also helps in reducing the coupling with the remaining external currents (Sand and Haegi 1975; Gernhardt *et al* 1984).

Since toroidal magnetic field varies as R^{-1}, where R is the major radius, the size of the compensating coil depends upon the R-value of the centre of the diamagnetic coil for obtaining the condition of linking equal toroidal flux with diamagnetic and compensating coils. Further, the cross-section of the compensating coil has radial extent different from that of the diamagnetic coil since the compensating coil encircles the diamagnetic coil in conventional schemes. The result is that it is possible to obtain the exact compensation only when the two coils are held in static position. When there is mechanical vibration unequal linking of toroidal flux with the two coils is caused. This results in large errors.

Sand and Haegi (1975) constructed compensated diamagnetic coils which are sensitive only to second order of amplitude of mechanical vibration. The coil system consists of eccentric outer and inner circular coils. By providing a particular eccentricity the diamagnetic coil can be made insensitive to the first order of amplitude of mechanical vibration. It is not really necessary to construct eccentric inner and outer coils because the adjustable equivalent eccentricity can also be achieved by providing two auxiliary coils, one mounted on the inner wall of the tokamak vessel and the other on the outer wall.

The diamagnetic coil system with circular cross-section is not suitable for tokamaks with square cross-section since there is not enough space available either within the vessel or outside for its mounting. The difficulty with outside mounting of circular diamagnetic coil is that central bore of the tokamak is normally occupied by the ohmic heating transformer and buckling cylinder. This difficulty can be overcome by deploying diamagnetic coil of rectangular cross-section.

In § 2 we show that compensated diamagnetic coil of rectangular cross-section has the same properties as that of circular cross-section. Section 3 shows that with the coils of rectangular cross-section it is possible to construct diamagnetic coil system with better compensation for the toroidal field.

2. Diamagnetic coil with rectangular cross-section

We assume that the plasma column which is encircled by a diamagnetic loop is of circular cross-section so that flux excluded by the plasma is described by equation (1). For a single turn loop the ratio of error signal δv due to mechanical vibration to the diamagnetic signal v at the output of rectangular diamagnetic coil is given by

$$\frac{\delta v}{v} = \frac{16 q_b^2}{\pi} \frac{R_0^2}{b^2} \frac{\Delta \tilde{R}_0}{R_0} \frac{1}{\beta_p - 1}, \tag{3}$$

where $2b$ is length of a side of the diamagnetic coil with square cross-section, R_0 the major radius of the centre of the coil, $\Delta \tilde{R}_0$ the amplitude of the mechanical vibration and q_b, the safety factor, is given by

$$q_b = bB_\phi / R_0 B_p, \tag{4}$$

where B_ϕ is the poloidal magnetic field. Figure 1 shows the compensated coil system. For this the ratio is

$$\frac{\delta v}{v} = \frac{16 q_b^2}{\pi} \frac{\Delta \tilde{R}_0}{R_0} \frac{1}{\beta_p - 1}. \tag{5}$$

For Aditya tokamak $q_b \simeq 2{\cdot}5$, $b = 30$ cm, $R = 75$ cm and $\beta_p = 0{\cdot}13$. A mechanical vibration amplitude of 5 mm gives $\delta v/v \simeq 3$ for a single turn coil, and $\sim 0{\cdot}5$ for the compensated coils. Although $\delta v/v$ is considerably reduced in the case of compensated coils, it is still large. Figure 1 also shows the arrangement of compensated coils which is less sensitive to mechanical vibration. It consists of eccentric outer and inner coils. For this system $\delta v/v$ is given by

$$\frac{\delta v}{v} = \frac{16 q_b^2}{\pi} \frac{\Delta \tilde{R}_0}{R_0} \frac{1}{\beta_p - 1} \left(1 - \frac{R_1^2 - R_2^2}{b_2^2 - b_1^2}\right), \tag{6}$$

putting

$$b_1 = b, \quad b_2 = b + \Delta b,$$

$$R_1 = R_0, \quad R_2 = R_0 + \Delta R_0. \tag{7}$$

In (6) we find that the term in the parenthesis reduces to zero to first order by adjusting eccentricity ΔR_0 equal to $\Delta b {\cdot} b/R$. This implies that

$$\frac{\delta v}{v} = 0\,(\Delta \tilde{R}_0^2 / R_0^2). \tag{8}$$

In practice, the success in achieving condition described by (8) depends upon mechanical precision which can be achieved in winding coils with good geometrical

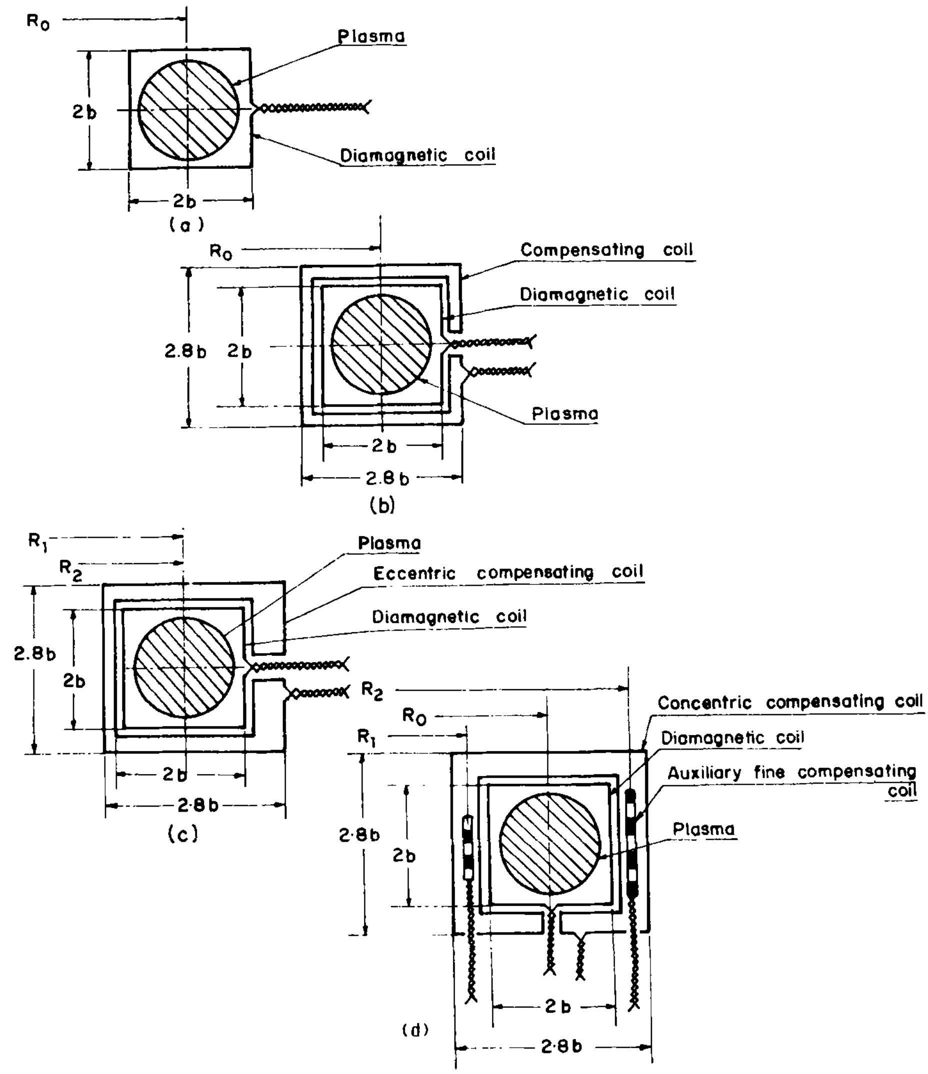

Figure 1. Schematics of measuring plasma diamagnetism with (a) a single turn rectangular coil, (b) concentric diamagnetic and compensating coil, (c) eccentric coil system and (d) auxiliary coils for external fine compensation adjustment and fine eccentricity adjustment.

symmetry and in providing the required eccentricity. So it is desirable to simulate the effect of eccentricity by external means. As shown in figure 1, this can be done by placing two auxiliary coils, one mounted on the inner wall of tokamak vessel and the other on outer wall.

The auxiliary coils are constructed such that they link equal flux when they are held in static position, that is, $\sigma_i B_\phi^i = \sigma_0 B_\phi^0$, where σ's are the area of auxiliary coils. When these coils are connected in opposition, the net signal is zero in static position. On the other hand, the mechanical vibration causes non-zero signal. The ratio of this signal to diamagnetic signal is

$$\frac{\delta v_{\text{aux}}}{v}=\frac{16q_b^2}{\pi}\cdot\frac{\sigma}{4b^2}\cdot\frac{R_0}{b(1-b^2/R_0^2)}\frac{1}{(\beta_p-1)}\frac{\Delta\tilde{R}_0}{R_0} \tag{9}$$

where we have used

$$\sigma_i/(R_0-b)=\sigma_0/(R_0+b)=\sigma/R_0.$$

Multiplying equation (9) by $4b^3(1-b^2/R^2)/\sigma R_0$ and subtracting the result from the ratio for conventional concentric coils, given by (5), we obtain compensation accurate to second order in $\Delta\tilde{R}_0/R_0$ which is the same as can be achieved with eccentric coil system.

Our results of diamagnetic coil of rectangular cross-section are compared in table 1 with the results of Sand and Haegi (1975) for coils with circular cross-section. As was expected, apart from numerical factors, the same kind of equations govern different configurations of diamagnetic coil irrespective of whether its cross-section is circular or rectangular. This shows that employing rectangular diamagnetic coil on a tokamak vessel of square cross-section can work well. We have not obtained significantly different result since we have followed the same principle of compensation as followed by Waelbroek *et al* (1962) and Sand and Haegi (1975).

3. Non-encircling compensating coil

Figure 2a shows another scheme of compensation. The two compensating coils, one above and another below the diamagnetic coil, are provided to compensate *in situ* the signal from diamagnetic coil. A notable feature of this scheme is that the compensating coil does not encircle the diamagnetic coil and the radial coordinates are the same for inner and outer sides of the two coils. The result is that in an ideal case where the toroidal field varies as R^{-1} only, the compensation will be exact even when the coil

Table 1. Comparison between circular and rectangular cross-section diamagnetic coils.

Configuration	Ratio of spurious signal to diamagnetic signal ($\delta v/v$)	
	Circular (diameter = 2b)	Square (side = 2b)
Single turn loop	$4q_b^2\frac{R_0^2}{b^2}\frac{\Delta\tilde{R}_0}{R_0}\frac{1}{(\beta_p-1)}$	$\frac{16q_b^2}{\pi}\frac{R_0^2}{b^2}\frac{\Delta\tilde{R}_0}{R_0}\frac{1}{(\beta_p-1)}$
Concentric compensated coil system	$4q_b^2\frac{\Delta\tilde{R}_0}{R_0}\frac{1}{(\beta_p-1)}$	$\frac{16q_b^2}{\pi}\frac{R_0^2}{b^2}\frac{\Delta\tilde{R}_0}{R_0}\frac{1}{(\beta_p-1)}$
Eccentric compensated coil system	$4q_b^2\frac{\Delta\tilde{R}_0}{R_0}\frac{1}{(\beta_p-1)}\left(1-\frac{R_2^2-R_1^2}{b_2^2-b_1^2}\right)$	$\frac{16q_b^2}{\pi}\frac{\Delta\tilde{R}_0}{R_0}\frac{1}{(\beta_p-1)}\left(1-\frac{R_2^2-R_1^2}{b_2^2-b_1^2}\right)$
Eccentric coil system satisfying equation (7)	$O(\Delta\tilde{R}_0^2/R_0^2)$	$O(\Delta\tilde{R}_0^2/R_0^2)$
Concentric compensated coil system & auxiliary coils	$O(\Delta\tilde{R}_0^2/R_0^2)$	$O(\Delta\tilde{R}_0^2/R_0^2)$

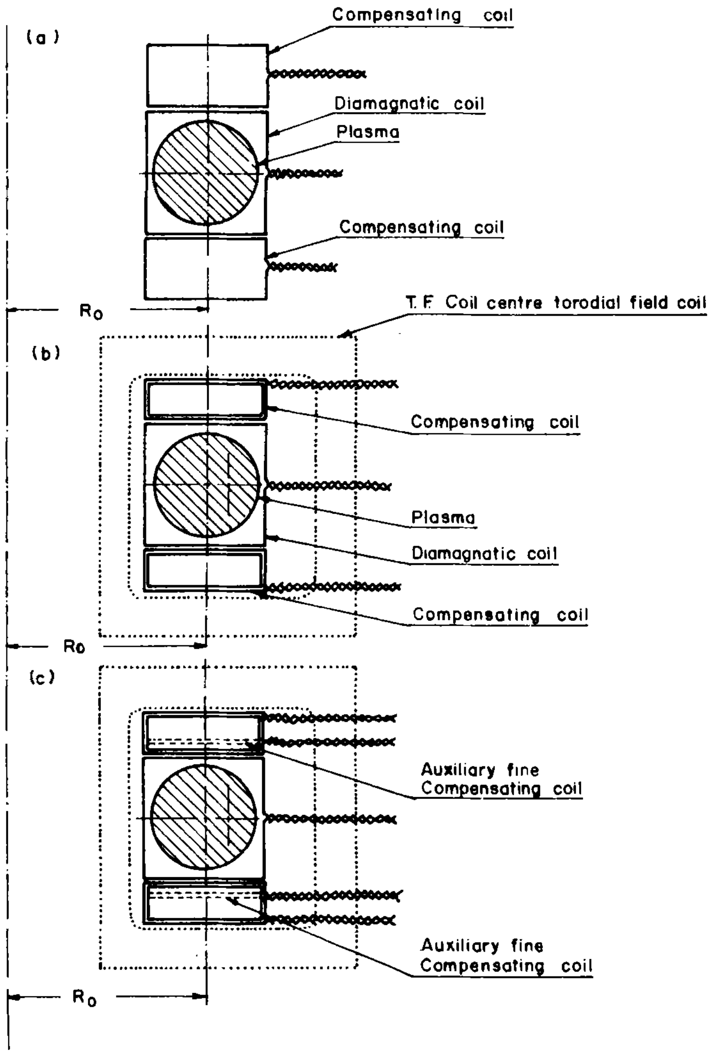

Figure 2. Schematics of measuring plasma diamagnetism with a rectangular coil compensated by (**a**) two one-turn non-encircling coils, (**b**) two two-turn non-encircling coils and (**c**) two two-turn non-encircling coils and an auxiliary coil for initial external fine adjustment. The dashed lines indicate the dimensions of toroidal field coils of Aditya.

system suffers mechanical vibration. It must be emphasized here that the exact compensation can be achieved even without providing eccentricity, or auxiliary coils.

In practice, the toroidal field is a function of ϕ and Z as well since it is provided by discrete field coils. The ϕ-variation is not a serious problem since the diamagnetic coil is localized in ϕ. On the other hand, the Z-variation will cause loss of exact compensation in the presence of mechanical vibration in the case where the nature of radial dependence of the toroidal magnetic field is dependent on Z; the explicit dependence of the toroidal field on Z can be taken care of by adjusting the vertical (in Z-direction) extent of the coils in such a way that the two sets of coils, diamagnetic and compensating, have the same mutual inductances to the toroidal field coils.

To see the effect of discrete coils on the compensation we take the Aditya tokamak as an illustrative example. Figure 3 shows the radial variation of the toroidal field for different values of Z in the plane half way between any two consecutive coils of the set of 18 toroidal field coils to be deployed on Aditya. By fitting polynomials in R^{-1} and Z to the calculated magnetic field curves shown in figure 3, we can approximate the magnetic field at any location within the cross-section of the toroidal field coils by

$$B_\phi = \frac{B_{\phi 0}}{1+\gamma x^2}\left(\frac{R_0}{R} + \frac{\alpha x^2}{1+\beta x^2}\frac{R_0^2}{R^2}\right) \tag{10}$$

where $x = Z/Z_0$. In this equation $2Z_0$ is the vertical extent of the diamagnetic coil and R_0 the major radius of the centres of the coils. The adjustable parameters α, β and γ correspond to the best fit to the calculated toroidal magnetic field values.

Using (10), we obtain the toroidal flux of toroidal field coils linked with diamagnetic coil as

$$\phi_{\rm dia} = 2B_{\phi 0}Z_0R_0\left[\ln\left(\frac{R_0+b}{R_0-b}\right)\frac{1}{\gamma^{1/2}}\tan^{-1}\gamma^{1/2} + \frac{2b}{R_0^2-b^2}\cdot\frac{1}{\gamma-\beta}\cdot\left(\frac{1}{\gamma^{1/2}}\tan^{-1}\gamma^{1/2} - \frac{1}{\beta^{1/2}}\tan^{-1}\beta^{1/2}\right)\right] \tag{11}$$

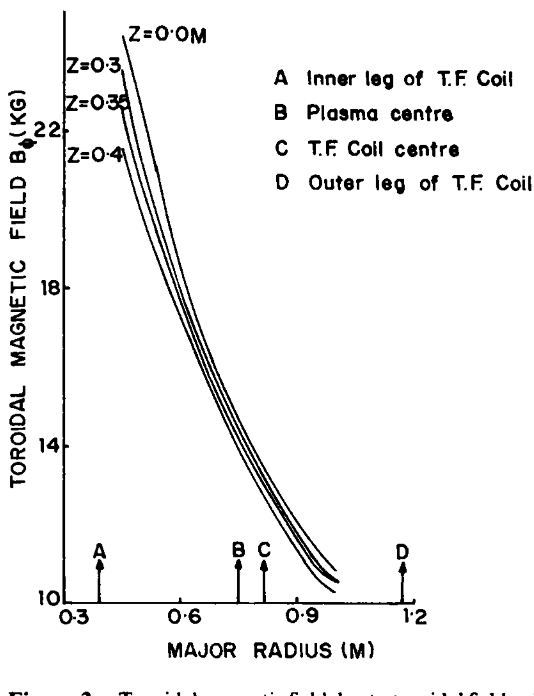

Figure 3. Toroidal magnetic field due to toroidal field coils on Aditya varies as a function of R for different values of Z.

and through compensating coil as

$$\phi_{\text{comp.}} = 2B_{\phi 0}Z_0R_0\left\{\ln\left(\frac{R_0+b}{R_0-b}\right)\frac{1}{\gamma^{1/2}}(\tan^{-1}\gamma^{1/2}x_1 - \tan^{-1}\gamma^{1/2}) + \frac{2\alpha b}{R_0^2-b^2}\cdot\frac{1}{\gamma-\beta}\left[\frac{1}{\gamma^{1/2}}(\tan^{-1}\gamma^{1/2}x_1 - \tan^{-1}\gamma^{1/2}) - \frac{1}{\beta^{1/2}}(\tan^{-1}\beta^{1/2}x_1 - \tan^{-1}\beta^{1/2})\right]\right\}, \tag{12}$$

where $(x_1 - 1)$ specifies the vertical dimension of compensating coils in units of one-half of the vertical extent of the diamagnetic coil. Subtracting (12) from (11), we obtain

$$\Delta\phi = 2B_{\phi 0}Z_0R_0\left\{\ln\left(\frac{R_0+b}{R_0-b}\right)\cdot\frac{1}{\gamma^{1/2}}(2\tan^{-1}\gamma^{1/2} - \tan^{-1}\gamma^{1/2}x_1 + \frac{2b}{R_0^2-b^2}\cdot\frac{1}{\gamma-\beta}\left[\frac{1}{\gamma^{1/2}}(2\tan^{-1}\gamma^{1/2} - \tan^{-1}\gamma^{1/2}x_1) - \frac{1}{\beta^{1/2}}(2\tan^{-1}\beta^{1/2} - \tan^{-1}\beta^{1/2}x_1)\right]\right\}. \tag{13}$$

By equating $\Delta\phi = 0$ in (13) we obtain the condition of exact compensation as

$$x_1 \simeq 2\left(1+\gamma+\alpha\beta+\alpha\beta\cdot\frac{b^2}{R_0^2}\right). \tag{14}$$

This gives vertical dimension of compensating coils. We notice that the vertical dimension of rectangular compensating coil is not exactly equal to Z_0 because of dependence of toroidal magnetic field on Z. The Z-dependence introduces terms which are to the first order determined by γ and to second order by $\alpha\beta$. An additional term which is of second order in b/R_0 is actually a fourth order term since $1 \gg \gamma > \alpha > \beta$. It is this term which causes loss of exact compensation during mechanical vibration of the diamagnetic and compensating coils. To obtain estimate of the error signal produced by the loss of compensation due to mechanical vibration, we differentiate (13) with respect to R_0 and make use of (1) and (4) to give the ratio of error signal to diamagnetic signal as

$$\frac{\delta v}{v} = \frac{8q_b^2}{\pi}\frac{1}{\beta_p - 1}\alpha\beta\frac{\Delta\tilde{R}_0}{R_0}, \tag{15}$$

which depends upon the first order of $\Delta\tilde{R}_0/R_0$. This might give the impression that eccentric compensating coils yield better compensation [see (8)]. However, we must note the fact that $\alpha\beta$ is less than $\Delta\tilde{R}_0/R_0$ even when mechanical vibration amplitude is as small as a few millimeters since $\gamma < 0{\cdot}1$, $\alpha < 0{\cdot}01$ and $\beta < 0{\cdot}001$. Thus, $\delta v/v < 0$ $(\Delta\tilde{R}_0^2/R_0^2)$. Hence a non-encircling compensating coil, having the same radial extent and coordinates as that of diamagnetic coil, can achieve better degree of compensation even without making provision for external fine adjustment. However, it is preferable to provide an auxiliary coil for initial external fine adjustment so as to satisfy equation (14). This can be done by another rectangular coil of very small vertical dimension of the order of γ (figure 2c).

In principle, the compensation can be achieved by a single one-turn rectangular compensating coil. However, the limited size of toroidal field coils prevents from doing so in practice as the vertical dimension of the compensating coil would extend beyond the vertical extent of the toroidal field coils where the toroidal magnetic field falls very rapidly. In case of Aditya tokamak even the configuration of two equally sized one-turn compensating coils, one below and one above the diamagnetic coil, is not possible for the same reason. Therefore it is necessary to build compensating coils of two turns as shown in figure 2 (b and c).

It is important to note that the scheme of non-encircling coil for compensation would not work in case of circular coils. This is because it suffers from the same difficulties as in the case of encircling compensating coil since radial extent and coordinates would be different for the two coils.

For obtaining reliable measurements with this system we still need to eliminate the other sources of errors. The main sources of errors are due to eddy currents excited in the vessel wall and non-zero coupling to poloidal field. The signal output of the diamagnetic coil system (including compensating coil) may be corrected for eddy currents by the method suggested by Rothman (1967). In this method, the compensated diamagnetic signal is processed by an analog circuit which simulates the solution to the circuit equations of the vessel wall and diamagnetic loop. For minimizing poloidal coupling it becomes necessary to mount the diamagnetic coil in such a way that it presents least area for linking with the poloidal flux. This would require high mechanical precision. However, the difficulty that coupling to the poloidal field does not remain constant during mechanical vibration necessitates that we measure the coupling to the poloidal field windings separately and infer coupling to the poloidal field of the plasma current by repeating the measurements with a reversed toroidal field or reversed plasma current.

Providing proper electrostatic screening and high common mode rejection in the signal processing circuits will enhance the reliability and precision of measurements.

The system of diamagnetic coil with non-encircling compensating coil described here will be constructed and mounted on Aditya.

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