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Magnetic moments of octet baryons in a chiral potential model

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Abstract. Incorporating the lowest-order pionic correction, the magnetic moments of the nucleon octet have been calculated in a chiral potential model. The potential, representing phenomenologically the nonperturbative gluon interactions including gluon self-couplings, is chosen with equally mixed scalar and vector parts in a power-law form. The results are in reasonable agreement with experiment.

Keywords. Quark; gluon; nucleon octet; chiral symmetry; magnetic moment; baryon-pion absorption vertex function; pion-nucleon coupling constant.

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1. Introduction

The study of magnetic moments of octet baryons provides some understanding of the structure of these hadrons. Starting from the naive quark-model (Lipkin 1973) approach, a great deal of progress has been made in this area of study (Chodos *et al* 1974; DeGrand *et al* 1975; Brown and Rho 1979; Brown *et al* 1980; Isgur and Karl 1980). The level of accuracy of the above predictions is nevertheless far from certain experimental standards.

The subsequent developments recognize the importance of chiral $SU(2) \times SU(2)$ symmetry for hadrons in strong interaction so as to incorporate the role of charged pion cloud in determining the magnetic moments of these hadrons. The cloudy bag model (Theberge *et al* 1980, 1981; Thomas *et al* 1981; Thomas 1983) has been quite successful in this respect. However it is not entirely free from objections because of the approximation of static spherical bag boundary to which it owes much of its success and simplicity. This is because of the fact that it is difficult to believe the spherical bag boundary remaining static and unperturbed even after the creation of a pion. Furthermore, cloudy bag model (CBM) does not explicitly exclude the pions from the bag volume for a number of reasons, although in any bag type model like this, restoration of chiral symmetry requires the introduction of additional pion field in the region exterior only to the bag boundary. Therefore the very inclusion of pions in the interior region is rather more or less ad hoc. Finally the bag formation for quark confinement, except for some suggestive arguments in its favour, has not yet been derived strictly speaking from any first principle theory. Therefore, the rigid spherical bag boundary of CBM which is nonetheless arbitrary and phenomenological, can always be replaced by a suitable and effective confining potential for individual quarks, keeping at the same time its good features and successful predictions.

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The chiral potential models (Tegen *et al* 1982; Tegen and Weise 1983; Tegen *et al* 1983) are no doubt some attempts in this direction. In such models the effective potential of individual quarks which is basically due to the interaction of quarks with the gluon field is assumed phenomenologically in the scalar harmonic or cubic form. The term in the quark Lagrangian density corresponding to this effective scalar potential is chirally odd through all space and hence requires the introduction of an additional pionic component everywhere in order to preserve chiral symmetry. However if one chooses the Lorentz structure of the effective potential in the form of an equal admixture of scalar and vector parts then the vector part of the potential would of course have no role to play in the chiral symmetry breaking. But on the other hand with the scalar part in equal proportion at every point it would render solvability of the Dirac equation of the independent quarks by reducing it to Schrödinger-like form with no spin-orbit splitting as required by the experimental baryon spectrum. Such a Lorentz structure of the two-body confining potential has already been observed phenomenologically (Appelquist *et al* 1978; Beavis *et al* 1979; Barik and Jena 1980, 1981) in the study of the fine-hyperfine splittings of the heavy meson spectra. This kind of conclusion was also reached in a gauge-invariant formalism of Eitchen and Feinberg (1979). In view of this potentials of such Lorentz character with equally mixed scalar and vector parts in linear (Ferreira 1977; Ferreira and Zagury 1977) harmonic (Ferreira *et al* 1980; Barik and Dash 1985, 1986) and non-coulombic power-law (Barik and Das 1983a,b, 1986) form have been used by many authors in recent past for the study of baryons. With such a potential in the power-law form we have earlier investigated the magnetic moments of baryons (Barik and Das 1983a,b), the electromagnetic form factors of nucleons (Barik and Das 1986) as well as the weak electric and magnetic form factors in the semileptonic baryon decays (Barik *et al* 1985). However in these studies we had not taken into account the possible corrections due to the centre of mass motion and the quark-pion coupling that arises because of the restoration of chiral symmetry in SU(2) sector. Instead we had insisted on our potential parameters and the quark masses to yield the magnetic moments of the nucleon octet and the nucleon form factor in as good an agreement as possible with experiment. But in such approaches the charge radius of the neutron can never be made nonzero apart from getting not so very satisfactory results for μ_{Σ^+} and μ_{Ξ^-}. Even if one gets an overall good fit to the data, the physical baryon, in principle, can never be realized as the bare quark core only without its surrounding charged pion cloud according to the requirements of chiral symmetry in SU(2)-flavour sector. Therefore the electromagnetic properties of the baryons must receive a contribution from the surrounding charged pion cloud over and above the contribution of the quark-core which may also get modified due to quark-pion coupling at the vertex. Realizing this we feel it necessary to reinvestigate here the magnetic moments of the octet baryons with the same potential model of equally mixed scalar and vector parts in power law form to examine in detail the role of the charged pion cloud surrounding the quark-core of these baryons.

2. Basic framework

In such a scheme, the quarks in a baryon-core are assumed to be independently confined by an average flavour-independent potential of the form,

$$V'(r) = (1+\gamma^0)V(r), \tag{1}$$

when $V(r) = (a^{\nu+1} r^{\nu} + V_0)$,

which represents phenomenologically the dominant non-perturbative gluon interactions and the gluon self-couplings. Then leaving behind for the moment any further residual interactions like the quark-pion interaction to be treated perturbatively, one can write the Lagrangian density in zeroth order for independent quarks in a baryon as,

$$\mathscr{L}_q^0(x) = \bar{\Psi}_q(x)[\tfrac{i}{2}\gamma^{\mu}\overleftrightarrow{\partial}_{\mu} - m_q - V'(r)]\Psi_q(x). \tag{2}$$

Considering all the quarks in a baryon-core in their ground $1S_{1/2}$-state, the normalized quark orbitals $\Psi_q(\mathbf{r})$ satisfying the Dirac equation derivable from $\mathscr{L}_q^0(x)$ as,

$$[\gamma^0 E_q - \gamma\cdot\mathbf{p} - m_q - V'(r)]\Psi_q(\mathbf{r}) = 0 \tag{3}$$

can be written in a two-component form as,

$$\Psi_q(\mathbf{r}) = N_q\begin{pmatrix} \varphi_q(\mathbf{r}) \quad \chi\uparrow \\ \frac{\boldsymbol{\sigma}\cdot\mathbf{p}}{\lambda_q}\varphi_q(\mathbf{r}) \quad \chi\uparrow \end{pmatrix}, \tag{4}$$

where $\lambda_q = E_q + m_q$ and

$$N_q = \left(\frac{\lambda_q}{2(E_q - V_0 - a^{\nu+1}\langle r^{\nu}\rangle)}\right)^{1/2} \tag{5}$$

is the overall normalization constant. $\langle r^{\nu}\rangle$ is the expectation value of r^{ν} in the state $\varphi_q(\mathbf{r})\cdot\varphi_q(\mathbf{r})$ satisfies the Schrödinger equation

$$\varphi_q'' + \frac{2}{r}\varphi_q' + \lambda_q[E_q - m_q - 2V(r)]\varphi_q = 0. \tag{6}$$

With $$\varphi_q(\mathbf{r}) = f(r)Y_0^0(\theta,\phi) = \frac{1}{\sqrt{4\pi}}\frac{U_q(r)}{r}, \tag{7}$$

equation (6) reduces to

$$U_q''(r) + \lambda_q[E_q - m_q - 2V(r)]U_q(r) = 0. \tag{8}$$

This can be transformed into a convenient dimensionless form

$$U_q''(\rho) + (\varepsilon_q - \rho^{\nu})U_q(\rho) = 0, \tag{9}$$

where $\rho = r/r_{0q}$ is a dimensionless variable with

$$r_{0q} = [2\lambda_q a^{\nu+1}]^{-1/(\nu+2)} \tag{10}$$

and $$\varepsilon_q = (E_q - m_q - 2V_0)\left(\frac{2a^{\nu+1}}{\lambda_q^{\nu/2}}\right)^{-2/(\nu+2)} \tag{11}$$

Equation (9) is the basic eigenvalue equation. It can be solved for ε_q and U_q by the standard numerical method. The individual quark binding energy E_q can then be obtained as

$$E_q = 2V_0 + m_q + ax_q, \tag{12}$$

where x_q is the root of the equation

$$x_q^{\frac{\nu+2}{\nu}}\left[x_q+\frac{2}{a}(m_q+V_0)\right]=2^{2/\nu}\varepsilon_q^{\frac{\nu+2}{\nu}} \tag{13}$$

that can be solved by a standard numerical method.

3. Magnetic moments of bare quark-core

The magnetic moment of a baryon in its ground state primarily consists of contributions from its bare quark-core in terms of the corresponding constituent quark moments which are defined as,

$$\mu_q=\tfrac{1}{2}\int d^3r(\mathbf{r}\times\mathbf{J}(\mathbf{r}))_z, \tag{14}$$

where $$\mathbf{J}(\mathbf{r})=e_q\Psi_q^\dagger(\mathbf{r})\boldsymbol{\alpha}\Psi_q(\mathbf{r}) \tag{15}$$

e_q is the charge of the quark in the units of proton charge. Then μ_q can be expressed as

$$\mu_q=\tfrac{1}{2}e_q(I_A-I_B), \tag{16}$$

where $$I_A=\int d^3r\, r\sin\theta\cos\Phi\Psi_q^\dagger(\mathbf{r})\alpha_y\Psi_q(\mathbf{r}), \tag{17}$$

$$I_B=\int d^3r\, r\sin\theta\sin\Phi\Psi_q^\dagger(\mathbf{r})\alpha_x\Psi_q(\mathbf{r}). \tag{18}$$

Substitution of (4) in (17) and (18) and further simplification yields

$$I_A=-I_B=N_q^2/\lambda_q. \tag{19}$$

Then $$\mu_q=(2M_pN_q^2/\lambda_q)e_q\,\text{n.m}\quad\text{(nuclear magneton)}, \tag{20}$$

where M_p is the mass of the proton. The bare core contribution to the magnetic moment of a baryon B is given by

$$\mu_B^0=\langle B\uparrow|\sum_q\mu_q\sigma_z^q|B\uparrow\rangle, \tag{21}$$

where $|B\uparrow\rangle$ represents the regular SU(6) state of the baryon. This yields the bare baryon magnetic moments in terms of the constituent quark moments μ_q which can be given by the following usual expressions

$$\begin{aligned}
&\mu_p^0=\tfrac{1}{3}(4\mu_u-\mu_d),\quad \mu_{\Sigma^+}^0=\tfrac{1}{3}(4\mu_u-\mu_s),\quad \mu_{\Xi^-}^0=\tfrac{1}{3}(4\mu_s-\mu_d),\\
&\mu_n^0=\tfrac{1}{3}(4\mu_d-\mu_u),\quad \mu_{\Sigma^0}^0=\tfrac{1}{3}(2\mu_u+2\mu_d-\mu_s),\quad \mu_{\Xi^0}^0=\tfrac{1}{3}(4\mu_s-\mu_u),\\
&\mu_\Lambda^0=\mu_s,\quad \mu_{\Sigma^-}^0=\tfrac{1}{3}(4\mu_d-\mu_s),\quad \mu_{(\Lambda,\Sigma^0)}^0=\frac{1}{\sqrt{3}}(\mu_d-\mu_u).
\end{aligned} \tag{22}$$

There would be a sizable spurious contribution to the energy E_q from the motion of the centre of mass of the three-quark system. Unless this aspect is duly accounted for, a concept of the independent motion of the quarks inside the core will not lead to a physical baryon state of definite momentum. Accounting for the c.m. motion appropriately, a correction to the magnetic moment would result which can be calculated in the same way as in MIT bag model (Bartelski *et al* 1983; Eich *et al* 1983) and we may take over their result to write the c.m. corrected magnetic moment as,

$$\mu_B^{0\prime} = \frac{3\mu_B^0 + Q_B \dfrac{M_p}{M_B}(1-\delta_B)}{(1+\delta_B+\delta_B^2)}, \tag{23}$$

where $\quad \delta_B^2 = \langle M_B^2/E_B^2 \rangle = [1 - \langle \mathbf{P}_B^2 \rangle / E_B^2],$

and $\quad \delta_B = \langle M_B/E_B \rangle = \frac{1}{2}(1+\delta_B^2),$ (24)

when $\mathbf{P}_B$ is the centre of mass momentum, $E_B = \sum_q E_q$ is the relativistic energy of the quark core and Q_B is the charge of the baryon. Therefore the physical mass of the bare baryon core is $M_B = (E_B^2 - \langle \mathbf{P}_B^2 \rangle)^{1/2} \cdot \langle \mathbf{P}_B^2 \rangle$ is evaluated with the usual approximation $\langle \mathbf{P}_B^2 \rangle = \sum_q \langle \mathbf{P}^2 \rangle_q$ where $\langle \mathbf{p}^2 \rangle_q$ is the mean square average of the individual quark momenta with respect to $\Psi_q(\mathbf{r})$. One obtains

$$\begin{aligned} \langle \mathbf{p}^2 \rangle_q = {} & 2N_q^2[E_q(E_q - m_q) - (3E_q - m_q - 2V_0)V_0 \\ & - (3E_q - m_q - 4V_0)a^{\nu+1} r_{0q}^{\nu} \langle \rho^\nu \rangle + 2a^{2(\nu+1)} r_{0q}^{2\nu} \langle \rho^{2\nu} \rangle]. \end{aligned} \tag{25}$$

Evaluation of $\langle \mathbf{p}^2 \rangle_u$ and $\langle \mathbf{p}^2 \rangle_s$ would lead us to a preliminary estimate of the magnetic moments of octet-baryons. The correct estimate of magnetic moments can be obtained by incorporating the effect of the coupling of pions to the bare nucleon.

4. Pion coupling and magnetic moments

In a potential model like this, the quark confinement implies a breaking of chiral symmetry which is widely believed to be a good symmetry for the hadrons. The lack of chiral invariance is evident from the fact that the scalar term proportional to $G(r) = V(r) + m_q$ in $\mathscr{L}_q^0$ is chirally odd. Of course, the vector part poses no problem in this respect. To restore this chiral symmetry in the PCAC limit, one can introduce in the usual manner an elementary pseudovector pion field $\mathbf{\Phi}$ of small mass ($m_\pi = 0{\cdot}140$ GeV).

One can regard the pion as a phenomenological object without internal structure and its interaction with the quarks can be described by a Lagrangian density linear in $\mathbf{\Phi}$:

$$\mathscr{L}_I^\pi(x) = -\frac{i}{f_\pi} G(r) \overline{\Psi}_q(x) \gamma^5 (\boldsymbol{\tau} \cdot \mathbf{\Phi}) \Psi_q(x), \tag{26}$$

where $f_\pi = 93$ MeV is the pion decay constant. Then following the Hamiltonian technique as in CBM one can describe the pion coupling effects in low order perturbation theory.

It is to be noted that, at the quark core level (N, Δ), $(\Delta, \Sigma, \Sigma^*)$ and (Ξ, Ξ^*) are separately mass degenerate. The mass degeneracy is removed by obtaining corrections for electrostatic and magnetostatic energies due to one-gluon exchange interactions, pionic self-energy of the baryons, and energy due to centre of mass motion. However, for the study of magnetic moments of octet baryons we do not consider the effect of the residual one-gluon exchange interactions. We also ignore the possible corrections from the virtual pion-cloud which may arise from the requirement of preserving $SU(3)_L \times SU(3)_R$ symmetry in general, because of the so much heavier kaon mass being involved in the process. Hence for the present we only consider the longest range (i.e. pionic) corrections.

The pionic self-energy of the baryons from the single-loop self-energy diagram can be obtained as,

$$\Sigma_B(E) = \sum_k \sum_{B'} \frac{V^{BB'}(\mathbf{k}) V^{BB'}(\mathbf{k})}{(E - w_k - M^0_{B'})}, \tag{27}$$

where
$$\sum_k = \sum_{\text{isospin}} \int d^3\mathbf{k}/(2\pi)^3 \tag{28}$$

and $V^{BB'}(\mathbf{k})$ is the general baryon-pion absorption vertex function found in this model as

$$V_j^{BB'}(\mathbf{k}) = i\sqrt{4\pi}\frac{f_{BB'\pi}}{m_\pi}\frac{ku(k)}{(2w_k)^{1/2}}(\sigma^{BB'}\cdot\hat{k})\tau_j^{BB'}. \tag{29}$$

Here B and B' refer to any of the ground states of the octet and decuplet baryons, $\sigma_j^{BB'}$ and $\tau_j^{BB'}$ are spin and isospin matrices and $w_k^2 = (\mathbf{k}^2 + m_\pi^2)$. The form factor $u(k)$ can be found here as (Barik and Das 1986)

$$u(k) = \frac{10N_q^2}{3\lambda_q}\left[\langle (V(r)+m_q)j_0(kr)\rangle + \left\langle r\frac{dV(r)}{dr}\frac{j_1(kr)}{kr}\right\rangle\right], \tag{30}$$

where $j_0(kr)$ and $j_1(kr)$ represent the zeroth order and first order spherical Bessel functions, respectively.

The physical baryon state $|\tilde{B}\rangle$ can be written upto one-pion level as

$$|\tilde{B}\rangle = z_B^{1/2}|B\rangle + \sum_{B'} C_{B'\pi}|B'\pi\rangle, \tag{31}$$

where $|B\rangle$ represents the bare baryon state, z_B and $|C_{B'\pi}|^2$ are the probabilities of finding the state $|B\rangle$ and $|B'\pi\rangle$ respectively in the physical baryon state. The normalization requirement yields

$$z_B = 1 - \tfrac{1}{3}I_1 \sum_{B'} C_{BB'} f^2_{BB'\pi}, \tag{32}$$

where
$$C_{BB'} = (\sigma^{BB'}.\sigma^{B'B})(\tau^{BB'}.\tau^{B'B}), \tag{33}$$

and
$$I_1 = \frac{1}{\pi m_\pi^2}\int_0^\infty dk \frac{k^4 u(k)}{w_k^3}. \tag{34}$$

It is difficult to evaluate the integral I_1 with $u(k)$ expressed in the form given in (30). We therefore express it in a convenient form

$$u(k) = (1 - \alpha k^2 + \beta k^4)\exp(-\gamma k^2), \tag{35}$$

where $(\alpha, \beta, \gamma) = (0{\cdot}11595, 0{\cdot}222, 4{\cdot}0816)$ with appropriate units to make $u(k)$ dimensionless. The values of α, β and γ are ascertained by fitting this expression numerically with the original one provided by (30). I_1 is then evaluated by the standard numerical method and is equal to 0·6387.

The z-component of the magnetic moment operator of a physical baryon state can be decomposed into isoscalar μ^s and iso-vector μ^v parts as,

$$\hat{\mu}_z = (\mu^s + \tau_3 \mu^v)\sigma_3. \tag{36}$$

Then the expectation value of $\hat{\mu}_z$ in a given baryon state yields an expression for the magnetic moment μ_B^c due to the baryon current, which is (Nogami and Ohtsuka 1982)

$$\mu_B^c = \langle \tilde{B} | \hat{\mu}_z | \tilde{B} \rangle$$
$$= z_B \left[\mu_B^0 + \tfrac{1}{3} I_1 \sum_{B'B''} D_{B(B'B'')} f_{BB'\pi} f_{BB''\pi} \right], \tag{37}$$

where

$$D_{B(B'B'')} = \sum_{i,\alpha} (\sigma_i^{BB'} \tau_\alpha^{BB'}) \mu_{B'B''}^0 (\sigma_i^{B''B} \tau_\alpha^{B''B}). \tag{38}$$

B', B'' refer to appropriate intermediate states which are mentioned in table 1. $D_{B(B'B'')}$

Table 1. Baryon-pion coupling constants, isoscalar and isovector parts of the transition moments $\mu_{BB'}^0$ ($\mu_{BB}^0 = \mu_B^0$) in SU(3) limit and the spin-isospin reduced matrix elements for various baryon-states. (Here, $\sum_i \sigma_i^{BB'} \sigma_j^{B'} \sigma_i^{B'B} = \xi \sigma_j^B$ and $\sum_\alpha \tau_\alpha^{BB'} \tau_\beta^{B'} \tau_\alpha^{B'B} = \eta \tau_\beta^B$)

BB'	$f_{BB'\pi}/f_{NN\pi}$	$\mu_{BB'}^{0s}/\mu_p^0$	$\mu_{BB'}^{0v}/\mu_p^0$	$\sigma^{BB'}\cdot\sigma^{B'B}$	$\tau^{BB'}\cdot\tau^{B'B}$	ξ	η
NN	1	1/6	5/6	3	3	−1	−1
$N\Delta$	$6\sqrt{2}/5$	0	$\sqrt{2}$	2	2	10/3	10/3
ΔN	$6\sqrt{2}/5$	0	$\sqrt{2}$	1	1	1/3	1/3
$\Delta\Delta$	1/5	1/6	1/6	15	15	11	11
$\Lambda\Lambda$	0	−1/3	0	3	—	−1	—
$\Lambda\Sigma$	$-2\sqrt{3}/5$	0	$-1/\sqrt{3}$	3	3	−1	1
$\Sigma\Lambda$	$-2\sqrt{3}/5$	0	$-1/\sqrt{3}$	3	1	−1	1
$\Sigma\Sigma$	4/5	1/3	2/3	3	2	−1	1
$\Lambda\Sigma^*$	−6/5	0	−1	2	3	10/3	1
$\Sigma^*\Lambda$	−6/5	0	−1	1	1	1/3	1
$\Sigma\Sigma^*$	$-2\sqrt{3}/5$	$-1/\sqrt{3}$	$-1/\sqrt{3}$	2	2	10/3	1
$\Sigma^*\Sigma$	$-2\sqrt{3}/5$	$-1/\sqrt{3}$	$-1/\sqrt{3}$	1	2	1/3	1
$\Sigma^*\Sigma^*$	2/5	0	1/3	15	2	11	1
$\Xi\Xi$	−1/5	−1/2	−1/6	3	3	−1	−1
$\Xi\Xi^*$	$-2\sqrt{3}/5$	$-1/\sqrt{3}$	$-1/\sqrt{3}$	2	3	10/3	−1
$\Xi^*\Xi$	$-2\sqrt{3}/5$	$-1/\sqrt{3}$	$-1/\sqrt{3}$	1	3	1/3	−1
$\Xi^*\Xi^*$	1/5	−1/6	1/6	15	3	11	−1

are evaluated with the help of the appropriate spin-isospin factors provided in table 1, following the prescriptions given by Nogami and Ohtsuka (1982) to obtain the dressed core magnetic moments for the nucleon octet as,

$$\mu_N^c = z_N \,[\mu_N^0 + (-\mu_N^{0s} + \tfrac{1}{3}\tau_3 \mu_N^{0v}) f_{NN\pi}^2 \, I_1 + (\tfrac{20}{9}\mu_\Delta^{0s} + \tfrac{100}{27}\tau_3 \mu_\Delta^{0v}) I_1 f_{N\Delta\pi}^2 + (\tfrac{32}{7}\tau_3 \mu_{N\Delta}^{0v} f_{NN\pi} f_{N\Delta\pi} I_1)], \tag{39}$$

$$\mu_\Lambda^c = z_\Lambda \,[\mu_\Lambda^0 - \mu_\Sigma^{0s} I_1 f_{\Lambda\Sigma\pi}^2 + \tfrac{10}{3}\mu_{\Sigma^*}^{0s} I_1 f_{\Lambda\Sigma^*\pi}^2 + \tfrac{8}{3}\mu_{\Sigma\Sigma^*}^{0s} I_1 f_{\Lambda\Sigma\pi} f_{\Lambda\Sigma^*\pi}], \tag{40}$$

$$\begin{aligned}\mu_\Sigma^c = z_\Sigma \,[&\mu_\Sigma^0 + (-\tfrac{2}{3}\mu_\Sigma^{0s} - \tfrac{1}{3}\tau_3 \mu_\Sigma^{0v}) I_1 f_{\Sigma\Sigma\pi}^2 - \tfrac{1}{3}\mu_\Lambda^{0s} I_1 f_{\Sigma\Lambda\pi}^2 \\ &+ (\tfrac{20}{9}\mu_{\Sigma^*}^{0s} + \tfrac{10}{9}\tau_3 \mu_{\Sigma^*}^{0v}) I_1 f_{\Sigma\Sigma^*\pi}^2 + (\tfrac{16}{9}\mu_{\Sigma\Sigma^*}^{0s} \\ &+ \tfrac{8}{9}\tau_3 \mu_{\Sigma\Sigma^*}^{0v}) I_1 f_{\Sigma\Sigma\pi} f_{\Sigma\Sigma^*\pi} - \tfrac{8}{9}\tau_3 \mu_{\Lambda\Sigma^*}^{0v} I_1 f_{\Sigma\Lambda\pi} f_{\Sigma\Sigma\pi}],\end{aligned} \tag{41}$$

$$\begin{aligned}\mu_\Xi^c = z_\Xi \,[&\mu_\Xi^0 + (-\mu_\Xi^{0s} + \tfrac{1}{3}\tau_3 \mu_\Xi^{0v}) I_1 f_{\Xi\Xi\pi}^2 + (\tfrac{10}{3}\mu_{\Xi^*}^{0s} - \tfrac{10}{9}\tau_3 \mu_{\Xi^*}^{0v}) I_1 f_{\Xi\Xi^*\pi}^2 \\ &+ (\tfrac{8}{3}\mu_{\Xi\Xi^*}^{0s} - \tfrac{8}{9}\tau_3 \mu_{\Xi\Xi^*}^{0v}) I_1 f_{\Xi\Xi\pi} f_{\Xi\Xi^*\pi}],\end{aligned} \tag{42}$$

$$\begin{aligned}\mu_{(\Lambda\Sigma^0)}^c = (z_\Lambda z_\Sigma)^{1/2} \,[&\mu_{\Lambda\Sigma^0}^0 - \tfrac{1}{3}\mu_{\Lambda\Sigma^0}^{0v} I_1 f_{\Lambda\Sigma\pi}^2 + \tfrac{1}{3}\mu_\Sigma^{0v} I_1 f_{\Sigma\Sigma\pi} f_{\Lambda\Sigma\pi} \\ &+ \tfrac{4}{9}\mu_{\Lambda\Sigma^*}^{0v} I_1 f_{\Lambda\Sigma^*\pi} f_{\Lambda\Sigma\pi} - \tfrac{4}{9}\mu_{\Sigma\Sigma^*}^{0v} I_1 (f_{\Lambda\Sigma^*\pi} f_{\Sigma\Sigma\pi} + f_{\Lambda\Sigma\pi} f_{\Sigma\Sigma^*\pi}) \\ &- \tfrac{10}{9}\mu_{\Sigma^*}^{0v} I_1 f_{\Lambda\Sigma^*\pi} f_{\Sigma\Sigma^*\pi}].\end{aligned} \tag{43}$$

Here $\mu_{B'B''}^0$ ($\mu_{BB}^0 \equiv \mu_B^0$) are transition moments describing the $B'B''\gamma$-coupling. First of all we use the SU(6) values of μ_B^0 and $\mu_{B'B''}^0$ in the above expressions, when the isovector transition moments $\mu_{B'B''}^{0v}$ are obtained from the relation $\mu_{B'B''}^{0v} = \mu_N^{0v}(f_{B'B''\pi}/f_{NN\pi})$. The isoscalar and isovector parts of $\mu_{B'B''}^0$ and that of μ_B^0 in the exact SU(3) limit are summarized in table 1. Substituting the values of μ_B^{0s}, μ_B^{0v}, $\mu_{B'B''}^{0s}$ and $\mu_{B'B''}^{0v}$ in (39) to (43) and factoring out the bare-core SU(3) value μ_B^0 for the corresponding baryon from these expressions one obtains the dressed core magnetic moment of a baryon in general as,

$$\mu_B^c = z_B (1 + s_B)\mu_B^0, \tag{44}$$

when z_B can also be written from (32) and (33) as,

$$z_B = (1 - \tilde{s}_B). \tag{45}$$

where the factors s_B and $\tilde{s}_B$ as found here for each baryon can be listed as follows:

$$(s_p, s_n, s_\Lambda) \equiv (\tfrac{29}{5}, 6, \tfrac{108}{25}) I_1 f_{NN\pi}^2,$$

$$(s_{\Sigma^+}, s_{\Sigma^0}, s_{\Sigma^-}) \equiv (\tfrac{92}{75}, \tfrac{36}{25}, \tfrac{4}{5}) I_1 f_{NN\pi}^2,$$

$$(s_{\Xi^0}, s_{\Xi^-}, s_{(\Lambda\Sigma^0)}) \equiv (\tfrac{18}{25}, \tfrac{33}{25}, \tfrac{36}{25}) I_1 f_{NN\pi}^2,$$

and

$$(\tilde{s}_N, \tilde{s}_\Lambda, \tilde{s}_\Sigma, \tilde{s}_\Xi) = (\tfrac{171}{25}, \tfrac{108}{25}, \tfrac{12}{5}, \tfrac{27}{25}) I_1 f_{NN\pi}^2. \tag{46}$$

However to incorporate the SU(3)-symmetry breaking effects we can use our model predictions for the bare core moments μ_B^0 listed in table 2, in (44) instead of the SU(3) values. The dressed core magnetic moments μ_B^c computed with $f_{NN\pi}^2 = 0{\cdot}08$, are presented in table 2, along with the c.m. corrected values $\mu_B^{c'}$ obtained according to (23).

Table 2. Calculated values of magnetic moments for octet baryons (in n.m.) with possible corrections to the bare quark-core moment μ_B^0 due to the effects of pion cloud and c.m. motion.

B	μ_B^0	μ_B^c	$\mu_B^{c'}$	$\delta\mu_B^\pi$	μ_B	Expt.
p	2·4102	2·0325	2·2197	0·4883	2·7081	2·7928
n	−1·6068	−1·3657	−1·4738	−0·4883	−1·9621	−1·913
Λ	−0·5862	−0·5576	−0·6036	0·0	−0·6036	$-0{\cdot}613 \pm 0{\cdot}004$
Σ^+	2·3378	2·1797	2·3811	0·1831	2·5643	$2{\cdot}33 \pm 0{\cdot}13$
Σ^0	0·731	0·6885	0·7453	0·0	0·7453	$0{\cdot}61 \pm 0{\cdot}08$*
Σ^-	−0·8758	−0·7998	−0·8873	−0·1831	−1·0705	$-1{\cdot}11 \pm 0{\cdot}031$
						$-1{\cdot}23 \pm 0{\cdot}03$
Ξ^0	−1·3172	−1·2903	−1·3993	0·0444	−1·3549	$-1{\cdot}25 \pm 0{\cdot}14$
Ξ^-	−0·5138	−0·5181	−0·5819	−0·0444	−0·6263	$-0{\cdot}69 \pm 0{\cdot}04$
(Λ, Σ^0)	−1·3916	−1·2353	−1·333	−0·3076	−1·6406	$-1{\cdot}82^{+0{\cdot}18}_{-0{\cdot}25}$

* Values computed according to the definition $\mu_{\Sigma^0} = \frac{1}{2}(\mu_{\Sigma^+} + \mu_{\Sigma^-})$ exp which is quite often referred to as the measured value.

Finally we consider the contributions from the corresponding pion current which arises due to the coupling of photon to the pion field surrounding the quark core of the system. This provides additional contributions to the isovector part of the magnetic moments. In view of the assumed mass degeneracy of the baryon and intermediate baryon states, such contributions due to pionic current as obtained by Nogami and Ohtsuka (1982) would be found here as,

$$\delta\mu_N^\pi = \tau_3 \tfrac{88}{25} I_2 f_{NN\pi}^2; \quad \delta\mu_\Lambda^\pi = 0; \quad \delta\mu_\Sigma^\pi = \tau_3 \tfrac{33}{25} I_2 f_{NN\pi}^2;$$

$$\delta\mu_\Xi^\pi = \tau_3 \tfrac{8}{25} I_2 f_{NN\pi}^2; \quad \delta\mu_{(\Lambda,\Sigma^0)}^\pi = -\frac{96}{25\sqrt{3}} I_2 f_{NN\pi}^2 \tag{47}$$

where

$$I_2 = \frac{M_p}{\pi m_\pi^2} \int_0^\infty \mathrm{d}k \frac{k^4 u^2(k)}{w_k^4}. \tag{48}$$

I_2 is evaluated in the same manner as I_1 in (34) and is equal to 1·7342. The magnetic moment of a physical baryon state is then expressed as

$$\mu_B = (\mu_B'^c + \delta\mu_B^\pi). \tag{49}$$

5. Results and discussion

The computation of the magnetic moments of octet baryons in such a scheme would ultimately depend on the choice of the Lagrangian mass parameters (m_u, m_d, m_s) and the potential parameters (a, V_0, v) which would lead to the individual quark binding energies (E_u, E_d, E_s) and the corresponding quark-wave function through the eigenvalue equations (9) to (13). In our earlier work (Barik and Das 1983a) we had taken the potential parameters to be the same as those obtained in the study of mesons (Barik and Jena 1982) and estimated the quark masses to obtain a reasonable description of the static properties of nucleons including the magnetic moment μ_Λ. However in a Lagrangian formulation adopted here we choose to fix the quark mass parameters in the current quark limit as $m_u = m_d = 5$ MeV and $m_s = 200$ MeV. Also following a non-relativistic study of Ω^--baryon (Richard 1981) in a power-law potential model, we fix the potential parameter $\nu = 0{\cdot}1$. That leaves us with only two parameters a and V_0 to be estimated for which we take the spin-isospin average mass of N and Δ together with the mass of Ω^- baryon as the two basic guiding inputs, keeping in view the possible centre of mass corrections as well as the pionic corrections. We must point out here that we have not attempted to obtain a best chi-square fit to the available data but instead have obtained a suitable choice of the parameters to provide an overall satisfactory description of the nucleon octet under investigation. Thus we find that with a suitable choice of the potential parameters as,

$$(a, V_0, \nu) = (1{\cdot}8779 \text{ GeV}, \ -2{\cdot}1152 \text{ GeV}; 0{\cdot}1)$$

and with the numerical solution provided by the scaled eigenvalue equation (9) through $\varepsilon_q = 1{\cdot}2364$; the individual quark binding energies and the corresponding quark-wavefunction normalizations can be computed from (12), (13) and (5) respectively as:

$$(E_u = E_d, E_s) = (0{\cdot}540 \text{ GeV}; 0{\cdot}6383 \text{ GeV}),$$

$$(N_u^2 = N_d^2, N_s^2) = (0{\cdot}7, 0{\cdot}7855).$$

The constituent quark magnetic moments are then computed by using (20). These are obtained as,

$$(\mu_u, \mu_d, \mu_s) = (1{\cdot}6068, \ -0{\cdot}8034, \ -0{\cdot}58618) \text{ n.m.}$$

which enable the bare-core values of the magnetic moments for octet baryons to be calculated according to (22). Then taking the experimental value of $f_{NN\pi}^2$ as 0·08, z_B, $\mu_B^{c'}$, $\delta\mu_B^\pi$ and finally μ_B are computed and the results are presented in table 2. The results are in good agreement with the corresponding experimental data. So far as the magnetic moments of Σ^+ and Ξ^- are concerned, the results obtained here with $\mu_{\Sigma^+} = 2{\cdot}5776$ n.m. and $\mu_{\Xi^-} = -0{\cdot}6278$ n.m. certainly show an improvement over our earlier calculations (Barik and Das 1983a) as $\mu_{\Sigma^+} = 2{\cdot}7586$ n.m. and $\mu_{\Xi^-} = -0{\cdot}4994$. Again one can notice from table 2 that the effect of the surrounding pion cloud in determining the magnetic moments of the baryons is quite significant in modifying the bare core value μ_B^0 to μ_B^c together with the additional contribution $\delta\mu_B^\pi$ which is quite sizable for nucleons in particular.

It should be mentioned here that we have not considered the higher order pionic contribution to the magnetic moment. Moreover other possible corrections to the baryon magnetic moments arising out of configuration mixing and seaquark effects are not considered. Nevertheless it seems quite reasonable to conclude that in the present phenomenological chiral potential model, the lowest order pionic corrections lead to the satisfactory evaluation of magnetic moments of octet baryons.

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