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Quarkonium spectroscopy with vacuum polarization correction

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Abstract. The quarkonium spectroscopy has been studied by considering a non-relativistic potential which includes the QCD vacuum polarization corrections. The potential consists of a short-range 2-loop QCD potential matched to a Martin-type power law potential for large distances. The Poggio-Schnitzer correction to the leptonic decay width has also been included. The energy levels, leptonic decay widths and E1 transition rates of ψ and Υ families have been calculated and have been found to be in good agreement with experimental results. The toponium spectroscopy has also been studied for the range of M_t values suggested by the recent jet events observed by the UA1 collaboration. The contribution of the decay through a virtual Z^0 has also been included in the calculation. The potential seems to provide a very good non-relativistic description of the quarkonium systems.

Keywords. Quarkonia; vacuum polarization corrections; leptonic decay widths; E1 transition rates; quark-antiquark potential.

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1. Introduction

The vacuum polarization corrections to the annihilation rates of heavy vector mesons to lepton pairs have recently been studied by many authors (Billoire and Morel 1978; Celmaster *et al* 1978; Eichten *et al* 1976; Eichten and Gottfried 1977; Poggio and Schnitzer 1978, 1979a and b). It has been pointed out that even for the non-relativistic treatment of the quark antiquark ($Q\bar{Q}$) systems, two different cases may arise, depending on the characteristic length scales of the bound $Q\bar{Q}$ states, where the corrections are to be treated differently. Thus for cases where the $Q\bar{Q}$ spectroscopy is determined essentially by the potential at distances less than about 0·1 fm, the corrections can be calculated by the perturbative quantum chromodynamics (QCD). But for extended $Q\bar{Q}$ bound states, the spectroscopy is determined mostly by the potential at larger distances and the non-perturbative effects of the confinement potential must also be considered. It is, therefore, expected that the vacuum polarization correction to the annihilation rates of ψ and Υ families cannot be determined by perturbative QCD alone. Therefore, one can hope to obtain only model-dependent results. The problem has been studied with the help of a Bethe-Salpeter equation which also provides a framework for estimating the relativistic corrections (Durand and Durand 1982; Keung and Muzinich 1983; McClary and Byers 1983; Moxhay and Rosner 1983). One may, therefore, consider a simultaneous expansion in two parameters, the velocity squared β^2 and the QCD coupling constant α_s. Poggio and Schnitzer have shown that most of the next to leading order corrections in α_s and the quark velocity to the leptonic decay (in a static linear confinement potential) can be accounted for by replacing ϕ (0) by $\phi(1/M_Q)$. They have also pointed out that for extended $Q\bar{Q}$ systems where the confinement potential dominates, one should not consider a running coupling constant in the one-gluon exchange term. Rather, one should include the vacuum polarization corrections in the confinement potential so that there is no double counting of the QCD screening. This prescription, though simple, is incomplete in the sense that it is not clear which approach should be appropriate for the known quarkonia. A simple uniform prescription for all cases, even if phenomenological, will be useful for practical applications, particularly because the known heavy quarkonia fall in neither of the two extreme categories. The purpose of the present paper is to suggest a simple prescription which may be applicable to all heavy quarkonia. The results obtained are consistent with the observed quarkonium spectroscopy and may be used to predict results for the toponium states.

We first write down the non-relativistic $Q\bar{Q}$ potential in the form

$$V_{Q\bar{Q}}(r) = f(r)V_{\mathrm{QCD}}(r) + [1 - f(r)]\, V_M(r), \tag{1}$$

where $f(r)$ is the Woods-Saxon function

$$f(r) = \frac{1}{1+\exp[(r-a)/s]}, \tag{2}$$

and $V_{\mathrm{QCD}}(r)$ is the $Q\bar{Q}$ potential calculated by including the 2-loop gluon and light quark vacuum polarization correction terms (equation (5)). The potential $V_M(r)$ is the potential as determined for the range $0{\cdot}1\ \mathrm{fm} \lesssim r < 1\ \mathrm{fm}$ by the ψ and Υ spectroscopy. Since there is no compulsion to prefer any particular form for this confinement potential, we have chosen the Martin (1980, 1981) form

$$V_M(r) = A + Br^{0{\cdot}1}, \tag{3}$$

with the condition that

$$V_{\mathrm{QCD}}(a) = V_M(a). \tag{4}$$

We may study the $Q\bar{Q}$ bound states by solving the Schrödinger equation with the potential (1), with the additional provision for including the suggestions of Poggio and Schnitzer for calculating the leptonic widths, i.e., we replace

$$|\phi(0)|^2 \quad \text{by} \quad |\phi(1/M_Q)|^2$$

in the Van Royen-Weisskopf (1967a and b) formula. This may be looked upon as a phenomenological prescription with some qualitative justification. For a very heavy quarkonium, e.g. toponium,

$$\phi(1/M_Q) \approx \phi(0),$$

and one deals with the potential given essentially by the one-gluon exchange term with a running coupling constant. The confining part of the potential is not important for the system. For cases where $V_M(r)$ dominates the $Q\bar{Q}$ spectroscopy, the first term on the right side of (1) has very little effect and the Poggio-Schnitzer prescription is valid. When distances $r \sim a$ become relevant, the choice (4) minimizes the double counting of screening, if any. An interpolation is now possible for cases where both the short range and the confining terms are important. We have shown in this paper that such an interpolation is possible and indeed consistent with the experimental results. The remarkable success of the Poggio-Schnitzer prescription for light vector and pseudoscalar mesons, as has been shown by Krasemann (1980), fits with this scheme. The choice of the particular form for the potential (1) is motivated by another consideration. It gives a simple parametrization suitable for studying the details of the cross-over region. The parameters a and s may be used to fix roughly the position and the extension of the cross-over region. These parameters will also be helpful to fit the fine-hyperfine splittings of the $Q\bar{Q}$ system, which will be discussed elsewhere.

The presentation of the paper is as follows. In the next section, we discuss the $Q\bar{Q}$ potential and use it to study the spectroscopy of the $c\bar{c}$ and $b\bar{b}$ systems. The predictions for the $t\bar{t}$ system for a range of M_t values, suggested by the recent jet events (Arnison *et al* 1984), are also presented. In § 3, we consider the leptonic widths as well as rates for some other decay modes. We have considered, in particular, the contribution of the Z^0 exchange for toponium decay. For the excited states, we have made use of some general properties of Schrödinger equation to calculate the Poggio-Schnitzer correction factors. This can be done for heavy quarkonia, without referring to any potential explicitly, once we know the correction factor for the $1S$ state. Our conclusions are summarized in the last section.

2. The $Q\bar{Q}$ potential

The extensive work on the $c\bar{c}$ and $b\bar{b}$ spectroscopy have succeeded in determining uniquely the $Q\bar{Q}$ static potential within the range $0{\cdot}1\text{ fm} \lesssim r \lesssim 1\text{ fm}$. In fact, the different potentials (Applequist *et al* 1975; Buchmüller *et al* 1980; Eichen *et al* 1975; Levine and Tomozawa 1979; Quigg and Rosner 1979; Richardson 1979; Stanley and Robson 1980) that fit the data reasonably well all agree within this range. Since distances much larger than 1 fm will not be relevant for the problems under consideration, we have chosen Martin's form (1980, 1981) for the potential $V_M(r)$. For small distances, the $Q\bar{Q}$ potential should be given accurately by the one-gluon exchange term with a running coupling constant. This potential upto 2-loop corrections has been considered by a number of authors (Buras *et al* 1977; Buras 1980; Billoire 1980; Fischler 1977). The potential may be written as

$$V_{\mathrm{QCD}}(r) = -\frac{4\pi C_2(R)}{b_0 r \ln(1/\Lambda^2_{\overline{MS}} r^2)} \times \left[1 + \left(2\gamma_E + \frac{c}{b_0}\right)\frac{1}{\ln(1/\Lambda^2_{\overline{MS}} r^2)} - \frac{b_1}{b_0^2}\frac{\ln\ln(1/\Lambda^2_{\overline{MS}} r^2)}{\ln^2(1/\Lambda^2_{\overline{MS}} r^2)}\right] \quad (5)$$

where γ_E is the Euler constant and

$$b_0 = \tfrac{11}{3} C_2(G) - \tfrac{2}{3} N_f \quad (6)$$

$$b_1 = \tfrac{34}{3}[C_2(G)]^2 - \tfrac{10}{3} C_2(G) N_f - 2C_2(R) N_f \quad (7)$$

$$c = \tfrac{31}{9} C_2(G) - \tfrac{10}{9} N_f. \quad (8)$$

In above, $C_2(R)$ and $C_2(G)$ are the invariant quadratic casimir operators, which for $SU_C(3)$ equal 4/3 and 3 respectively. N_f gives the number of quark flavours relevant for the problem, which we choose to be equal to 4. In the standard $\overline{MS}$ scheme, it is customary to choose N_f as the number of quark flavours with mass $\leqslant \mu$, where μ is the renormalization mass. The QCD scale parameter $\Lambda_{\overline{MS}}$ has been chosen as equal to 200 MeV.

We combine the potential $V_{QCD}(r)$ with the large distance potential $V_M(r)$ as in (1) so that the condition (4) is satisfied. This gives, with r in fm,

$$A = -7{\cdot}392, \quad B = 8{\cdot}080 \tag{9}$$

and $a = 0{\cdot}072325$ fm. Because of our choice of (1) and (4), the results will show only a very weak dependence on the parameter s which gives the scale of the crossover region. We have chosen $s = 0{\cdot}01$ fm. The combination (1) will ensure that the potential is given by one of the two terms in their respective domains and the potential in the crossover region is a smooth interpolation between them. Combinations of potentials have already been considered by some authors (Bhanot and Rudaz 1978; Deo and Barik 1983). We have followed in particular that of Igi and Hikasa (1983) with some modification.

We have solved the Schrödinger equation numerically with the potential (1) and the energy levels for the $c\bar{c}$ and $b\bar{b}$ systems are shown in table 1. We have chosen $M_b = 4{\cdot}837$ GeV and $M_c = 1{\cdot}44$ GeV. The experimental results are also shown. The general agreement of our results with the experimental results is better than those obtained with other potentials considered in this connection.

In table 2, we have presented the results for some toponium levels with M_t ranging from 30 GeV to 50 GeV as suggested by the recent UA1 experiments (Arnison *et al* 1984). The variation of the binding energy of any level with M_t is found to be almost linear. The levels are, however, still not coulombic. It is, therefore, essential that one should consider the perturbative QCD potential like (1) and not just a coulomb-like potential while studying the toponium system. Since the root mean square radius R, given by $R^2 = \langle 1S|r^2|1S\rangle$, is about 0·07 fm for $M_t \sim 40$ GeV, we can probe the potential in the true perturbative region. The higher excited states depend also on the long range part of the potential.

Table 1. Some $b\bar{b}$ and $c\bar{c}$ energy levels calculated with potential (1). The levels marked with an asterisk indicate the centre of gravity values.

State	Mass of $b\bar{b}$ system (GeV) Experimental	Mass of $b\bar{b}$ system (GeV) Calculated	Mass of $c\bar{c}$ system (GeV) Experimental	Mass of $c\bar{c}$ system (GeV) Calculated
1*S*	9·460	9·461	3·097	3·098
2*S*	10·023	10·038	3·686	3·708
3*S*	10·355	10·374	4·030	4·063
4*S*	10·575	10·615	4·415	4·319
1*P*	*9·882	9·882	*3·549	3·539
2*P*	*10·261	10·265		3·945
1*D*		10·154		3·828
2*D*		10·445		4·135

Table 2. Some toponium levels for different t-quark mass.

Mass of t-quark (GeV)	Toponium mass (GeV)			
	$1S$	$2S$	$1P$	$2P$
30	59·115	59·686	59·572	59·924
35	69·047	69·627	69·520	69·870
40	78·985	79·575	79·475	79·823
45	88·926	89·528	89·435	89·781
50	98·871	99·487	99·400	99·744

It may be useful to compare our results with those obtained with Richardson (1979) potential and the logarithmic potential $V(r) = A \ln (R/R_0)$, considered by Quigg and Rosner (1979). The difference in energies, $E(2S)-E(1S)$, in the three cases are 590, 630 and 560 MeV respectively for $M_t = 40$ GeV and 616, 645 and 560 MeV respectively if $M_t = 50$ GeV.

For the $t\bar{b}$ system, one gets the following results. For $M_t = 40$ GeV and $M_b = 4{\cdot}837$ GeV, the binding energy for $1S$ and $2S$ states of $t\bar{b}$ system will be $-0{\cdot}416$ GeV and 0·150 GeV respectively.

The E1 transition rates of the quarkonia are given by

$$\Gamma(n^3P_j \to \gamma + n'^3S_1) = \frac{4}{9}\alpha Q_q^2 \omega^3 \left(\int_0^\infty R_{n0} R_{n'1} r^3 \, \mathrm{d}r\right)^2, \tag{10}$$

$$\Gamma(n^3S_1 \to \gamma + n^3P_j) = \frac{4}{3}\frac{2j+1}{9}\alpha Q_q^2 \omega^3 \left(\int_0^\infty R_{n0} R_{n'1} r^3 \, \mathrm{d}r\right)^2, \tag{11}$$

where ω is the photon energy, given by

$$\omega = \frac{M^2(v_1) - M^2(v_2)}{2M(v_1)}, \quad M(v_1) > M(v_2). \tag{12}$$

In calculating the $E1$ transition rates, we have used the experimental values of the photon energy, as we have not considered the fine structure interactions. The results are shown in table 3.

It is already known that the $\psi' \to \gamma + \chi$ rate is suppressed by a factor of 2–3 in comparison with non-relativistic estimates. A number of factors help in restoring the agreement with the experimental results. First, relativistic effects can affect the dipole matrix element significantly. The overlap integral $\langle 1P|r|2S\rangle$ may be particularly sensitive to the relativistic corrections, as it is the sum of two contributions of opposite signs due to the presence of a node in the $2S$ wavefunctions. A shift in the wavefunction caused by the relativistic corrections serves to reduce the value of the matrix element. This alone is, however, not enough. One should also consider the coupled channel effects for ψ', which also reduce the overlap between the two states (Eichten *et al* 1978, 1980). Our results for the transition rate for $\psi' \to \gamma + \chi$ agree with other non-relativistic calculations. For the upsilon system the relativistic effects are less important and we get fairly good agreements with the experimental results. Thus $\sum_i \Gamma(\Upsilon' \to \gamma + \chi_{bi}) = 4{\cdot}9 \pm 1{\cdot}8$ keV, experimentally, whereas we get the value 4·38 keV.

Table 3. E1 transition rates for $b\bar{b}$ and $c\bar{c}$ systems. Experimental values of ω are used in the calculations.

	$b\bar{b}$ system		$c\bar{c}$ system	
Transition	ω (MeV)	Transition width (keV)	ω (MeV)	Transition width (keV)
$1^3P_0 \to 1^3S_1$	404·4	32·434	303·2	176·470
$1^3P_1 \to 1^3S_1$	425·4	37·783	388·7	371·283
$1^3P_2 \to 1^3S_1$	444·6	43·131	429·4	500·945
$2^3S_1 \to 1^3P_0$	148·9	0·927	261·0	62·736
$2^3S_1 \to 1^3P_1$	127·2	1·770	171·8	53·924
$2^3S_1 \to 1^3P_2$	107·4	1·686	127·7	37·006
$2^3P_0 \to 1^3S_1$	743·8	5·638		
$2^3P_1 \to 1^3S_1$	763·3	6·092		
$2^3P_2 \to 1^3S_1$	779·0	6·477		
$2^3P_0 \to 2^3S_1$	207·8	13·133		
$2^3P_1 \to 2^3S_1$	228·4	17·365		
$2^3P_2 \to 2^3S_1$	245·0	21·451		

Table 4. Leptonic decay widths with and without Poggio-Schnitzer (PS) correction for $b\bar{b}$ and $c\bar{c}$ systems.

State	$\lvert\psi(0)\rvert^2$ (GeV^3)	Experimental (keV)	Uncorrected (keV)	PS corrected (keV)
Υ (9·460)	4·714	1·100 ± 0·120	1·247	0·995
Υ (10·023)	2·602	0·507 ± 0·051	0·611	0·466
Υ (10·355)	1·857	0·362 ± 0·050	0·408	0·304
Υ (10·575)	1·466	0·240 ± 0·053	0·308	0·225
ψ (3·097)	0·743	4·600 ± 0·390	7·332	4·847
ψ (3·686)	0·413	2·050 ± 0·210	2·845	1·601
ψ (4·030)	0·296	0·750 ± 0·100	1·697	0·868
ψ (4·415)	0·241	0·490 ± 0·130	1·224	0·583

3. The decay widths

While calculating the leptonic decay widths of the quarkonia we apply the Poggio-Schnitzer (PS) correction to the Van Royen-Weisskopf relation:

$$\Gamma(v \to e^+e^-) = \frac{4\alpha^2 Q_q^2}{M_{q\bar{q}}^2} |\phi(0)|^2, \tag{13}$$

where Q_q is the quark charge and $M_{q\bar{q}}$ is the mass of the quarkonium $q\bar{q}$. The uncorrected and the corrected values of the leptonic widths for ψ and Υ families are shown in table 4. It is seen that the PS correction factor is about 0·6 to 0·7 for the $b\bar{b}$ and $c\bar{c}$ systems and leads to better agreement with the experimental results. In calculating the correction factors for the excited levels, we make use of some general features of the Schrödinger's equation. We first show that once the correction factor is known for the $1S$ state, we may calculate very easily the correction factors for all nS states of a heavy quarkonium.

Suppose $R_1(r)$ is the $1S$ state and $R_n(r)$ is the nS state of a heavy quarkonium. We define

$$K_n(r) = R_n(r)/R_1(r). \tag{14}$$

It is now easy to check that $K_n(r)$ satisfies the equation

$$K_n''(r) + \left(\frac{2}{r} + \frac{2R_1'}{R_1}\right) K_n'(r) + \alpha_n K_n(r) = 0, \tag{15}$$

where primes denote differentiation w.r.t. r and

$$\alpha_n = \frac{2\mu}{\hbar^2}(E_n - E_1). \tag{16}$$

Equation (15) is very convenient for studying the small distance behaviour of the excited states. Note that the equation does not involve the potential $V(r)$ explicitly and will, therefore, be useful for potentials singular at the origin. For a small r, we may neglect $R'(r)/R(r)$ and obtain the solution

$$K_n(r) = K(0)\frac{\sin\sqrt{\alpha_n}\,r}{\sqrt{\alpha_n}\,r}. \tag{17}$$

Hence, if $1/M_q$ is sufficiently small so that the above approximation is good, we have

$$\left|\frac{\phi_{nS}(1/M_q)}{\phi_{nS}(0)}\right|^2 = \frac{\hbar^2 M_q^2 \sin^2([2\mu(E_n - E_1)]^{1/2}/M_q\hbar)}{2\mu(E_n - E_1)}\left|\frac{\phi_{1S}(1/M_q)}{\phi_{1S}(0)}\right|^2. \tag{18}$$

The left side of (18) gives the Poggio-Schnitzer correction factor. Equation (15) may be useful in many applications. It may be noted that (17) is again a good solution near the maximum of the $1S$ wavefunction.

To see if the above approximation is applicable to the $Q\bar{Q}$ states, we have compared the exact numerical values of $\phi_{nS}(1/M_q)/\phi_{nS}(0)$ for the $c\bar{c}$ and $b\bar{b}$ states with those predicted by the relation (18). Our conclusion is that the Poggio-Schnitzer correction factors can be calculated easily by the relation (18) for the ψ and Υ families and also for heavier quarkonia. The calculated leptonic widths are consistent with the experimental results.

It may be pointed out that the relative Poggio-Schnitzer correction factors depend on the product of the mass of the quark $M_q = 2\mu$ and the level spacings ΔE and are insensitive to details of the potential. However, since ΔE is fixed, a different PS factor can result only if M_q is chosen differently. This shows that the relative PS correction factor should be almost the same in all potential models which fit the observed quarkonia masses.

The annihilation of the toponium, because of its predicted mass $\sim$ 80 GeV, presents a case where an interesting interplay of weak, electromagnetic and strong interactions is expected. In the calculation for a decay of the $^3S_1$ state, one should in general consider (i) the photon and the Z^0 exchange in the S-channel and the W exchange in the t-channel, (ii) the single quark weak decays of t, (iii) the three-gluon annihilation term and (iv) also the two-gluon plus photon term. For a toponium mass close to Z^0 mass, the neutral current decays dominate over strong and electromagnetic decays. For decays to lepton pairs, we need consider the interference of two amplitudes corresponding to the annihilation through (a) a virtual photon and (b) through a virtual Z^0. In the standard Salam-Weinberg model the contribution (b) can be obtained by noting that the weak neutral quark current is given by

$$J_\mu^0 = \frac{1}{2}\sum_i (\alpha_i \bar{q}_i \gamma_\mu q_i + \beta_i \bar{q}_i \gamma_\mu \gamma_5 q_i), \tag{19}$$

where i indicates the flavour. The vector and the axial vector coupling constants are given by

$$\alpha_i = 1 - \tfrac{8}{3}\sin^2\theta_w, \quad \beta_i = 1 \qquad \text{for} \quad i = u, c, t \tag{20}$$

and

$$\alpha_i = -1 + \tfrac{4}{3}\sin^2\theta_W, \quad \beta_i = -1 \qquad \text{for} \quad i = d, S, b. \tag{21}$$

For an electron, $\alpha_e = 4\sin^2\theta_W - 1$ and $\beta_e = -1$. Since only the vector part of the neutral current couples to the $^3S_1$ state, the interference between the two amplitudes depends only on the vector coupling constant α_t. We can write down the total leptonic decay width of the toponium state, including the Z^0 contribution (Khoze and Shifman 1983), as

$$\Gamma_{\text{total}}(T \to e^+e^-) = \Gamma_\gamma(T \to e^+e^-)\left[1 - \frac{2\alpha_e\alpha_t}{Q_t}y + \frac{(\alpha_e^2+\beta_e^2)\alpha_t^2}{Q_t^2}y^2\right], \tag{22}$$

where Q_t is the charge of the t-quark, Γ_γ is the pure electromagnetic leptonic decay width and

$$y = \frac{\sqrt{2}G_F}{16\pi\alpha}\Big/(M_z^{-2} - M(t\bar{t})^{-2}). \tag{23}$$

We have shown in table 5 the electromagnetic as well as the total leptonic decay widths of the $t\bar{t}$ system for a range of M_t values. The contribution of the Z^0 exchange term is

Table 5. Leptonic decay width of toponium states (e.m. and total) for various M_t.

Mass of t-quark (GeV)	1S $\lvert\psi(0)\rvert^2$ (GeV^3)	1S Leptonic decay width (keV) e.m.	1S Leptonic decay width (keV) Total	2S $\lvert\psi(0)\rvert^2$ (GeV^3)	2S Leptonic decay width (keV) e.m.	2S Leptonic decay width (keV) Total
30	136·99	3·712	3·707	63·72	1·694	1·693
35	194·02	3·854	3·969	85·78	1·675	1·732
40	264·73	4·018	4·898	110·93	1·659	2·070
45	350·52	4·197	19·116	138·97	1·642	9·396
50	452·67	4·385	31·363	169·69	1·624	9·701

negligible for ψ and Υ states, but dominates the leptonic decay widths for the toponium states, if M_t lies in the range suggested by the UA1 collaboration (Arnison *et al* 1984). Equation (22), however, should not be applied down to the Z^0-pole. It has been pointed out by Gusken *et al* (1985) that if the difference between the toponium and Z^0 masses and the width of Z^0 are of comparable magnitude, the situation changes and one may even expect a dip in the resonance excitation cross-section for $M(t\bar{t}) = M_z$. However, for $M(t\bar{t}) \geqslant M_z + \Gamma_z$, the line shape will resemble that obtained by the incoherent sum, given by (22) although there may be some distortion. However, the possibility remains that some excited states closer to the Z^0-boson pole may have a larger leptonic width than the ground state. It may be noted that the rates for decays into three gluons or two gluons and a photon are small and almost mass independent and are, therefore, difficult to observe against the enhanced rates for electro-weak decay.

In table 6, we have tabulated the decay widths of toponium to lighter quark-antiquark systems, $T \to q\bar{q}$. This is given by

$$\Gamma(T \to q\bar{q}) = 3\,\Gamma_\gamma(T \to e^+e^-)\left[Q_q^2 - \frac{2\alpha_t\alpha_q Q_q}{Q_t}y + \frac{(\alpha_q^2+\beta_q^2)y^2}{Q_t^2}\right], \tag{24}$$

where Q_q is the charge of the quark q and α_q and β_q are as given in (20) and (21).

4. Discussion

We have shown that a simple short-range QCD potential matched to a Martin type power potential gives results in agreement with the experimental values for a wide range of physical processes. The Poggio-Schnitzer correction is included while calculating the decay amplitudes of the bound states by assuming that the $Q\bar{Q}$ annihilate at an average distance $r \sim 1/M_Q$, which gives the relativistic size of the quark. The correction is, of course, negligible for very heavy quarkonia. Our aim has been to determine a theoretically motivated potential satisfying the constraints imposed by the available experimental results. Even though the large distance behaviour of the potential is not known, it is interesting to see that a static non-relativistic potential is capable of describing fairly accurately the general features of the $Q\bar{Q}$ spectroscopy.

It may be useful to study how the calculated results change as the parameters of the potential (1) are varied. We first consider the QCD scale parameter $\Lambda_{\overline{MS}}$ which has been chosen to be equal to 0·2 GeV. If we choose instead $\Lambda_{\overline{MS}} = 0{\cdot}15$ GeV and leave the

Table 6. $t\bar{t}$ decay width to low mass $q\bar{q}$ for various M_t.

Mass of t-quark (GeV)	$t\bar{t}$ decay width to $u\bar{u}$ (keV)		$t\bar{t}$ decay width to $d\bar{d}$ (keV)	
	1S	2S	1S	2S
30	7·754	3·609	4·512	2·145
35	13·207	5·982	11·049	5·103
40	37·100	16·660	40·883	18·580
45	390·420	199·130	492·120	252·000
50	569·410	169·120	733·760	217·710

other parameters unchanged (thus failing to satisfy the condition (4)), we get results which are qualitatively worse, particularly for the ground state. As expected, the higher states show a weaker dependence on $\Lambda_{\overline{MS}}$. Thus, the B.E. for the $1S$ state with $\Lambda_{\overline{MS}} = 0{\cdot}2$ GeV and 0·15 GeV are $-0{\cdot}213$ GeV and $-0{\cdot}204$ GeV respectively. The leptonic decay widths of nS states decrease by about 10% as $\Lambda_{\overline{MS}}$ is decreased from 0·2 GeV to 0·15 GeV.

Another parameter to consider here is s, which describes the scale of the crossover region. Our conclusion is that the quarkonia data favour a sharp crossover, i.e., a small value of $s \sim 0{\cdot}01$ fm. We have shown in figure 1 the potentials $V_{\mathrm{QCD}}(r)$, $V_M(r)$ and the total $V_T(r)$ with an inset which shows the nature of the cross-over on a magnified scale. It may be pointed out that the parameters of the potential may need minor adjustments when relativistic effects are considered. The gross features of the potential shown in figure 1, however, are unlikely to be altered. We shall consider the fine-hyperfine splittings of the $Q\bar{Q}$ system elsewhere. The success of the non-relativistic potential may be looked upon as an evidence in favour of QCD. This will be tested further in the near

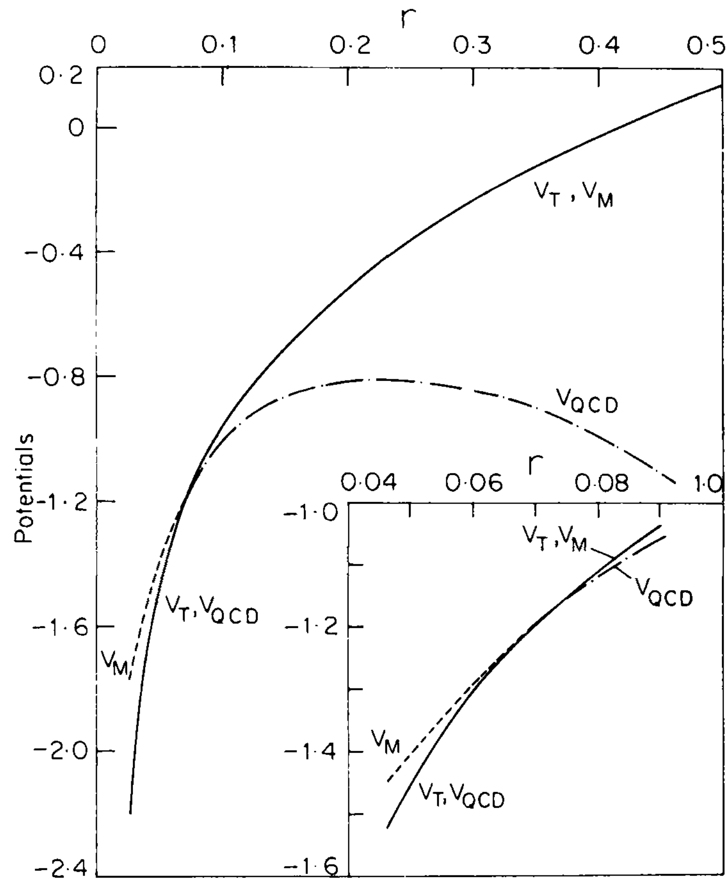

Figure 1. The potentials $V_{\mathrm{QCD}}(r)$, $V_M(r)$ and the total potential $V_T(r)$ as given by equation (1), are plotted against r. The units are Gev-fm. The inset shows the crossover region on a magnified scale.

future when the experimental results for the toponium family become available. The recent UA1 results indicate that the toponium should be observed at the e^+e^- colliders which are presently under construction (TRISTAN, SLC and LEP). It is expected that a variety of interesting and experimentally observable physical processes connected with the toponium system will be available for study when these colliders become operational.

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References

Applequist T, DeRújula A, Politzer H D and Glashow S L 1975 *Phys. Rev. Lett.* **34** 365
Arnison G *et al* (UA1 collaboration) 1984 *Phys. Lett.* **B147** 493
Bhanot G and Rudaz S 1978 *Phys. Lett.* **B78** 119
Billoire A and Morel A 1978 *Nucl. Phys.* **B135** 131
Billoire A 1980 *Phys. Lett.* **B92** 343
Buchmüller W, Grunberg G and Tye S H H 1980 *Phys. Rev. Lett.* **45** 103
Buras A J, Floratos E G, Ross D A and Sachrajda C T 1977 *Nucl. Phys.* **B131** 308
Buras A J 1980 *Rev. Mod. Phys.* **52** 199
Celmaster W, Georgi H and Machacek M 1978 *Phys. Rev.* **D17** 879
Deo B B and Barik B K 1983 *Phys. Rev.* **D27** 249
Durand B and Durand L 1982 *Phys. Rev.* **D25** 2312
Eichten E, Gottfried K, Kinoshita T, Kogut J, Lane K D and Yan T M 1975 *Phys. Rev. Lett.* **34** 369
Eichten E, Gottfried K, Kinoshita T, Lane K D and Yan T M 1976 *Phys. Rev. Lett.* **36** 500
Eichten E and Gottfried K 1977 *Phys. Lett.* **B66** 286
Eichten E, Gottfried K, Kinoshita T, Lane K D and Yan T M 1978 *Phys. Rev.* **D17** 3090
Eichten E, Gottfried K, Kinoshita T, Lane K D and Yan T M 1980 *Phys. Rev.* **D21** 203
Fischler W 1977 *Nucl. Phys.* **B129** 157
Grosse H and Martin A 1980 *Phys. Rep.* **C60** 341
Güsken S, Kühn J H and Zerwas P M 1985 SLAC-PUB-3580
Igi K and Hikasa K 1983 *Phys. Rev.* **D28** 565
Keung W Y and Muzinich I J 1983 *Phys. Rev.* **D27** 1518
Khoze V A and Shifman M A 1983 DESY preprint 83-105
Krasemann H 1980 *Phys. Lett.* **B96** 397
Levine R and Tomozawa Y 1979 *Phys. Rev.* **D21** 840
Martin A 1980 *Phys. Lett.* **B93** 338
Martin A 1981 *Phys. Lett.* **B100** 511
McClary R and Byers N 1983 *Phys. Rev.* **D28** 1692
Moxhay P and Rosner J L 1983 *Phys. Rev.* **D28** 1132
Poggio E C and Schnitzer H J 1978 *Phys. Rev. Lett.* **41** 1344
Poggio E C and Schnitzer H J 1979a *Phys. Rev.* **D19** 1557
Poggio E C and Schnitzer H J 1979b *Phys. Rev.* **D20** 1175
Quigg C and Rosner J L 1979 *Phys. Rep.* **C56** 168 and the references therein
Richardson J L 1979 *Phys. Lett.* **B82** 272
Stanley D P and Robson D 1980 *Phys. Rev.* **D21** 3180
Van Royen R and Weisskopf V F 1967a *Nuovo Cimento* **50** 617
Van Royen R and Weisskopf V F 1967b *Nuovo Cimento* **51** 583