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Surface diffuseness of deformed and rotating nuclei

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Abstract. The surface diffuseness of deformed and rotating nuclei has been studied using the energy density formalism. It is shown that the surface diffuseness exhibits an anisotropy. This anisotropy in surface diffuseness can result in an anisotropic charged particle emission from highly spinning nuclei as has been seen in some recent experiments.

Keywords. Surface diffuseness; energy density formalism; rotating nuclei; deformed nuclei; proximity potential.

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1. Introduction

The liquid drop model (Myers and Swiatecki 1974) and its extension to rotating nuclei (Cohen *et al* 1974) based on an idealized incompressible and uniformly charged fluid with a constant surface tension has provided a quantitative understanding of many macroscopic features of nuclei having a large spin. A semi-microscopic justification of this classical model was given by Grammaticos (1978) making use of the energy density formalism, where the nuclear part of the energy is written as the volume integral of a suitably chosen energy density and the coulomb and rotational energies are calculated classically. In the present work, we have studied the equilibrium density distributions of deformed and rotating nuclei making use of a similar energy density formalism but also including explicitly an anisotropy of the surface diffuseness. It was found that for both deformed and rotating nuclei the surface diffuseness exhibits an anisotropy with respect to the direction of the symmetry and the rotation axis.

2. The energy density formalism

According to the theorem of Hohenberg and Kohn (1964), it is always possible to represent the total energy of a Fermion system, such as a nucleus, as the volume integral of an energy density which is a functional of the matter density and its derivatives only.

$$E = E[\rho, \nabla\rho, \Delta\rho \ldots] = \int \varepsilon[\rho, \nabla\rho, \Delta\rho \ldots]\, \mathrm{d}v. \tag{1}$$

While it is not always possible to find the exact expression for the energy density ε, semi classical approximations have been found useful in the past (Brack *et al* 1985). The ground state energy is then determined by minimizing E with respect to variations in the density distribution $\rho(r)$, subject to the conservation of the total number of nucleons (For the present investigations we consider only the total nucleon density ρ and neglect terms arising from the difference between proton and neutron densities). A futher simplification is achieved by writing an analytical expression with a few parameters for the density $\rho(r)$, and carrying out the energy minimization with respect to only these limited number of parameters.

Following Grammaticos (1978), we write for the nuclear part of the energy density

$$\varepsilon = a_1\rho^2 + a_2\rho^{2+\sigma} + a_3(\nabla\rho)^2/\rho + c\rho^{5/3}, \qquad (2)$$

where the constants a_1, a_2, a_3 and σ are determined from known properties of infinite and semi-infinite nuclear matter and are given below.

$$a_1 = -655{\cdot}76 \text{ MeV fm}^3, \quad a_2 = 769{\cdot}5 \text{ MeV fm}^2,$$

$$a_3 = 3{\cdot}5 \text{ MeV fm}^2, \quad \sigma = 1/3.$$

The constant c is given by

$$c = (3/5)(3\pi^2/2)^{2/3}\hbar^2/2m. \qquad (3)$$

The coulomb and the rotational energies are given as

$$E_{\text{coul}} = (3/5)Z(Z-1)e^2/(r_0 A^{1/3}) \qquad (4)$$

and

$$E_{\text{rot}} = I^2/(2J). \qquad (5)$$

Here I is the spin of the nucleus and J is its moment of inertia about the rotation axis.

For the nuclear density distribution, Grammaticos uses a two-parameter form

$$\rho = \rho_0/[1+\exp\{\alpha_0(r-R(\theta,\varphi))\}], \qquad (6)$$

where ρ_0 and α_0 are the equilibrium central density and surface diffuseness for a spherical nucleus with no spin. $R(\theta, \varphi)$ describes the deformed nuclear shape for which the Hill-Wheeler parametrization (Hill and Wheeler 1953) is taken. The semi-axes of the ellipsoid a, b, c were given by

$$a = R\exp(5/4\pi)^{1/2}\beta\cos(\gamma+2\pi/3), \qquad (7)$$

$$b = R\exp(5/4\pi)^{1/2}\beta\cos(\gamma-2\pi/3), \qquad (8)$$

$$c = R\exp(5/4\pi)^{1/2}\beta\cos\gamma, \qquad (9)$$

where β and γ are Bohr parameters describing the shape of the ellipsoid. R is obtained from nucleon number conservation $\int d^3r\rho(r) = A$. For a given spin the total energy is minimized with respect to the parameters α_0 and ρ_0 and the shape parameters β and γ.

The above formalism implicitly assumes that surface diffuseness is isotropic, independent of θ and φ. But for deformed nuclei, the local curvature is a function of θ and φ and it is likely that the surface diffuseness can exhibit dependence on the local curvature. Similarly for rotating nuclei, the centrifugal force can introduce an anisotropy with respect to the rotation axis. Thus for deformed and rotating nuclei, there is a need to include explicitly, in the minimization, a surface diffuseness anisotropy parameter. We, therefore, use a three-parameter form

$$\rho = \rho_0/[1+\exp\{(\alpha_0-\alpha_2\sin^2\theta)(r-R(\theta,\varphi))\}]. \tag{10}$$

The parameter α_2 represents the angular variation in the surface diffuseness about the nuclear symmetry axis which is also assumed to be the rotation axis. The calculations have been carried out for oblate shapes of the nucleus and the minimum energy configurations were determined for different deformation parameters and spins.

3. Results and discussion

Figure 1 shows plots of the calculated energies versus the diffuseness anisotropy parameter α_2 for spherical and oblate shapes of a typical nucleus of ^{150}Sm with no spin. For the spherical shape of the nucleus the energy minimum is indeed at $\alpha_2 = 0$ indicating no anisotropy. However, for highly deformed shapes, the minimum shifts to small non-zero values of α_2 which seems to indicate that the surface diffuseness is greater in a direction perpendicular to symmetry axis. Figure 2 shows plots of the calculated energies versus the parameter α_2 for spherical and different oblate shapes of the nucleus ^{150}Sm for a typical nuclear spin of 70 $\hbar$. It can be seen that not only the ground state configuration is deformed with a value of $\beta \approx 0{\cdot}3$ but the anisotropy parameter α_2 also has a definite non-zero value. The sign of α_2 suggests that the surface diffuseness is greater in a direction perpendicular to rotation axis which is also assumed to be the symmetry axis in the present investigation. The present investigations also

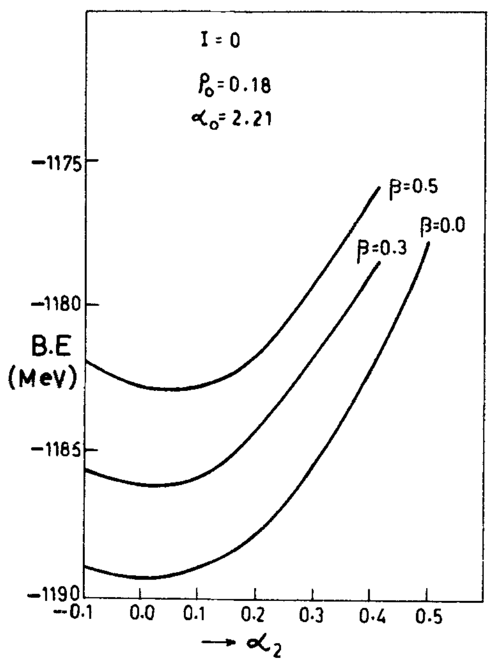

Figure 1. Binding energy versus surface diffuseness anisotropy for different deformations of a typical nucleus ^{150}Sm without spin.

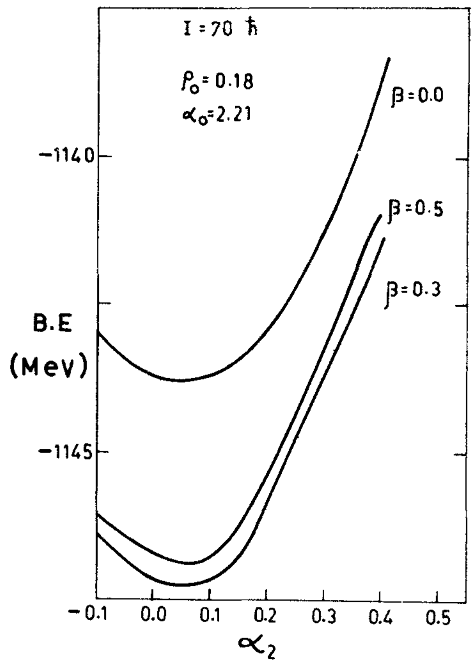

Figure 2. Binding energy versus surface diffuseness anisotropy for different deformations of ^{150}Sm with a typical spin of $70\hbar$.

show that this new degree of freedom does not appreciably change the absolute value of the total energy of minimum configuration. But the anisotropy in the surface diffuseness is expected to affect the interaction potential between two deformed and rotating nuclei depending on their relative orientations. For example, in the framework of the proximity potential (Blocki *et al* 1977) both the universal proximity function and the strength of the interaction depend on the diffuseness of the nuclear surface. It is, therefore, interesting to investigate the effect of the surface diffuseness anisotropy for deformed and rotating nuclei on the ion-ion interaction potential energies which play a central role in heavy ion collision dynamics. The nuclear potential is given by (Birkelund *et al* 1979)

$$V_N(\zeta) = 4\pi b\gamma R\phi(\zeta), \tag{11}$$

where $\phi(\zeta)$ is the universal proximity function

and $$R = C_1C_2/(C_1+C_2). \tag{12}$$

γ is the surface energy coefficient and C_1 and C_2 are the half density radii for the two interacting ions. C_i ($i = 1, 2$) are given by

$$C_i = R_i[1-(b/R_i)^2 \ldots], \tag{13}$$

where b is related to surface diffuseness and is usually taken to be unity and R_i is sharp surface radius. The separation distance ζ of two surfaces is given by

$$\zeta = (r - C_1 - C_2)/b. \tag{14}$$

The coulomb potential was calculated using phenomenological potential given below

$$V_C(r) = 1{\cdot}438 Z_1 Z_2/r, \qquad r > R_C, \tag{15}$$

$$V_C(r) = V_0 - kr^n, \qquad r < R_C, \tag{16}$$

where $R_C = R_1 + R_2$ is the nuclear charge radius. The expression for V_0, n and k are taken from Birkelund *et al* (1979).

Using the above expressions we calculated the nuclear interaction potential for the system $\alpha + {}^{150}Sm$. Figure 3 shows the dependence of the interaction potential on the surface diffuseness parameter of ^{150}Sm. It can be seen that the interaction barrier depends on the surface diffuseness. If, as shown in the present work, the surface diffuseness is anisotropic, with respect to the spin direction for a spherical nucleus, the corresponding interaction barrier also shows an anisotropy with respect to the spin direction. This would result, for example, in an anisotropic charged particle emission with respect to spin direction, particularly for low energy charged particles. It is interesting to note that such an anisotropic emission of charged particles from highly spinning nuclei has been experimentally observed by Dilmanian *et al* (1982). The authors had interpreted these results to imply that those nuclei are deformed at high spins. The present calculations clearly demonstrate that the above conclusion is not unambiguous since a spinning nucleus even in its spherical shape induces an anisotropic surface diffuseness and therefore an anisotropic charged particle emission. One has to, therefore, include both the effects, namely deformation and the surface diffuseness anisotropy, in a calculation of the interaction potential and the corresponding barrier penetrability.

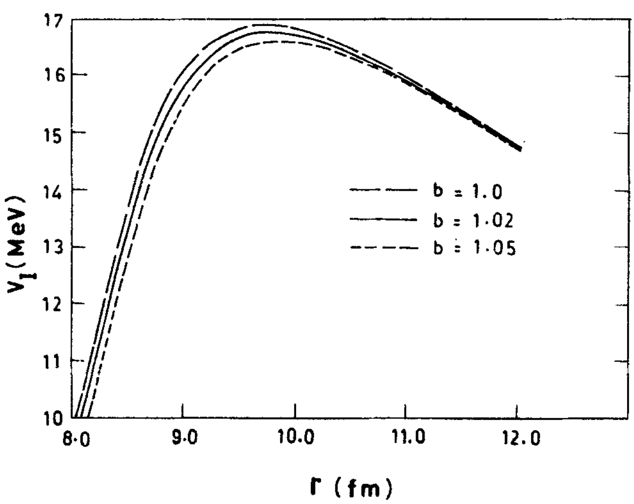

Figure 3. Interaction barrier for the system of ^{150}Sm and α for different surface diffuseness parameter of ^{150}Sm.

In summary, we have shown that for both deformed and rotating nuclei the surface diffuseness exhibits an anisotropy with respect to the direction of the symmetry axis and the rotation axis. These have interesting implications on low energy charged particle emission from nuclei having high spins.

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