Pramāṇa – J. Phys., Vol. 27, No. 5, November 1986, pp. 629–635. © Printed in India.

Analogue of Aharonov-Bohm effect in spin gauge theory

P C NAIK
Institute of Physics, University of Tokyo, Komaba, Meguroku, Tokyo 153, Japan
Present address: Department of Physics, S.C.S. College, Puri 752 001, India

MS received 3 September 1985; revised 31 May 1986

Abstract. We predict the possibility of observing analogue of the Aharonov-Bohm effect due to long range interaction between spins originating in the framework of spin gauge theory developed earlier. The effect is predicted for both electrons (fermions) and photons. Appropriate experimental set-ups are suggested.

Keywords. Aharonov-Bohm effect; axial vector gauge field; restricted Lorentz gauge symmetry; circularly-polarized laser beam.

PACS Nos 03·65; 12·90; 07·60

1. Introduction

The Aharonov-Bohm (AB) effect (Aharonov and Bohm 1959) on electron waves has received much attention from the point of view of both theory and experiment. Arguments, both for and against this effect have surfaced every now and then (Bocchieri *et al* 1978; Roy 1980; Peshkin 1981; Bocchieri *et al* 1984). However, the recent experiments by Tonomura *et al* (1983) with electron holography provide a firm evidence in support of the existence of this effect. The question of locality vs non-locality and field vs potential might remain theoretically debatable, but the fact that two coherent electron beams passing around a long solenoidal current develop a phase difference proportional to the magnetic flux enclosed within the path is experimentally established. This is essentially the central guiding theme of the discussions in the present paper.

The AB effect in electromagnetic interaction is mediated by the Abelian vector gauge field, i.e. the photon. The possibility of occurrence of a generalized AB effect in non-Abelian massless SU(2) gauge theory was suggested by Wu and Yang (1975). Their suggestion has been transcribed into experiment by Zeilinger *et al* (1983) who came out with a null conclusion which is not surprising. The SU(2) gauge fields in the electro-weak framework become massive $W^{\pm}$ and Z, thus quenching the phenomenon.

The case referred to here is that of another interaction proposed earlier (Naik and Pradhan 1981) and studied somewhat in detail (Naik 1980). This interaction arises while gauging the Lorentz group with a restricted choice of gauge parameters. The outcome is a massless axial vector gauge field that mediates long range Coulomb-like interaction between spins. A naive generalization of the ideas in electromagnetic case would suggest that an analogue of the AB effect in this case could be observed. In the present work we pursue such an idea to show that the effect is not only possible but is easier to observe.

In §2 we calculate the phase shift for fermions and photons. Section 3 deals with the possibility of realizing appropriate solenoidal current and definite prediction for the experiment.

2. Phase change for fermions and photons

The calculation of phase change in the electromagnetic AB effect is well known. The invariance under $U(1)$ gauge transformations provides a prescription for introducing vector potential into a Hamiltonian theory. The momentum $\mathbf{p}$ is replaced by $(\mathbf{p}-e\mathbf{A}/c)$ wherever it occurs in the Hamiltonian. In quantum theory, this means

$$p_\mu\psi \to \left(p_\mu - \frac{eA_\mu}{c}\right)\psi \tag{1}$$

in the Schrödinger's equation. The effect of this transformation on the wave function can be written as

$$\psi_0(x) \to \psi(x) = \exp\left(\frac{ie}{\hbar c}\int_{x_0}^{x} A_\mu \, \mathrm{d}x^\mu\right)\psi_0(x), \tag{2}$$

where x_0 is some arbitrary initial reference point and the integration runs over some path P, between x_0 and x. On the right side of (2) $\psi_0(x)$ is a formal solution of the free equation, but does not obey the single-valuedness criterion whenever the exponential factor is multivalued. When an electron wave is split into two parts and made to go around a solenoid with uniform magnetic field inside, along the axis (figure 1), at the recombination point the two waves develop a phase difference

$$\eta = \frac{e}{\hbar c}\oint \mathbf{A}\cdot \mathrm{d}\mathbf{l} = \frac{e}{\hbar c}\int_s \mathbf{B}\cdot \mathrm{d}\mathbf{s}. \tag{3}$$

Since the net magnetic flux enclosed within the area is non-zero, the result is a shift of the interference fringe pattern which is experimentally observed. The extension of this argument to the non-Abelian SU(2) case ignoring the non-linearities is straightforward.

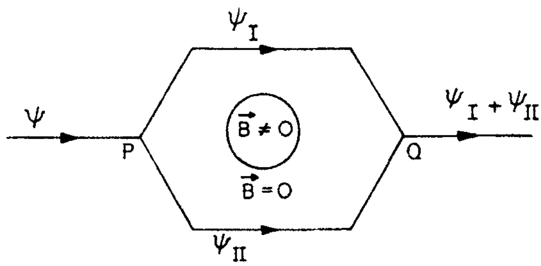

Figure 1. The conventional AB effect. An electron beam is split coherently at P and recombined at Q. A magnetic flux through the region between the two component beams ψ_I and ψ_{II} causes a phase-shift in the interference pattern.

In the case of our restricted Lorentz gauge symmetry (Naik and Pradhan 1981) for the fermions,

$$p_\mu \psi \rightarrow (p_\mu + \tfrac{3}{4} g \gamma_5 a_\mu)\, \psi. \tag{4}$$

The presence of an a_μ potential, as in the earlier case is equivalent to

$$\psi_0(x) \rightarrow \psi(x) = \exp\left(\frac{ig'}{\hbar c} \int_{x_0}^{x} a_\mu \, \mathrm{d}x^\mu\right) \psi_0(x), \tag{5}$$

where $g' \equiv \frac{3}{4} g \gamma_5$. So, if one starts with a beam of fermions with definite polarization, and lets it to pass around certain analogue of the solenoid of the AB experiment with only axial vector potential outside, one would expect a phase shift given by

$$\eta = \frac{g'}{\hbar c} \int_s (\nabla \times a) \cdot \mathrm{d}s. \tag{6}$$

Besides, one would expect some spin-flip too, since g' is a matrix. In writing (6) we use the Abelian-Stoke's theorem and ignore the nonlinearities of our theory in view of the small coupling strength which is of the order of 10^{-10}. Under the approximation mentioned, transformation (5) is the local chiral transformation on the fermion.

For photons, the chiral transformation on photon state is the quantum version of a duality transformation of the classical field variables (Zwanziger 1968). The analogue of Dirac equation for the photon is obtained by combining **E** and **B** as

$$\boldsymbol{\psi} = \mathbf{E} + i\mathbf{B}$$

to finally read

$$-cS_j p_j \boldsymbol{\psi} = i\hbar \frac{\partial \boldsymbol{\psi}}{\partial t} \tag{7}$$

where $(S_i)_{jk} = i\,\varepsilon_{ijk}$, the spin-1 matrix (Good 1957). In order to reproduce dimensionally correct Lagrangian the field variables are redefined as

$$\boldsymbol{\phi} = \boldsymbol{\psi}/(8\pi\varepsilon)^{\frac{1}{2}}, \tag{8}$$

where ε is the photon energy (Kursonoglu 1962). One may write (7) alternatively as

$$S_\mu p^\mu \boldsymbol{\phi} = 0. \tag{9}$$

To preserve the invariance under local chiral transformations, (9) would be modified to

$$S_\mu (p^\mu + g' a^\mu)\, \boldsymbol{\phi} = 0. \tag{10}$$

It may be noticed that the interaction term $g'\, \boldsymbol{\phi}^\dagger\, (S_\mu a^\mu)\, \boldsymbol{\phi}$, upon contraction of the axial vector field operators, would give the same spin-spin potential as derived from our spin gauge theory (Naik and Pradhan 1981). It is easy to see that for static case the phase change on photon state is again given by (6).

3. Experimental realization

In view of feebleness of the coupling strength g, the Lagrangian density for the axial vector field in linear approximation is given by

$$\mathscr{L}_G = -\tfrac{1}{2}(\partial_\mu a^\nu)^2 \tag{11}$$

along with the Lorentz condition $\partial_\mu a^\mu = 0$. The equation of motion that follows from the Lagrangian density (11) is the familiar wave equation for a_μ as for the electromagnetic potential A_μ. Therefore one may define the fields, for $\phi \equiv a_0 = 0$ as

$$\mathscr{E} = -\nabla \times a; \quad \mathscr{B} = -\partial a/\partial t. \tag{12}$$

Now the Maxwell's equations in terms of the $\mathscr{E}$ and $\mathscr{B}$ fields can be written down simply on replacing **E**, **B**, **J** and ρ by $\mathscr{B}$, $-\mathscr{E}$, **j** and ζ respectively (Pradhan *et al* 1971). Here **j** is the axial current density and ζ, the chiral charge density.

The Maxwell's equations for the axial vector potential lead to

$$\left(\nabla^2 - \frac{\partial^2}{\partial t^2}\right) a = -\mathbf{j}. \tag{13}$$

We may write

$$\mathbf{j}(\mathbf{x}, t) = g\rho(\mathbf{x})\,\mathbf{s}(t) \tag{14}$$

where $\mathbf{s}(t)$ is the spin average and not a matrix and ρ, the number density of the spin carriers. The static solution of (13) is given by

$$\begin{aligned} a_i(\mathbf{x}, t) &= \frac{1}{4\pi}\int \frac{\mathrm{d}^3 x' j_i(\mathbf{x}', t)}{|\mathbf{x}-\mathbf{x}'|} \\ &= \frac{g s_i(t)}{4\pi}\int \frac{\mathrm{d}^3 x' \rho(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|}. \end{aligned} \tag{15}$$

Again $\rho(\mathbf{r}) = \delta(\mathbf{r})$ for a point particle and this leads to

$$a(\mathbf{x}, t) = \frac{g}{4\pi x}\,\mathbf{s}(t). \tag{16}$$

Equation (16) gives us

$$\begin{aligned} \mathscr{E} &= \frac{g}{4\pi}\left(\frac{\mathbf{r}\times\mathbf{s}}{r^3}\right) \\ \mathscr{B} &= 0. \end{aligned} \tag{17}$$

In (17) $\mathscr{E}$ is the static spin field.

It is further well known that for right (left) circularly polarized photons σ_+ (σ_-) the spin points in (opposite to) the direction of momentum. So the $\mathscr{E}$ field produced by a polarized photon moving in certain medium can be written as

$$\mathscr{E} = \frac{g'}{4\pi v}\left(\frac{\mathbf{r} \times \mathbf{v}}{r^3}\right) \tag{18}$$

where v is the speed of photon in the medium and g' a pseudoscalar quantity. Let us imagine the passage of $\sigma_\pm$ photon beam in a long solenoidal path in optical fibre in which the speed of light remains constant. One would obtain the field due to a photon current element at an external point given by

$$\Delta\mathscr{E} = \frac{I(\mathbf{r} \times \mathbf{dl})}{4\pi v r^3} \tag{19}$$

where $I = Ng'vA$, N being the number density of photons and A, the area of cross-section through which the photon current passes. Henceforth it needs no more argument to state that the $\mathscr{E}$ field due to a long solenoidal photon current is confined inside and is parallel to the axis of the solenoid. It is given by

$$|\mathscr{E}| = In/vl \tag{20}$$

where n/l is the number of turns of the solenoid per unit length. The number density N of photons can be further related to the energy density by writing $N = u/h\nu$. Equation (20), therefore, can finally be cast into the form

$$|\mathscr{E}| = (g'EnA)/h\nu cl, \tag{21}$$

where $E = cu$ is the laser intensity. The expression for phase change in terms if the laser flux $F = EA$,

$$\eta = g'^2 Fns/(2\pi\hbar^2 c^2 vl), \tag{22}$$

s being the area of the circular cross-section of the solenoid.

The experimental design should now be clear. A circularly polarized laser beam is made to take solenoidal path through a long optical fibre. Polarized electrons, neutrons or laser beams are then scattered around such solenoid. One would expect to observe interference fringe-shift as a function of intensity of the laser beam passing through the solenoid (figure 2).

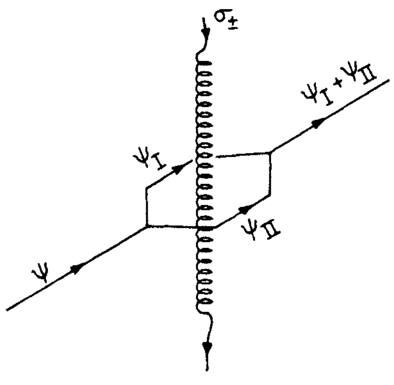

Figure 2. Schematic diagram of the set-up for observing AB effect due to long range spin-spin interaction. $\sigma_\pm$ laser beams through an optical fibre taking solenoidal path replaces the electron current. ψ represents polarized fermion (electron or neutron) or photon beam.

At this stage, we have definite prediction regarding this experiment. The fringe-shift will definitely depend upon the polarization of the interfering beams, the polarization of the laser current and the direction of such current. Reversal of the spin polarization or the direction of passage of the laser current will reverse the fringe-shift; but reversal of any two of them simultaneously will restore the original shift again. However, when accompanied with the intensity fluctuation, one would expect some spin-flip in the interfering beams also after the scattering.

4. Conclusion

We have predicted the possibility of observing the analogue of the AB effect by scattering polarized fermions or photons around a long solenoid of optical fibre carrying polarized laser beams. Definite reversal of fringe shifts are predicted depending upon polarization and direction of flow of laser current. The possibility of working with photons as interfering particles suggest that the AB effect predicted here may be observed even with a Michelson interferometer by placing the optical fibre solenoid between the interfering beams.

The coupling strength g'^2 occurring in the expression for phase-shift is to be identified with that of the spin-spin interaction estimated to be of the order of 10^{-10} (Naik and Pradhan 1981). The experiment suggested here provides alternative methods of its determination through a number of variations.

Acknowledgements

Most of this work was done at the University of Tokyo during the author's stay there. The author is thankful to the Ministry of Education, Japan for financial support. He is indebted to Prof. K Kawarabayashi for kind hospitality at Tokyo and for helpful discussions on the problem. This work was enkindled as reflections on certain deliberations at the International symposium on Foundations of Quantum Mechanics, Tokyo, Aug. 1983. The author acknowledges with thanks brief discussions with Profs C N Yang, Y Aharonov, A Zeilinger and J Anandan. The author is thankful to Prof. T Pradhan for valuable discussions and for having gone through the manuscript. Helpful discussions with Prof. S P Mishra, A Khare and J Maharana are gratefully acknowledged.

References

Aharonov Y, Bohm D 1959 *Phys. Rev.* **115** 485

Bocchieri P and Loinger A 1978 *Nuovo Cimento* **A47** 475; Bocchieri P and Loinger A 1984 *Lett. Nuovo Cimento* **39** 148 and references therein

Ferari E 1977 in *Tachyons, monopoles and related topics* (ed.) E Recami (Amsterdam: North Holland) p. 200

Good R H Jr 1957 *Phys. Rev.* **105** 1914

Kurusonoglu B 1962 *Modern Quantum Theory* (San Francisco: W H Freeman) ch. II

Naik P C 1980 *Quantum electrodynamics with axial photon*; Ph.D. thesis, Utkal University

Naik P C and Pradhan T 1981 *J. Phys.* **A14** 2795

Peshkin M 1981 *Phys. Rep.* **80** 375
Pradhan T, Basu U and Lahiri A 1971 SINP report Th/71–16
Pradhan T and Agarwal B K 1972 SINP report TH/72–3
Roy S M 1980 *Phys. Rev. Lett.* **44** 111
Tonomura A, Umezaki H, Matsuda T, Osakabe N, Endo J and Sugita Y 1983 *Phys. Rev. Lett.* **51** 331
Wu T T and Yang C N 1975 *Phys. Rev.* **D12** 3845
Zeilinger A, Horne M A and Shull C G 1983 *Proc. Int. Symposium on Foundations of Quantum Mechanics,* Tokyo, Aug. 29–31, Tokyo (Tokyo: Phys. Soc. Jpn.)
Zwanziger D 1968 *Phys. Rev.* **176** 1489