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The odd-even shifts in odd-odd deformed nuclei based on residual interaction studies

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Abstract. Residual interaction calculations have been made for predicting the sign and the magnitude of the odd-even shifts observed in the rotational levels of the $K = 0$ bands in the doubly-odd rare earth nuclei. It is shown that, contrary to the conclusions reached in earlier studies, the same zero-range spin-dependent residual interaction can reproduce the odd-even shifts as well as the GM splitting energies. This has been made possible with the inclusion of the phase factors for the total intrinsic spin and the total parity of the two-quasi-particle states in the Newby matrix elements. Predictions are made for the odd-even shifts for other $K = 0$ bands arising from several configurations not confirmed or observed so far.

Keywords. Odd-even shift; residual interaction; zero-range force; Nilsson wavefunction; odd-odd deformed nuclei.

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1. Introduction

The level structures of odd-odd nuclei are perhaps the most scarcely studied, and the least understood, facets of nuclear structure physics. The complexity of the highly dense spectra and the undefined residual neutron-proton interaction V_{np} have been the main deterrent factors leading to this situation. In the case of the deformed nuclei, the difficulties are somewhat less formidable since the two-particle band quantum number K is restricted to only two values $K^{\pm} = |\Omega_p \pm \Omega_n|$ and the relative ordering of these two members is determined by the well-established Gallagher-Moszkowski (1958) (GM) rule favouring spin-spin coupling. However, even in these cases, the only quantities investigated in some detail including the residual n-p interaction contribution are the splitting energies E_{GM} of the GM doublets (Pyatov 1963; de Pinho and Picard 1965; Jones *et al* 1971; Neiburg *et al* 1972; Boisson *et al* 1976; Elmore and Alford 1976) and, to a lesser extent, the odd-even shifts (Asaro *et al* 1960; Newby 1962) usually labelled as the Newby terms E_N. While the former could be satisfactorily described through these investigations, many doubts, as specified in the following, have persisted in respect of the evaluation of the odd-even shifts appearing for the levels in the $K = 0$ bands in odd-odd deformed nuclei (Pyatov 1963; de Pinho and Picard 1965; Jones *et al* 1971; Neiburg *et al* 1972; Boisson *et al* 1976; Elmore and Alford 1976). Even till the beginning of eighties, no serious efforts at quantitative prediction of the bandhead energies of the two-quasi-particle states, and thus of the complete spectra, of these nuclei was reported. Only 'very approximate predictions' using the simple assumption of the energy of the two-particle state being the sum of the single particle energies in neighbouring nuclei were attempted. For instance the predicted bandhead energies in the nucleus $^{236}_{93}Np_{143}$

were quoted (Lindner *et al* 1981) as $E(K^\pi = 2^+) = 55$–200 keV, $E(K^\pi = 3^+) = 125$–265 keV etc, each band having a series of rotational levels. Faced with this situation, we developed a formulation (Sood and Singh 1982), using a zero range residual interaction (Pyatov 1963), which enabled us to satisfactorily describe, and quantitatively predict, the bandhead energies of the two-particle states of doubly-odd nuclei. This approach has since been widely tested and applied to an extended series of such nuclei of the rare-earth as well as the actinide regions (Sood 1983, 1984a,b,c,d, 1985a,b; Sood and Singh 1983, 1984a,b). Here we report on the results of the application of this formulation (Sood and Singh 1982) to the outstanding problem of the evaluation of the odd-even shifts for $K = 0$ bands. A preliminary report on this problem was presented earlier (Sood and Ray 1985).

The problem was first brought into focus by Asaro *et al* (1960). They examined the properly symmetrized two-particle Nilsson model wavefunctions and concluded that the residual *n-p* interaction 'indeed gives rise to an energy term with sign alternating with spin I'. This proposal was examined in some detail by Newby (1962) who framed certain selection rules and also presented numerical results with central as well as tensor forces. Pyatov (1963) adopted the zero-range interaction for exploratory studies of the GM splittings as well as the odd-even shifts both for odd-odd and even-even nuclei. However, it was concluded about a decade later when calculations were performed by Jones *et al* (1971) with a sufficient data base (20 E_{GM} and 7 E_N) that 'the zero-range force is not suitable for the calculation of the odd-even shift, although it may be quite successfully applied to the calculation of the energy splitting'. Similar conclusions were arrived at by Neiburg *et al* (1972). Boisson *et al* (1976) and Elmore and Alford (1976), in parallel studies using a much wider data base as well as a variety of forces (including zero/short/long range forces), concluded that 'it is not possible to fit both splitting energies and odd-even shifts with the same central effective *n-p* interaction'.

We have reinvestigated this problem with a view to seek a prescription which may yield a simultaneous fit to both the splitting energies and the odd-even shifts with the same central force. For our calculations we employ the zero range force adopted in many earlier investigations. For meaningful comparison with earlier results we adopt the parameter value $\alpha W = 0{\cdot}84$ MeV arrived at by Elmore and Alford (1976), from a least squares fit to the splitting energies using a data base of 78 values. This parameter value agrees with that derived by Jones *et al* (1971) and is quite close to that of Boisson *et al* (1976). In the following we present an expression for the odd-even shift Newby term which, using this interaction parameter derived from fitting the splitting energies, yields good fits to the odd-even shifts thus resolving a dilemma that has persisted over the past two decades.

2. Theoretical formulation

For a given two-particle configuration, the Newby matrix element is defined by (Newby 1962)

$$E_N = -\mathscr{P}\langle \chi_{\Omega_p}\chi_{-\Omega_n}|V_{np}|\chi_{-\Omega_p}\chi_{\Omega_n}\rangle, \tag{1}$$

where $\Omega_p = \Omega_n$ and $\mathscr{P}$ is the total parity of the nuclear state. For the residual *n-p* interaction, we adopt the zero range spin-dependent central potential given by Pyatov

(1963) as

$$V_{np} = -4\pi g \delta (r_p - r_n) [(1-\alpha) + \alpha(\sigma_p \cdot \sigma_n)], \tag{2}$$

where g is the interaction parameter, α determines the fractional strength of spin-spin force in V_{np}, and σ_p and σ_n are the proton and neutron spin operators. Equation (2) thus includes, in addition to a spin-independent (Wigner) term, an explicitly spin-dependent term. Using product type two-particle wavefunctions in terms of the Nilsson single-particle wavefunction

$$\chi_\Omega = \sum_{l,\Sigma} a_{l,\Omega-\Sigma} R_{nl}(r) Y_{l,\Omega-\Sigma}(\theta, \phi) f_{1/2,\Sigma} \tag{3}$$

we write (1) as

$$E_N = -\mathscr{P}[(1-\alpha) WB_0 + \alpha WB_\sigma], \tag{4}$$

where the new parameter W is related to g of (2) through the oscillator frequency,

$$W = (2v^3/\pi)^{1/2} g, \tag{5}$$

and has therefore the dimensions of energy; B_0 and B_σ are the Newby matrix elements corresponding to the spin-independent and spin-dependent parts respectively. The analytical expressions for these matrix elements are obtained to be the following:

$$\begin{aligned} B_0 = & \sum_{l_1} \sum_{l_2} \sum_{l'_1} \sum_{l'_2} [2l_1+1)(2l'_1+1)(2l_2+1)(2l'_2+1)]^{1/2} \\ & \times F^0(n_1 l_1; n'_1 l'_1 | n_2 l_2; n'_2 l'_2) \sum_L \frac{1}{2L+1} \langle l_1 l_2 00 | L0 \rangle \langle l'_1 l'_2 00 | L0 \rangle \\ & \times \sum_\Sigma [a_{l_1\Omega_1-\Sigma}\, a_{l_2\Omega_2-\Sigma}\, a_{l'_1\Omega_1+\Sigma}\, a_{l'_2\Omega_2+\Sigma} \langle l_1 l_2 (-\Omega_1+\Sigma)(\Omega_2-\Sigma) | \\ & L(-\Omega_1+\Omega_2) \rangle \langle l'_1 l'_2 (\Omega_1+\Sigma)(-\Omega_2-\Sigma) | L(\Omega_1-\Omega_2) \rangle + a_{l_1\Omega_1-\Sigma}\, a_{l_2\Omega_2+\Sigma} \\ & \times a_{l'_1\Omega_1+\Sigma}\, a_{l'_2\Omega_2-\Sigma} \langle l_1 l_2 (-\Omega_1+\Sigma)(\Omega_2+\Sigma) | L(-\Omega_1+\Omega_2+2\Sigma) \rangle \\ & \times \langle l'_1 l'_2 (\Omega_1+\Sigma)(-\Omega_2+\Sigma) | L(\Omega_1-\Omega_2+2\Sigma) \rangle], \end{aligned} \tag{6}$$

$$\begin{aligned} B_\sigma = & \sum_{l_1} \sum_{l_2} \sum_{l'_1} \sum_{l'_2} [(2l_1+1)(2l'_1+1)(2l_2+1)(2l'_2+1)]^{1/2} \\ & \times F^0(n_1 l_1; n'_1 l'_1 | n_2 l_2; n'_2 l'_2) \sum_L \frac{1}{2L+1} \langle l_1 l_2 00 | L0 \rangle \langle l'_1 l'_2 00 | L0 \rangle \\ & \times \sum_\Sigma [2a_{l_1\Omega_1-\Sigma}\, a_{l_2\Omega_2-\Sigma}\, a_{l'_1\Omega_1-\Sigma}\, a_{l'_2\Omega_2-\Sigma} \langle l_1 l_2 (-\Omega_1+\Sigma)(\Omega_2-\Sigma) | \\ & \times L(-\Omega_1+\Omega_2) \rangle \langle l'_1 l'_2 (\Omega_1-\Sigma)(-\Omega_2+\Sigma) | L(\Omega_1-\Omega_2) \rangle + a_{l_1\Omega_1-\Sigma}\, a_{l_2\Omega_2+\Sigma} \\ & \times a_{l'_1\Omega_1+\Sigma}\, a_{l'_2\Omega_2-\Sigma} \langle l_1 l_2 (-\Omega_1+\Sigma)(\Omega_2+\Sigma) | L(-\Omega_1+\Omega_2+2\Sigma) \rangle \\ & \times \langle l'_1 l'_2 (\Omega_1+\Sigma)(-\Omega_2+\Sigma) | L(\Omega_1-\Omega_2+2\Sigma) \rangle - a_{l_1\Omega_1-\Sigma}\, a_{l_2\Omega_2-\Sigma} \\ & \times a_{l'_1\Omega_1+\Sigma}\, a_{l'_2\Omega_2+\Sigma} \langle l_1 l_2 (-\Omega_1+\Sigma)(\Omega_2-\Sigma) | L(-\Omega_1+\Omega_2) \rangle \\ & \times \langle l'_1 l'_2 (\Omega_1+\Sigma)(-\Omega_2-\Sigma) | L(\Omega_1-\Omega_2) \rangle], \end{aligned} \tag{7}$$

where we have replaced the subscripts p and n by the numbers 1 and 2. The quantities F^0 are radial Slater integrals. We prove in the Appendix that

$$B_0 = 0, \tag{8}$$

$$B_\sigma = A_0 - A_\sigma, \tag{9}$$

where A_0 and A_σ are the GM-matrix elements corresponding to the spin-independent and spin-dependent parts respectively, the analytical expressions for which are (Sood and Singh 1982)

$$\begin{aligned} A_0 = &\sum_{l_1}\sum_{l_2}\sum_{l'_1}\sum_{l'_2}[(2l_1+1)(2l'_1+1)(2l_2+1)(2l'_2+1)]^{1/2} \\ &\times F^0(n_1 l_1; n'_1 l'_1 | n_2 l_2; n'_2 l'_2)\sum_L \frac{1}{2L+1}\langle l_1 l_2 00|L0\rangle\langle l'_1 l'_2 00|L0\rangle \\ &\times \sum_\Sigma [a_{l_1\Omega_1-\Sigma}\, a_{l_2\Omega_2-\Sigma}\, a_{l'_1\Omega_1-\Sigma}\, a_{l'_2\Omega_2-\Sigma}\langle l_1 l_2(\Omega_1-\Sigma)(-\Omega_2+\Sigma)| \\ &\times L(\Omega_1-\Omega_2)\rangle\langle l'_1 l'_2(\Omega_1-\Sigma)(-\Omega_2+\Sigma)|L(\Omega_1-\Omega_2)\rangle + a_{l_1\Omega_1-\Sigma}\, a_{l_2\Omega_2+\Sigma} \\ &\times a_{l'_1\Omega_1-\Sigma}\, a_{l'_2\Omega_2+\Sigma}\langle l_1 l_2(\Omega_1-\Sigma)(-\Omega_2-\Sigma)|L(\Omega_1-\Omega_2-2\Sigma)\rangle \\ &\times \langle l'_1 l'_2(\Omega_1-\Sigma)(-\Omega_2-\Sigma)|L(\Omega_1-\Omega_2-2\Sigma)\rangle], \end{aligned} \tag{10}$$

$$\begin{aligned} A_\sigma = &\sum_{l_1}\sum_{l_2}\sum_{l'_1}\sum_{l'_2}[(2l_1+1)(2l_2+1)(2l'_1+1)(2l'_2+1)]^{1/2} \\ &\times F^0(n_1 l_1; n'_1 l'_1 | n_2 l_2; n'_2 l'_2)\sum_L \frac{1}{2L+1}\langle l_1 l_2 00|L0\rangle\langle l'_1 l'_2 00|L0\rangle \\ &\times \sum_\Sigma [a_{l_1\Omega_1-\Sigma}\, a_{l_2\Omega_2+\Sigma}\, a_{l'_1\Omega_1-\Sigma}\, a_{l'_2\Omega_2+\Sigma}\langle l_1 l_2(\Omega_1-\Sigma)(-\Omega_2-\Sigma)| \\ &L(\Omega_1-\Omega_2-2\Sigma)\rangle\langle l'_1 l'_2(\Omega_1-\Sigma)(-\Omega_2-\Sigma)|L(\Omega_1-\Omega_2-2\Sigma)\rangle - a_{l_1\Omega_1-\Sigma} \\ &\times a_{l_2\Omega_2-\Sigma}\langle l_1 l_2(\Omega_1-\Sigma)(-\Omega_2+\Sigma)|L(\Omega_1-\Omega_2)\rangle\{a_{l'_1\Omega_1-\Sigma}\, a_{l'_2\Omega_2-\Sigma} \\ &\times \langle l'_1 l'_2(\Omega_1-\Sigma)(-\Omega_2+\Sigma)|L(\Omega_1-\Omega_2)\rangle - 2a_{l'_1\Omega_1+\Sigma}\, a_{l'_2\Omega_2+\Sigma} \\ &\times \langle l'_1 l'_2(\Omega_1+\Sigma)(-\Omega_2-\Sigma)|L(\Omega_1-\Omega_2)\rangle\}]. \end{aligned} \tag{11}$$

Substituting from (8) and (9) in (4) we obtain

$$E_N = -\mathscr{P}\alpha W(A_0 - A_\sigma). \tag{12}$$

We recall (Sood and Singh 1982) that the corresponding expression for the GM splitting energy is

$$E_{GM} = \pm 2\alpha W|A_\sigma|, \tag{13}$$

where the +ve (−ve) sign is taken for parallel (antiparallel) pair of intrinsic spins. Here we remember that for parallel pair of intrinsic spins the $K=0$ band is spin-singlet ($\Sigma = 0$, antiparallel-coupling) and for antiparallel pair, it is spin-triplet ($\Sigma = 1$, parallel-coupling). Thus we can rewrite (12) as (noting that A_0 is always negative)

$$E_N = \tfrac{1}{2} E_{\mathrm{GM}} \left[\frac{|A_0|}{|A_\sigma|} - 1 \right] \mathscr{P}. \tag{14}$$

Accordingly we find that the Newby odd-even shift and the GM splitting energy are related to each other, and hence it appears reasonable to expect that both can be described using the same interaction parameters. From this consideration we suggest, combining (13) and (14), the following expression for the odd-even shift:

$$E_N = (-)^{\Sigma} \mathscr{P} \alpha W [|A_0| - |A_\sigma|], \tag{15}$$

where we have included the phase factors related to the total parity $\mathscr{P}$ in common with earlier studies, and also to the total intrinsic spin Σ of the two-particle state. The latter has been introduced to remedy the anomaly pointed out by Elmore and Alford (1976) that the predicted sign for the spin-triplet ($\Sigma = 1$) bands in most cases does not agree with the experiment.

3. Results and discussion

The experimental odd-even shift is determined (e.g. Elmore and Alford 1976) from the lowest two observed members of $K = 0$ bands using the expression

$$E_N = \tfrac{1}{2}(-)^I [E_0(I) - E_0(I+1)] + \frac{\hbar^2}{2\mathscr{I}} [(-)^I (I+1) + a_p a_n \delta_{\Omega_n \frac{1}{2}}], \tag{16}$$

where the last term contains the decoupling coefficients a_p and a_n for the case when $\Omega_p = \Omega_n = 1/2$, and E_0 denotes the unperturbed level energy. If the bands were unperturbed (not admixed by Coriolis mixing etc) and the same residual *n-p* interaction was valid for a given configuration, one should have observed the same values of the splitting energy as well as the odd-even shift for the same configuration appearing in different nuclei. Occasionally we find wide divergences in these values obtained from the observed level energies. For instance the odd-even shifts deduced for ^{156}Tb and ^{158}Tb for the configuration $(3/2[411]_p - 3/2[521]_n)$ (Elmore 1974) are not only different but even of opposite sign from its value deduced for ^{160}Tb (Kern *et al* 1974). The case of ^{176}Lu, which corresponds to the only known exception to the GM rule, also falls in the category for which the observed and the predicted odd-even shifts are widely divergent. Disregarding these few anomalous cases, we list in table 1 the known cases of $K = 0$ bands for odd-odd nuclei of the rare earth region. In this table we compare our predictions using (12) for the zero range potential within the framework of the earlier approach alongside the predicted values of Elmore and Alford (1976) for the central force with a Gaussian radial dependence that has a combination of short and long range parts and our predictions using the newly proposed equation (15) in comparison with the respective experimental values. For objective comparison we adopt the deformation δ corresponding to the deformation parameter ε_2 (neglecting the hexadecapole deformation ε_4) used by Elmore and Alford (1976) obtained through the relation (Bow 1970)

$$\delta \approx \varepsilon_2 [1 + \varepsilon_2 / 3]. \tag{17}$$

Table 1. Theoretical and experimental odd-even shifts in odd-odd nuclei of the rare-earth region.

Nucleus	Configuration		$\|\Sigma\|$	$\mathscr{P}$	Calc. E_N(keV)			Expt.
	Proton	Neutron			EA	SR(A)	SR(B)	E_N(keV)
^{158}Tb	3/2[411]	3/2[402]	1	+	31	28	−28	−32
^{160}Tb	3/2[411]	3/2[521]	0	−	−164	−174	−39	−17
^{166}Ho	7/2[523]	7/2[633]	0	−	−238	−189	−33	−32
^{168}Tm	1/2[411]	1/2[400]	1	+	51	57	−57	−29
	1/2[411]	1/2[521]	0	−	−158	−188	−50	−27
^{170}Tm	1/2[411]	1/2[521]	0	−	−158	−188	−49	−38
^{170}Lu	7/2[404]	7/2[633]	1	+	39	24	−24	−42
^{172}Tm	1/2[411]	1/2[521]	0	−	−154	−189	−48	−25
^{172}Lu	7/2[404]	7/2[633]	1	+	39	24	−24	−55[a]
	1/2[541]	1/2[521]	0	+	66	81	66	74
^{174}Lu	7/2[404]	7/2[633]	1	+	39	24	−24	−28
^{182}Ta	7/2[404]	7/2[503]	1	−	−27	−22	22	28
^{184}Ta	7/2[404]	7/2[503]	1	−	−12	−26	26	35
^{188}Re	9/2[514]	9/2[505]	1	+	23	18	−18	−54

[a] See Toth *et al* (1979)

Presently we are not attempting to quantitatively match the predicted and the experimental values but are only interested in testing the validity of the interaction parameters, which are derived by fitting the splitting energies, for the description of odd-even shifts. Accordingly we adopt the parameter $\alpha W = 0{\cdot}84$ MeV obtained from a least squares fit to the splitting energies and do not vary it from nucleus to nucleus, or from configuration to configuration. Unless otherwise indicated, data on the odd-even shifts are taken from the table of isotopes. (Lederer and Shirley 1978) or directly from the references therein. The matrix elements A_0 and A_σ are calculated at the appropriate deformation of the nucleus (Elmore and Alford 1976). The values of κ and μ are given the mass dependence suggested by Nilsson *et al* (1969).

From table 1, it is clear that our results labelled SR(A) obtained using (12) have the same sign and about the same magnitude as those of Elmore-Alford (1976) labelled EA. In both the cases, the theoretical values of E_N for the spin-singlet ($\Sigma = 0$) bands are too large in comparison to the experimental values, although the sign is correctly predicted. On the other hand, for spin-triplet($\Sigma = 1$) bands we have about the right magnitude, but the opposite sign. Thus, we find a common discrepancy in both the models. This discrepancy can be removed by introducing another phase factor for the total intrinsic spin of the nuclear state in the manner shown in (15). Our theoretical results labelled SR(B) in table 1 obtained using (15) are in good agreement with the experimental values both in sign and in magnitude.

Encouraged by the degree of agreement, we predict, as given in table 2, the values of odd-even shifts for some configurations expected below 1 MeV excitation energy, but not yet confirmed or observed, in some rare-earth nuclei. It may be recalled that we have not included the effects of coriolis mixing in our calculations and as such the theoretical values should be compared with estimates from unperturbed energies. The large

Table 2. Predicted odd-even shifts for some configurations not yet confirmed or observed in some rare-earth nuclei.

Configuration		Calc. E_N	
Proton	Neutron	(keV)	Expected in nuclei
5/2[532]	5/2[642]	−47	$^{156-158}$Tb
	5/2[523]	−44	$^{158-162}$Tb
5/2[413]	5/2[642]	−37	$^{156-160}$Tb
	5/2[523]	−45	$^{158-162}$Tb
	5/2[512]	34	^{164}Tb
3/2[411]	3/2[532]	75	$^{156-158}$Tb
	3/2[651]	33	^{156}Tb
	3/2[402]	−28	^{156,160}Tb, $^{158-162}$Ho, $^{160-164}$Tm
	3/2[521]	−39	^{162}Tb, $^{158-162}$Ho, $^{160-164}$Tm
7/2[523]	7/2[633]	−33	$^{162-164}$Tb, $^{164,168-170}$Ho, $^{166-172}$Tm, $^{168-172}$Lu
	7/2[514]	−26	^{174}Tm, ^{176}Lu
1/2[411]	1/2[400]	−57	$^{158-162}$Ho, $^{160-166}$Tm
	1/2[521]	−50	$^{158-170}$Ho, $^{162-166,174}$Tm, $^{166-174}$Lu, $^{172-176}$Ta
	1/2[510]	50	$^{168-174}$Tm, ^{178}Lu, $^{180-184}$Ta, $^{186-188}$Re
7/2[404]	7/2[633]	−24	$^{164-166}$Ho, $^{166-172}$Tm, ^{168}Lu, $^{172-178}$Ta
	7/2[514]	−31	$^{170-178}$Lu, $^{176-182}$Ta
	7/2[503]	24	^{186}Ta
5/2[402]	5/2[642]	20	^{158}Tb, $^{162-164}$Ho, $^{162-166}$Tm, ^{170}Lu, ^{176}Ta
	5/2[523]	42	$^{162-164}$Ho, $^{162-166}$Tm, ^{166,170}Lu
	5/2[512]	−25	$^{168-172}$Tm, $^{170-176}$Lu, $^{176-178}$Ta, $^{180-186}$Re
1/2[541]	1/2[521]	66	$^{162-166}$Ho, $^{164-172}$Tm, $^{168-170,174}$Lu, $^{172-178}$Ta, $^{176-182}$Re
	1/2[510]	−49	$^{168-170}$Tm, ^{170,176}Lu, ^{178}Ta, $^{180-184}$Re
9/2[514]	9/2[624]	−25	$^{176-178}$Lu, $^{176-184}$Ta, $^{180-182}$Re

deviations of the theoretical values from the experimental ones observed in some cases may be due to the coriolis band-mixing effect. In addition to the shift due to a residual n-p interaction, the members of a $K = 0$ rotational band will also be perturbed by the coriolis coupling. The coriolis interaction only mixes bands which differ by one unit in K value. Therefore if there is a $K=1$ band near the $K=0$ band, then the energy levels of the $K=0$ band will be perturbed. However, if the effect is weak these shifts can be taken into account by simply renormalizing the moment of inertia.

4. Summary and conclusions

The aim of this paper was to reproduce the empirical odd-even shifts using the same n-p residual interaction potential which was employed in evaluating the GM-splitting energies in doubly-odd axially-symmetric deformed nuclei of the rare-earth region. It is found that a central force having a Gaussian radial dependence that includes both short and long range components and a zero-range spin-dependent delta-function force including only a single phase factor for the total nuclear parity of the state give the same results, which do not agree with the experimental odd-even shifts. However, with the inclusion of phase factors for the total intrinsic spin and total parity of the nuclear state in the Newby matrix element, it is found that the same zero-range spin-dependent central potential, which has been used to fit the measured splitting energies, can reproduce the empirical odd-even shifts reasonably well. It has also been shown in Appendix that the spin-independent (Wigner) term does not contribute to the odd-even shift; only the spin-dependent term contributes. Since our calculations show that $|A_0| > |A_\sigma|$, the direction of the odd-even shift due to spin-spin force depends only on the total intrinsic spin and total parity of the state.

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Appendix 1. Relation between the Newby- and GM-matrix elements

Let us introduce the following abbreviated notation:

$$\sum = \sum_{l_1}\sum_{l_2}\sum_{l'_1}\sum_{l'_2}\sum_{L}\sum_{\Sigma}, \quad F^0 = F^0(n_1l_1; n'_1l'_1|n_2l_2; n'_2l'_2),$$

$$a_i^{\pm} = a_{l_i\Omega_i\pm\Sigma}, \qquad A_i^{\pm} = a_{l'_i\Omega_i\pm\Sigma},$$

$$S(ll'L) = \frac{1}{2L+1}[(2l_1+1)(2l'_1+1)(2l_2+1)(2l'_2+1)]^{1/2},$$

$$C_0 = \langle l_1l_200|L0\rangle\langle l'_1l'_200|L0\rangle,$$

$$C_1 = \langle l_1l_2(\Omega_1-\Sigma)(-\Omega_2-\Sigma)|L(\Omega_1-\Omega_2-2\Sigma)\rangle \times \langle l'_1l'_2(\Omega_1-\Sigma)(-\Omega_2-\Sigma)|L(\Omega_1-\Omega_2-2\Sigma)\rangle,$$

$$C_2 = \langle l_1l_2(\Omega_1-\Sigma)(-\Omega_2+\Sigma)|L(\Omega_1-\Omega_2)\rangle \times \langle l'_1l'_2(\Omega_1-\Sigma)(-\Omega_2+\Sigma)|L(\Omega_1-\Omega_2)\rangle,$$

$$C_3 = \langle l_1l_2(\Omega_1-\Sigma)(-\Omega_2+\Sigma)|L(\Omega_1-\Omega_2)\rangle \times \langle l'_1l'_2(\Omega_1+\Sigma)(-\Omega_2-\Sigma)|L(\Omega_1-\Omega_2)\rangle,$$

$$C_4 = \langle l_1l_2(-\Omega_1+\Sigma)(\Omega_2+\Sigma)|L(-\Omega_1+\Omega_2+2\Sigma)\rangle \times \langle l'_1l'_2(\Omega_1+\Sigma)(-\Omega_2+\Sigma)|L(\Omega_1-\Omega_2+2\Sigma)\rangle. \tag{A.1}$$

Using the symmetry relation,

$$\langle l_1l_2m_1m_2|L(m_1+m_2)\rangle = (-)^{l_1+l_2-L}\langle l_1l_2 -m_1-m_2|L(-m_1-m_2)\rangle, \tag{A.2}$$

of the Clebsch-Gordan coefficients, we can write

$$C_3 = (-)^{l_1+l_2-L} \langle l_1 l_2 (-\Omega_1+\Sigma)(\Omega_2-\Sigma)|L(-\Omega_1+\Omega_2)\rangle \times \langle l'_1 l'_2 (\Omega_1+\Sigma)(-\Omega_2-\Sigma)|L(\Omega_1-\Omega_2)\rangle$$
$$= \langle l_1 l_2 (-\Omega_1+\Sigma)(\Omega_2-\Sigma)|L(-\Omega_1+\Omega_2)\rangle \times \langle l'_1 l'_2 (\Omega_1+\Sigma)(-\Omega_2-\Sigma)|L(\Omega_1-\Omega_2)\rangle, \quad (A.3)$$

because l_1+l_2-L is an even integer. Similarly

$$C_2 = \langle l_1 l_2 (-\Omega_1+\Sigma)(\Omega_2-\Sigma)|L(-\Omega_1+\Omega_2)\rangle \times \langle l'_1 l'_2 (\Omega_1-\Sigma)(-\Omega_2+\Sigma)|L(\Omega_1-\Omega_2)\rangle. \quad (A.4)$$

Thus in abbreviated notation, we can write

$$A_0 = \sum S(ll'L)F^0C_0[a_1^- a_2^- A_1^- A_2^- C_2 + a_1^- a_2^+ A_1^- A_2^+ C_1], \quad (A.5)$$

$$A_\sigma = \sum S(ll'L)F^0C_0[a_1^- a_2^+ A_1^- A_2^+ C_1 - a_1^- a_2^- A_1^- A_2^- C_2 + 2a_1^- a_2^- A_1^+ A_2^+ C_3), \quad (A.6)$$

$$B_0 = \sum S(ll'L)F^0C_0[a_1^- a_2^- A_1^+ A_2^+ C_3 + a_1^- a_2^+ A_1^+ A_2^- C_4]$$
$$= \sum S(ll'L)F^0 a_1^- a_2^- A_1^+ A_2^+ [C_0C_3 + C_0C_4], \quad (A.7)$$

interchanging the summation indices l_2 and l'_2 (since both have the same possible values). And

$$B_\sigma = \sum S(ll'L)F^0C_0[2a_1^- a_2^- A_1^- A_2^- C_2 + a_1^- a_2^- A_1^+ A_2^+ C_4 - a_1^- a_2^- A_1^+ A_2^+ C_3]. \quad (A.8)$$

Using the coupling rule for spherical harmonics

$$Y_{l_1m_1}(\theta\phi)Y_{l_2m_2}(\theta\phi) = \sum_L \left(\frac{(2l_1+1)(2l_2+1)}{4\pi(2L+1)}\right)^{1/2} \langle l_1 l_2 00|L0\rangle \times \langle l_1 l_2 m_1 m_2|L(m_1+m_2)\rangle Y_{L(m_1+m_2)}(\theta\phi), \quad (A.9)$$

and taking note of their orthogonality properties we obtain

$$4\pi\int Y^*_{l'_1m'_1} Y^*_{l'_2m'_2} Y_{l_1m_1} Y_{l_2m_2}\,d\Omega$$
$$= \sum \frac{1}{2L+1}[(2l_1+1)(2l'_1+1)(2l_2+1)(2l'_2+1)]^{1/2} \times \langle l_1 l_2 00|L0\rangle \langle l'_1 l'_2 00|L0\rangle \langle l_1 l_2 m_1 m_2|L(m_1+m_2)\rangle \times \langle l'_1 l'_2 m'_1 m'_2|L(m'_1+m'_2)\rangle \delta_{(m_1+m_2)(m'_1+m'_2)}. \quad (A.10)$$

Substituting $m_1 = -\Omega_1+\Sigma$, $m_2 = \Omega_2-\Sigma$, $m'_1 = \Omega_1+\Sigma$ and $m'_2 = -\Omega_2-\Sigma$, we get

$$\sum S(ll'L)C_0C_3 = 4\pi\int Y^*_{l'_1\Omega_1+\Sigma}Y^*_{l'_2(-\Omega_2-\Sigma)} \times Y_{l_1(-\Omega_1+\Sigma)}Y_{l_2(-\Omega_2+\Sigma)}\,\mathrm{d}\Omega, \tag{A.11}$$

since $\Omega_1 = \Omega_2$. Now applying the relation

$$Y^*_{lm}(\theta,\phi) = (-)^m Y_{l(-m)}(\theta,\phi) \tag{A.12}$$

to the 2nd and 4th factors on the right in (A.11) we obtain

$$\begin{aligned}\sum S(ll'L)C_0C_3 &= 4\pi\int Y^*_{l'_1\Omega_1+\Sigma}(-)^{-\Omega_2-\Sigma}Y_{l'_2\Omega_2+\Sigma}Y_{l_1(-\Omega_1+\Sigma)} \\ &\quad\times(-)^{\Omega_2-\Sigma}Y^*_{l_2(-\Omega_2+\Sigma)}\,\mathrm{d}\Omega \\ &= (-)^{2\Sigma}4\pi\int Y^*_{l'_1\Omega_1+\Sigma}Y^*_{l'_2(-\Omega_2+\Sigma)}Y_{l_1(-\Omega_1+\Sigma)} \\ &\quad\times Y_{l_2\Omega_2+\Sigma}\,\mathrm{d}\Omega;\quad \text{interchanging } l_2 \text{ and } l'_2 \\ &= -\sum S(ll'L)C_0C_4,\end{aligned} \tag{A.13}$$

since $\Sigma = \pm 1/2$. Substituting (A.13) in (A.7) and (A.8), we get

$$B_0 = 0, \tag{A.14}$$

$$B_\sigma = \sum S(ll'L)\,F^0C_0[2a_1^-\,a_2^-\,(A_1^-\,A_2^-\,C_2 - A_1^+\,A_2^+\,C_3)]. \tag{A.15}$$

Hence the spin-independent (Wigner) part does not contribute to the odd-even shift; only the spin-dependent part contributes.

Now, subtracting (A.6) from (A.5), we have

$$A_0 - A_\sigma = \sum S(ll'L)F^0C_0[2a_1^-\,a_2^-\,(A_1^-\,A_2^-\,C_2 - A_1^+\,A_2^+\,C_3)]. \tag{A.16}$$

Comparing (A.15) and (A.16), we get

$$B_\sigma = A_0 - A_\sigma. \tag{A.17}$$

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