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Exact potential minimization for a supergravity model

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Abstract. The low energy effective scalar potential arising from the supergravity model proposed by Nilles, Srednicki and Wyler is minimized exactly. Bounds are derived for the parameters of the theory from the requirement that SU(2) × U(1) be broken at the tree level. These results support earlier approximate results.

Keywords. Weinberg-Salam theory; supergravity; spontaneous symmetry breakdown.

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1. Introduction

The past few years have seen a lot of activity in the construction of realistic models of elementary-particle interactions based on $N = 1$ supergravity (Ellis 1984; Hall 1984; Arnowitt *et al* 1983). In this class of theories, supersymmetry is broken by gravitational effects. The effective Lagrangian at low energies contains some soft supersymmetry-breaking terms whose structure depends on the choice of the Kahler potential and the superpotential of the original super-gravity theory. These terms can lead to the breaking of electroweak SU(2) × U(1) at the tree level (Nilles *et al* 1983) or through one-loop radiative effects (Ellis 1984).

For the case of 'minimal' coupling of supergravity to matter, the smallest set of fields needed to break SU(2) × U(1) at the tree level consists of two doublets H, H' (with hypercharge $+\frac{1}{2}$, $-\frac{1}{2}$ respectively) and a singlet Y. If we assume that the theory possesses no dimensionful parameters, the low-energy superpotential g is essentially unique (Nilles *et al*-1983):

$$g = \lambda HH'Y + \tfrac{1}{3}\sigma Y^3. \tag{1}$$

The effective scalar potential V_{eff}, obtained after elimination of super-Higgs fields, can be written solely in terms of g, the gravitino mass $m_{3/2}$ and a numerical parameter A. The classical ground state is obtained by minimizing V_{eff} with respect to the fields. In the literature, this minimization has been performed approximately. This may be adequate for many purposes, but considering the theoretical importance of the model, an exact solution is clearly desirable.

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Here we present details of the minimization of the V_{eff} arising from (1). Our results are exact (except for some special values of λ and σ; see §2), and reproduce the approximate results quoted in the literature. The content of the paper is essentially technical. In §2 we enumerate the various possible solutions to the extremum conditions. The masses of the physical scalar particles are worked out in §3, where we obtain bounds on the value of A from the requirement of an acceptable symmetry-breaking pattern. Section 4 contains some concluding remarks.

2. Extremization of the effective scalar potential

The low energy effective potential V_{eff} for the theory with g as in (1) is given by (Nilles *et al* 1983)

$$V_{\text{eff}} = |\lambda|^2(|H|^2+|H'|^2)|Y|^2+|\lambda HH'+\sigma Y^2|^2$$
$$+m_{3/2}^2(|H|^2+|H'|^2+|Y|^2)+m_{3/2}[A(\lambda HH'Y+\tfrac{1}{3}\sigma Y^3)+\text{c.c.}]. \tag{2}$$

Writing $H_1'^0$ (H_2^0) for the upper (lower) components of $H'(H)$, the neutral fields are expanded as follows:

$$H_2^0 = \langle H_2^0\rangle+\frac{a_1+ib_1}{\sqrt{2}}, \qquad H_1^{0\prime} = \langle H_1'^0\rangle-\frac{a_2-ib_2}{\sqrt{2}}$$
$$Y=\langle Y\rangle+\frac{a_3+ib_3}{\sqrt{2}}. \tag{3}$$

We introduce dimensionless variables h and y by

$$\langle H_2^0\rangle = \pm\langle H_1'^0\rangle = \lambda^{-1}m_{3/2}h \qquad \langle Y\rangle = \lambda^{-1}m_{3/2}y. \tag{4}$$

The equality of $\langle H_2^0\rangle$ and $\langle H_1'^0\rangle$ upto a sign follows from the symmetry $H\leftrightarrow H'$ of the potential. The SU(2) × U(1) symmetry can be used to choose h and y to be real.

At an extremum, V_{eff} has a value obtained by replacing fields by their VEV's:

$$V_0 = \lambda^{-2}m_{3/2}^4\hat{V}_0$$
$$\hat{V}_0 = 2h^2y^2+(\pm h^2+\sigma'y^2)^2+2h^2+y^2+2A(\pm h^2y+\tfrac{1}{3}\sigma'y^3), \tag{5}$$

where $\sigma' = \sigma/\lambda$. The undetermined sign can be absorbed by the redefinition $\pm A\to A$, $\pm\sigma'\to\sigma'$ whereupon σ' can have either sign. In the following it is necessary to distinguish two cases: (I) $\sigma' = 0$ and (II) $\sigma'\neq 0$. The equations

$$\partial\hat{V}_0/\partial h = \partial\hat{V}_0/\partial y = 0$$

determine all extrema. They clearly admit of the trivial solution $h = y = 0$ with $V_0 = 0$. In full, the equations are

$$h(h^2+1+y^2+\sigma'y^2+Ay) = 0 \tag{6a}$$

$$[2y^2+Ay+1][(1+2\sigma')y+A] = 0 \qquad \text{for} \qquad h\neq 0. \tag{6b}$$

leading to the following non-trivial solutions:

Case I $(a_\pm)$ $h^2 = y^2 = \Delta_\pm^2,$

$$\hat{V}_0 = -\Delta_\pm^2(\Delta_\pm^2 - 1). \tag{7}$$

Case II $(a_\pm)$ $h^2 = (1-\sigma')y^2,\quad y^2 = \Delta_\pm^2,$

$$\hat{V}_0 = -\left(1 - \frac{2\sigma'}{3}\right)\Delta_\pm^2(\Delta_\pm^2 - 1); \tag{8a}$$

$(b_\pm)$

$$h^2 = 0,\qquad |y| = |\Delta_\pm/\sigma'|, \tag{8b}$$

$$\hat{V}_0 = -\frac{1}{3\sigma'^2}\Delta_\pm^2(\Delta_\pm^2 - 1),$$

(c) $h^2 = \sigma' y^2 - 1,\qquad y = -A/(1+2\sigma'),$

$$\hat{V}_0 = -\frac{2\sigma'}{3}\frac{A^4}{(1+2\sigma')^3} - \frac{A^2}{1+2\sigma'} - 1. \tag{8c}$$

Here $\Delta_\pm = \frac{1}{4}[|A| \pm (|A|^2 - 8)^{1/2}]$. The solutions given by (7) and (8a, b) exist only if $|A| \geqslant 2\sqrt{2}$. Which of the various extrema is the true classical vacuum depends on the values of σ' and A. We discuss this in detail in the next section.

3. Symmetry breaking

Of the various solutions to (6a) and (6b), the one with the lowest value of $\hat{V}_0$ (i.e. the absolute minimum) is to be identified as the classical vacuum. We now require that $SU(2) \times U(1)$ be broken to $U(1)$ at the tree level, i.e. we seek the conditions under which a symmetry-breaking extremum is the absolute minimum of V_{eff}. These conditions are obtained in the form of (σ'-dependent) bounds on the parameter A.

An absolute minimum must be a local minimum, i.e. eigenvalues of the scalar (mass)^2 matrices must be positive definite. The relevant mass terms are:

$$\tfrac{1}{2}m_{3/2}^2(A^T M_a^2 A + B^T M_b^2 B),$$

where

$$A = \begin{pmatrix}(a_1 - a_2)/\sqrt{2}\\ a_3\end{pmatrix}\qquad B = \begin{pmatrix}(b_1 + b_2)/\sqrt{2}\\ b_3\end{pmatrix},$$

$$\begin{aligned}
&(M_a^2)_{11} = 2h^2;\qquad (M_b^2)_{11} = 2(1 + y^2 + h^2),\\
&(M_a^2)_{12} = \sqrt{2}h(2y(1+\sigma') + A);\qquad (M_b^2)_{12} = \sqrt{2}h(2\sigma' y - A),\\
&(M_a^2)_{22} = 2h^2(1+\sigma') + 6y^2\sigma'^2 + 1 + 2Ay\sigma',\\
&(M_b^2)_{22} = 2h^2(1-\sigma') + 2y^2\sigma'^2 + 1 - 2Ay\sigma'.
\end{aligned} \tag{9}$$

Of the remaining scalar field combinations $\sqrt{2}(b_1 - b_2)$ and $\sqrt{2}\ (H_1 + H_2'^*)$ are 'eaten up' by the Z^0 and W^+ respectively, while $\sqrt{2}\ (a_1 + a_2)$ and $\sqrt{2}\ (H_1 - H_2'^*)$ correspond respectively to neutral and charged eigenstates of mass.

Let us discuss in detail the symmetry breaking solutions in (7) and (8).

Case I. The necessary and sufficient condition which breaks SU(2) × U(1) is clearly

$$|A| > 3. \tag{10}$$

The corresponding acceptable solution is a_+ of (7). The scalar masses are all determined from

$$\mathrm{Tr}\,M_a^2 = 1 + 4\Delta_+^2; \qquad \det M_a^2 = 2(2\Delta_+^4 + \Delta_+^2 - 1);$$
$$\mathrm{Tr}\,M_b^2 = 3(1 + 2\Delta_+^2); \qquad \det M_b^2 = 0.$$

and are given by $\sqrt{2}\,(1 + \Delta_+^2)^{1/2} m_{3/2}$, $(2\Delta_+^2 - 1)m_{3/2}$, $\sqrt{3} m_{3/2}\,(1 + 2\Delta_+^2)^{1/2}$ and 0. The vanishing of the last mass is expected from the Goldstone theorem because of the presence of an additional U(1) symmetry when $\sigma' = 0$. Note that $|A|$ is unbounded from above.

Case II. For $\sigma' < -\frac{1}{2}$, $|A| > 3$ is necessary and sufficient to break SU(2) × U(1). The acceptable solution is a_+ of (8a) i.e. $h^2 = (1 - \sigma')y^2 = (1 - \sigma')\Delta_+^2$. The scalar masses in this case are determined from

$$\mathrm{Tr}\,M_a^2 = 2\Delta_+^2\,(2 + 2\sigma'^2 - 3\sigma') + (1 - 2\sigma'),$$
$$\det M_a^2 = 2(1 - \sigma')\{2\Delta_+^4\,(1 - 2\sigma') + \Delta_+^2\,(1 + 2\sigma') - 1\},$$
$$\mathrm{Tr}\,M_b^2 = 6\Delta_+^2 + 3 + 2\sigma'(1 - \Delta_+^2 + 2\sigma'\Delta_+^2),$$
$$\det M_b^2 = 6\sigma'\{2\Delta_+^4\,(2\sigma' - 1) + (2\sigma' + 1)\Delta_+^2 + 1\}. \tag{11}$$

It is not difficult to see that for $\sigma' < -\frac{1}{2}$, $|A| > 3$ (i.e. $\Delta_+ > 1$) guarantees all scalar $(\text{mass})^2$ to be positive.

For $-\frac{1}{2} < \sigma' < 0$, SU(2) × U(1) cannot be broken and the global minimum of V_{eff} is solution (b_+) of (7). In the range $0 < \sigma' < 1$ we cannot identify which solution in (8) corresponds to the minimum. Finally for $\sigma' \geqslant 1$ a necessary condition for breaking SU(2) × U(1) is

$$|A| > (1 + 2\sigma')/\sigma'^{1/2}. \tag{12}$$

In this case, the acceptable solution corresponds to (7), viz.

$$y = -A/(1 + 2\sigma'), \qquad h^2 = \frac{\sigma' A^2}{(1 + 2\sigma')^2} - 1,$$

$$\hat{V}_0 = -\frac{2\sigma'}{3}\frac{A^4}{(1 + 2\sigma')^3} - \frac{A^2}{1 + 2\sigma'} - 1.$$

The $(\text{mass})^2$ matrix is more complicated in this case. We display exact results for the limiting case $\sigma' = 1$. Then (12) is just $|A| > 3$. The scalar masses are determined by

$$\mathrm{Tr}\,M_a^2 = \tfrac{2}{3}A^2 - 5, \qquad \det M_a^2 = \tfrac{2}{27}(A^2 - 9)^2,$$
$$\mathrm{Tr}\,M_b^2 = \tfrac{4}{3}A^2 + 1, \qquad \det M_b^2 = \tfrac{2}{9}(27 - A^2)A^2.$$

Clearly $|A| < \sqrt{27}$ to ensure det $M_b^2 > 0$. For the other values of σ' it is found that $|A|$ is bounded from above and consequently is greater than but of the order of $(1 + 2\sigma')/\sigma'^{1/2}$.

This concludes the summary of our results on the minimization of the potential in (2) in the cases that were analytically solvable.

We now briefly discuss the incorporation of leptons into the model considered here. This is done by modifying g as follows (Frere *et al* 1983).

$$g = \lambda HH'Y + \tfrac{1}{3}\sigma Y^3 + \lambda_e LH'\dot{E},$$

$$L = \begin{pmatrix} \nu \\ e^- \end{pmatrix}_L, \qquad E = e_L^c. \tag{13}$$

The price paid is the occurrence of the $U(1)_{e.m.}$ breaking minimum at

$$\langle Y \rangle = 0, \qquad \langle E \rangle = \langle e^- \rangle = \pm \langle H_1'^0 \rangle = \Delta_+.$$

The potential at this minimum is given by

$$V_0 = -\lambda_e^{-2} m_{3/2}^4 \Delta_+^2 (\Delta_+^2 - 1). \tag{14}$$

On the other hand, at the charge-preserving minimum we have from (7) and (8)

$$V_0 = -\lambda^2 m_{3/2}^4 \Delta_+^2 (\Delta_+^2 - 1) \qquad \text{(case I)},$$

$$V_0 \lesssim -\tfrac{4}{3}\sigma'^{-2} m_{3/2}^4 \Delta_+^4 \qquad (\text{case II}, \sigma \geqslant \lambda, |A| \gg 1).$$

We conclude that $\lambda \gtrsim \lambda_e$ is sufficient to make the charge-breaking minimum given by (14) the absolute minimum. This reproduces the results of Frere *et al* (1983) obtained by approximate minimization of V_{eff}. To get a physically meaningful symmetry-breaking pattern, one has to go beyond the model of (13). We have discussed this elsewhere (Joshipura *et al* 1984).

4. Concluding remarks

We have presented analytical results for the minimization of the V_{eff} of (2). Our results do not invalidate earlier approximate results, but support them. Thus our work is in the nature of making the earlier work technically complete. Considering the central role of the model studied here in supergravity model-building, this exercise seems not without value. We also hope that it will serve as an illustrative example. The complete results presented here can be used to discuss phenomenology of scalars when SU(2) × U(1) is broken at tree level.

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