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Feedback control of flute instability in mirrors by neutral beams

AVINASH
Plasma Physics Programme, Physical Research Laboratory, Ahmedabad 380 009, India

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Abstract. The feedback suppression of flute instabilities by neutral beam injection is studied. It is shown that the available neutral beam current in mirror is sufficient to simulate the min-B field and thereby keep the flute modes in control.

Keywords. Mirror machines; Neutral beam injection; feedback stabilization; flute modes; minimum-B; Loss-cone.

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1. Introduction

The first generation D-T mirror fusion reactors will function as an amplifier where the neutral beam injected power will be enhanced to give the total thermal output of the reactor. D-neutral beams are to be used in mirror machine reactors for density build-up and for raising the plasma temperature. The Q of a reactor may be defined as the ratio of fusion power output to injected power input (Ribe 1975). The injected power input may consist of neutral beam injected power and a significant fraction of reactor's electrical power output, which will be injected back in the reactor to energize the confining and stabilizing magnetic field coils. The recognized necessity of enhancing the Q of a mirror reactor has led to efforts to maximize the uses of available neutral beam power (Baldwin 1977). Here we propose a scheme by which the available neutral beam current in mirror reactors can be used to simulate the effects of min-B field and thereby keep the MHD and other drift instabilities in control. This scheme will thus dispense with the exigency of complex stabilizing fields which lead to a large incremental factor in the reactor size and cost considerations and may therefore enhance the Q.

The principle of injecting neutral beams to provide feedback-controlled volume sources of particle and momentum is already known (Chen and Furth 1969). It showed that such a scheme can be quite effective for low frequency plasma stabilization in reactors especially against drift instabilities. It was later shown that a similar scheme can be effective in suppressing trapped particle instabilities in Tokamaks (Sen and Sundaram 1976). The motivation of the present work is as follows. In mirror reactors, D-neutral beams will be injected perpendicular to the confining magnetic field. The neutral beam gets ionized by the mirror plasma to give rise to particle and momentum sources, which will bring about the density and temperature build-up of the fusion plasma. We wish to show that with an appropriate feedback modulation, these particle and momentum sources can also be used to quench the interchange instability of mirror plasmas. The strong momentum source provided by the neutral beam can be used to simulate the effects of the min-B field bringing about MHD stability.

The basic idea of the present stabilization scheme is to sense the growing perturbations of interchange instability by an independent system of sensors (optical or microwave beams) placed on the plasma surface. The signal is then amplified and phase-shifted by a calculated amount. With this signal an externally injected neutral beam is modulated which, on ionization from mirror plasma, gives particle and momentum sources in appropriate strength and phase to quench the interchange instability.

In §2 we have derived the modified dispersion relation for the interchange instability in the presence of these particle and momentum sources and have discussed various stabilization mechanisms. In §3 we have discussed the experimental feasibility of this scheme.

2. Calculations

For our theoretical model we consider a mirror geometry where $\hat{x}$-direction is the radial direction while $\hat{y}$ is the azimuthal direction. The plasma is imbedded in a mirror field $\mathbf{B} = B_0\hat{z}$. Because of the curvature of the field lines $\mathbf{R}$ it has an effective gravity $\mathbf{g}$ given by

$$\mathbf{g} = [v_\parallel^2 + \tfrac{1}{2}v_\perp^2]\frac{1}{R}\hat{x}, \tag{1}$$

acting in the $\hat{x}$-direction. The plasma has a density gradient given by

$$(1/n_0)(\mathrm{d}n_0/\mathrm{d}x) = 1/\lambda \tag{2}$$

in the direction opposite to that of $\mathbf{g}$, where n_0 is the equilibrium density of the plasma. We also assume an unstable interchange mode on the surface of the plasma as:

$$A = A_0 \exp[i\mathbf{k}\cdot\mathbf{y} - Wt] \tag{3}$$

where $\mathbf{k}$ and w are the wave number and frequency of the unstable mode. We study the stabilization of this unstable wave by an externally injected D-neutral beam in $\hat{x}$-direction. For simplicity we have taken the beam velocity in $\hat{y}$-direction to be zero.

As required by the feedback stabilization, we assume that the beam flux I_b per unit area, can be modulated arbitrarily in $\hat{y}$ direction and in time. We also recall that the neutral beam gets ionized by the target plasma to give particle and momentum sources. For beam energies $\geqslant$ 20 keV the dominant process is the proton ionization (Freeman and Jones 1974). In the equilibrium the particle source S maintains a constant plasma density by balancing the plasma losses from the open ends, that is

$$O = \frac{\mathrm{d}n_0}{\mathrm{d}t} = S - \left.\frac{\partial n_0}{\partial t}\right|_{\mathrm{E.L.}}, \tag{4}$$

where $S = I_{b0}\langle \sigma v \rangle / V_b$, $\left.\dfrac{\partial n_0}{\partial t}\right|_{\mathrm{E.L.}}$ represents end losses.

In (4) I_{b0} and V_b are the equilibrium beam flux and the beam velocity, while $\langle \sigma\cdot v \rangle$ is the rate coefficient for proton ionization. The plasma loss from the ends, represented by the second term in (4) is due to scattering of plasma particles into the loss-cone by electron-ion collisions. The electron-ion collision time is given by (Baldwin 1977)

$$\tau_{ie} = \frac{10^{13} \cdot T_e^{3/2}[\mathrm{keV}]}{n_0 \cdot \ln \Lambda}, \tag{5}$$

where $\ln \Lambda$ is the Spitzer's factor. Equations (4) and (5) together determine the equilibrium flux I_{b0} required to maintain an equilibrium plasma density n_0 as

$$I_{b0} = V_b \cdot n_0 [\langle \sigma \cdot v \rangle \cdot T_e^{3/2} \cdot 10^{12}]^{-1}. \tag{6}$$

This is of course assuming that the fluctuations due to microinstabilities have been stabilized by the motion of the target plasma along the field lines (Coensgen *et al* 1976). The momentum sources, on the other hand, cancel out in pairs. This is because beam injectors are placed in pairs at diametrically opposite points in the azimuthal directions around the plasma. And since plasma is azimuthally symmetric in equilibrium the momentum sources contributed by various injectors mutually cancel.

The first order perturbation in the source of the jth species S_j can be written as

$$I_{b1} n_{0j} \langle \sigma \cdot v \rangle V_b^{-1} + I_{b0} n_{1j} \langle \sigma \cdot v \rangle V_b^{-1}, \tag{7}$$

where I_{b1} and n_{1j} are the perturbed beam current density and the perturbed plasma density of the jth species respectively.

There also results a perturbed ion momentum source MV_bS_i where M is the ionic mass. We have neglected the electron momentum source as it is weak. In mirror machines the plasma transport across the magnetic field is brought about by the interchange flute mode which are on the surface of the plasma and our interest lies in stabilizing them. Hence the x-dependence of the source may be neglected.

We take the source for the jth species to be of the form

$$S_j = W_{fj} n_{1j}, \tag{8}$$

where n_{1j} is the density perturbation of the jth species. The gain and the phase θ of the feedback is given by the magnitude ($= |W|_{fj}$) and the argument of W_{fj} respectively. Since the electron and ion sources are caused by the ionization of the same neutral beam, we may take $W_{fe} = W_{fi} = W_f$.

To treat the interchange instability we use the standard two-fluid equations for the plasma after adding the sources of density and momentum as follows:

$$Mn_i \left[\frac{\partial n_i}{\partial t} + V_i \cdot \nabla V_i \right] = en_i \mathbf{E} + en_i \mathbf{V}_i \times \mathbf{B} + M \mathbf{V}_b S_i \tag{9}$$

$$0 = -en_e \mathbf{E} - en_e \mathbf{V}_e \times \mathbf{B} \tag{10}$$

$$\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e \mathbf{V}_e) = S_e, \tag{11}$$

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \mathbf{V}_i) = S_i. \tag{12}$$

In equilibrium we define a gravitational drift $\mathbf{V}_0 = -(g/\Omega_i)\hat{y}$. For simplicity the temperature effects are neglected. We consider electrostatic perturbations which vary as exp $i(\mathbf{K}\cdot\mathbf{Y}-Wt)$. In view of the low frequency of interchange modes, the approximations to be used are that of quasi-neutrality and the guiding centre approximation.

Equations (9) to (12) are linearized and combined in the usual fashion to give a quadratic equation for ω as

$$W^2-(KV_0+iW_f)W+g\frac{n_0'}{n_0}\left[1-\frac{V_bW_f}{V_0\Omega_i}\right]=0. \tag{13}$$

In the absence of the feedback i.e. $W_f=0$, (13) reduces to the usual quadratic equation for interchange mode which for $K^2V_0^2 \ll gn_0'/n_0$ grow absolutely ($W_R \ll W_i$) with a maximum growth rate $W_i=[gn_0'/n_0]^{1/2}$. In the presence of the feedback the two modes are

$$W=\tfrac{1}{2}(KV_0+iW_f)\pm\tfrac{1}{2}\left[(KV_0+iW_f)^2-4g\frac{n_0'}{n_0}\left(1-\frac{V_b}{V_0}\frac{W_f}{\Omega_i}\right)\right]^{1/2} \tag{14}$$

From this equation we find basically two different stabilizing mechanisms characterized by their phase differences which become important in different limits.

2.1 *Density smoothing*

This corresponds to the case when the neutral beam momentum V_b is very small i.e. $V_b \ll V_0$. Though such cases are not encountered in the present-day neutral beam injections, we still study them for the sake of completeness. To study this case we drop the momentum term from (14) ($V_b < V_0 \times \Omega_i/W_f$) in which case it becomes (after neglecting KV_0)

$$W \simeq i\frac{W_f}{2}\pm(-W_f^2-4gn_0'/n_0)^{1/2}. \tag{15}$$

Clearly the stabilizing phase angle in this case is $\theta=180°$. For gains $W_f^2 \gg 4gn_0'/n_0$ the two modes are

$$W_1=+i|W_f|[1+gn_0'/n_0\cdot W_f^2], \tag{16}$$

$$W_2=-i[gn_0'/n_0\cdot W_f^2]\cdot|W_f|. \tag{17}$$

By having sufficiently large gain W_f the growth time of the unstable mode $1/W_2$ can be made as long as desired, i.e. sufficiently longer than the desired confinement time.

The physical mechanism for the stabilization corresponding to this phase difference ($\theta=180°$) is the smoothing of the density perturbation by the neutral beam injected plasma. However this mechanism may be difficult to implement in practice because of the high gain requirements and small plasma replacement time τ_R as will be discussed shortly.

2.2 *Simulation of min-B field effects*

This corresponds to the case when the beam momentum is significant. i.e. $V_b \gg V_0$. This holds for the present day neutral beam injections. The stabilizing phase angle in this case is $\theta = 0$. The two modes are

$$W = \tfrac{1}{2}(KV_0 + iW_f) \pm \tfrac{1}{2}\left[(KV_0 + iW_f)^2 - \frac{4n_0'}{n_0}\left(1 - \frac{V_b}{V_0}\frac{W_f}{\Omega_i}\right)\right]^{1/2}. \quad (18)$$

From this equation it is easy to see that in the presence of the neutral beam power represented by V_b, there is an effective gravity g' given by

$$g' = g\left(1 - \frac{V_b}{V_0}\frac{W_f}{\Omega_i}\right). \quad (19)$$

Thus by an appropriate choice of beam parameter V_b and W_f the effective gravity g' can be made zero or its direction can be reversed. It is in this sense we say that min-B field has been simulated by the neutral beam power. The gain required to make $g' = 0$ is $W_f = V_0\Omega_i/V_b$. The two modes are

$$W_1 = 0, \quad W_2 = KV_0 + iW_f. \quad (20)$$

From (20) we see that the growth due to the interchange of plasma columns has been suppressed. However, one of the modes has a growth induced by the beam. This is because of the plasma dumped by the neutral beam. But this is of no concern as it is controllable. That is $W_f = V_0\Omega_i/V_b$ can be made as small as desired by having V_b sufficiently high.

The physical mechanism for the stabilization corresponding to this phase difference ($\theta = 0$) is the beam pressure against the crest of the density perturbation. This creates effects analogous to the effect of favourable gradients of zero-order magnetic field.

3. Results and discussion

To discuss the experimental feasibility we define a time called the plasma replacement time.

$$\tau_R = \frac{n_0}{n_1}\cdot\frac{1}{W_f}. \quad (21)$$

In applying neutral beam feedback technique to mirror reactors, care must be taken to ensure that the replacement time τ_R is sufficiently long compared to the desired confinement time τ. In this respect the first mechanism of density smoothing may not be quite feasible, because of the large gain requirements and consequently small τ_R. Besides in the present-day neutral beam injections, beam momentum is quite dominant. For this reason the second case of simulation of min-B field effects may be quite feasible. The τ_R in this case is $(n_0/n_1)(V_0\Omega_i/V_b)^{-1}$, which with $V_b \gg V_0$ can be made sufficiently long compared to the desired confinement time τ. It should be noted that by this method only one mode is suppressed at a time. Hence it should be utilized to suppress the mode causing maximum particle transport.

As a further indication of feasibility we may calculate the beam current requirement to stabilize a given level of fluctuations with a certain depth of modulation. The gain required for this purpose is $V_0\Omega_i/V_b$ which gives

$$S = W_f n_1 = \frac{V_0\Omega_i}{V_b} n_1.$$

Thus

$$S = \left[I_{b1} n_0 \frac{\langle \sigma \cdot v \rangle}{V_b} + I_{b0} n_1 \frac{\langle \sigma \cdot v \rangle}{V_b} \right], \tag{22}$$

which implies

$$I_{b0} = \frac{V_0 \Omega_i}{\langle \sigma \cdot V \rangle} \times \left[\frac{I_{b1}}{I_{b0}} \frac{n_0}{n_1} + 1 \right]^{-1} \frac{4 \cdot 8 \times 10^{-15}}{3} \frac{\text{Amp}}{\text{m}^2} \tag{23}$$

Thus the beam current density I_{b0} required to stabilize a given percentage of fluctuation n_1/n_0 with a certain depth of modulation can be obtained. It can be seen from (23) that higher the depth of modulation lesser is the beam current density requirements. Typically for present-day mirrors $V_0 \simeq 10^5$ cm/sec, $\langle \sigma \cdot v \rangle \sim 10^{-7}$, $\Omega_i = 10^7$ Hz. Now if we wish to stabilize 5% of fluctuations (i.e. $n_1/n_0 = 0{\cdot}05$) with a beam which has the same depth of modulation (i.e. $I_{b1}/I_{b0} = 0{\cdot}05$), then the beam current density requirement comes out to be 10^4 A/m^2 which is quite feasible with the neutral beam technology available today. It is interesting to note that this method of suppression places no constraint on the beam power.

This method is different from the dynamic feedback stabilization scheme given earlier (Avinash and Varma 1981) for the suppression of DCLC modes in mirrors, which required the introduction of material probes into the plasma to create modulated sources, because of which it was not found to be suitable for the stabilization of hot plasma. Since the present method does not require the introduction of material probes into the plasma it may be suitable for the stabilization of hot plasmas.

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