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Higher-order effects in large-angle coplanar symmetric $(e, 2e)$ processes at high energies

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Abstract. The triple differential cross-sections for the ionization of atomic hydrogen by fast electrons are considered in the case of a coplanar symmetric energy-sharing geometry. They are estimated in the modified Glauber (MG) approximation. It is found that MG results are significantly different from those in the second Born approximation only for $90° \leqslant \theta \leqslant 110°$. Outside this range they are almost identical.

Keywords. Ionization of hydrogen; triple differential cross-section; coplanar symmetric energy-sharing; large angle; higher order contribution; modified Glauber approximation.

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1. Introduction

The triple differential cross-sections (TDCS) measured in $(e, 2e)$ experiments provide the most sensitive probe for any model of single ionization by electron impact. In the case of coplanar asymmetric Ehrhardt-type geometry (Ehrhardt *et al* 1969), the crucial importance of second-order effects in accounting for all the main features (such as the angular positions, shapes and magnitudes of both the binary and recoil peaks) of the TDCS measurements, has been highlighted by Byron *et al* (1980 and 1982) and Ehrhardt *et al* (1982). In this geometry for a given incident electron of relatively high energy E_0 and momentum $\mathbf{k}_0$, a fast electron a of energy E_a and momentum $\mathbf{k}_a$ is detected at a small and fixed scattering angle θ_a in coincidence with a slow coplanar electron b of energy E_b and momentum $\mathbf{k}_b$. The coincidence rate is measured as a function of the scattering angle θ_b of the slow electron. On the other hand, in the coplanar symmetric kinematical arrangement, which is popular in $(e, 2e)$ spectroscopic studies, $E_a = E_b$, $\theta_a = \theta_b \simeq 45°$, $|\phi_a - \phi_b| = \pi$. It is found that in such a situation even the theories which are essentially first-order in character lead to reasonable results (McCarthy and Weigold 1976). However, for large $\theta_a (= \theta_b)$, it has been recently shown by Byron *et al* (1983) (in the case of hydrogen) and Pochat *et al* (1983) (in the case of helium) that the second-order Born term of the scattering amplitude becomes as important as and even more important than the first-order Born term. They have also shown that under these conditions the contribution of the second Born amplitude is governed by the initial and the final target states acting as intermediate states.

The present paper is aimed at investigating the contribution from still higher-order terms ($n > 2$) for large θ. The importance of these terms in the case of electron-induced excitations at large scattering angles has already been recognized. The calculation of higher-order terms is very difficult. However, a workable procedure which has been used in recent years with reasonable success, is the modified Glauber (MG) approximation (Byron and Joachain 1975; Gien 1976). Here the second-order term f_{G2} in the Glauber scattering amplitude f_G which is lacking in the off-shell intermediate scattering, is replaced by the second-order Born term. The MG scattering amplitude is given by

$$f_{\mathrm{MG}} = f_G + f_{B2} - f_{G2}. \tag{1}$$

The present authors (Baliyan and Srivastava 1985) recently used this approximation to analyze the coplanar asymmetric data of Lohmann *et al* (1984) for the ionization of hydrogen by 250 eV incident electrons. It is found to lead to better agreement with the measurements compared to the second Born approximation. This study is however limited to small θ_a and small E_b. Presently we are interested in the symmetric energy sharing and large $\theta_a (= \theta_b)$. One may object to the use of the Glauber approximation at large angles. However, several studies of elastic and inelastic scattering have shown that its predictions are quite satisfactory and the corresponding MG results show an improved agreement with the experimental data (see, for example, Gien 1979).

2. Calculation

We shall consider TDCS for the ionization of atomic hydrogen,

$$\frac{\mathrm{d}^3\sigma}{\mathrm{d}\Omega_a \mathrm{d}\Omega_b \mathrm{d}E_a} = \frac{k_a k_b}{k_0} |f_{\mathrm{MG}}|^2. \tag{2}$$

The amplitude f_G in (1) is evaluated by following the method of Roy *et al* (1981). The second Born amplitude f_{B2} is estimated by using the method of Byron *et al* (1983). It involves contributions coming from the ground and the continuum states as intermediate states. The procedure for evaluating f_{G2} is given in Baliyan and Srivastava (1985). Here we outline the main steps. The amplitude f_{G2} is given by Yates (1974):

$$f_{G2} = \frac{i}{\pi k_0} \int \frac{\mathrm{d}\mathbf{p}}{\mathbf{p}^2 |\mathbf{K} - \mathbf{p}|^2} \langle \phi_f | B(\mathbf{p}) B(\mathbf{K} - \mathbf{p}) \phi_i \rangle, \tag{3}$$

where $\mathbf{K} = \mathbf{k}_0 - \mathbf{k}_a$ and

$$B(\mathbf{p}) = 1 - \exp(i\mathbf{p} \cdot \mathbf{b}), \tag{4}$$

for a transition from the initial state ϕ_i to the final state ϕ_f. In the present case

$$\phi_i = \pi^{-1/2} \exp(-\lambda r), \quad \lambda = 1, \tag{5}$$

$$\phi_f = (2\pi)^{-3/2} \exp(-\pi\alpha/2) \Gamma\, 1 - i\alpha$$
$$\times \exp(i\mathbf{k}_b \cdot \mathbf{r})\, {}_1F_1(i\alpha, 1, -ik_b r - i\mathbf{k}_b \cdot \mathbf{r}), \quad \alpha = -1/k_b. \tag{6}$$

The cylindrical coordinates are used with the target-electron position vector written as $\mathbf{r} = \mathbf{b} + z\hat{\mathbf{z}}$, z-axis is taken perpendicular to the momentum transfer direction and $\mathbf{p}$ is a two-dimensional vector in the plane of $\mathbf{b}$. The matrix element in (3) is evaluated by expressing the confluent hypergeometric function in terms of the integral representation (Roy *et al* 1981):

$$ {}_1F_1(i\alpha, 1, z) = \frac{1}{2\pi i} \oint_C \mathrm{d}t\, t^{-1+i\alpha}(t-1)^{-i\alpha} \exp(zt). \tag{7} $$

The result is

$$ \begin{aligned} &\langle \phi_f | B(\mathbf{p}) B(\mathbf{K}-\mathbf{p}) | \phi_i \rangle \\ &= \frac{D}{2\pi i} \frac{\mathrm{d}}{\mathrm{d}\lambda} \oint_C \mathrm{d}t\, t^{-1-i\alpha}(t-1)^{i\alpha} \sum_{i=1}^{4} \frac{c_i}{A(\mathbf{u}_i) - B(\mathbf{u}_i)t} \\ &= D \frac{\mathrm{d}}{\mathrm{d}\lambda} \sum_{i=1}^{4} c_i \left(1 - \frac{B(\mathbf{u}_i)}{A(\mathbf{u}_i)}\right)^{i\alpha} \Big/ A(\mathbf{u}_i), \end{aligned} \tag{8} $$

where

$$ D = -i\, 2^{1/2} \Gamma\, 1 + i\alpha \exp(-\pi\alpha/2)/\pi, $$

$$ c_1 = c_2 = -c_3 = -c_4 = 1, $$

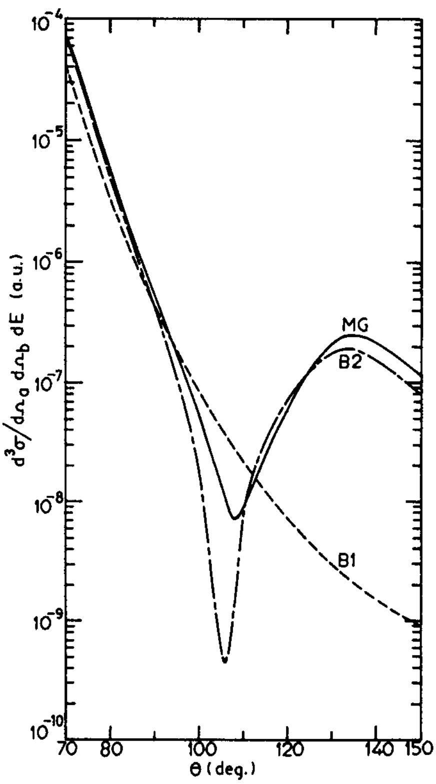

Figure 1. The triple differential cross section (in a.u.) for the ionization of atomic hydrogen for the case of coplanar symmetric energy-sharing geometry with $E_0 = 250$ eV, $E_a = E_b = 118{\cdot}2$ eV as a function of $\theta (= \theta_a = \theta_b)$. *B*1, first Born approximation; *B*2, second Born approximation; MG, modified Glauber approximation.

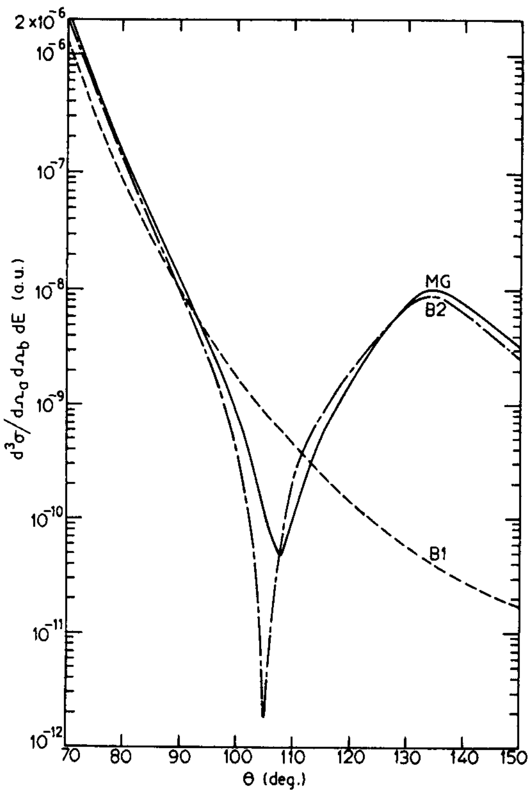

Figure 2. Same as for figure 1 but for $E_0 = 500$ eV, $E_a = E_b = 243{\cdot}2$ eV.

$$\mathbf{u}_1 = 0, \quad \mathbf{u}_2 = \mathbf{K}, \quad \mathbf{u}_3 = \mathbf{p}, \quad \mathbf{u}_4 = \mathbf{K} - \mathbf{p},$$

$$A(\mathbf{u}) = k_b^2 + \lambda^2 + \mathbf{u} \cdot (\mathbf{u} - 2\mathbf{k}_b),$$

$$B(\mathbf{u}) = 2(k_b^2 + i\lambda k_b - \mathbf{u} \cdot \mathbf{k}_b).$$

The integration over $\mathbf{p}$ in (3) is performed numerically.

3. Results

Figures 1 and 2 show our MG results along with those obtained in the first Born ($B1$) and the second Born ($B2$) approximations at incident electron energies $E_0 = 250$ eV and 500 eV respectively. Throughout this paper the first Born amplitude has been evaluated exactly while the second-order Born contribution has been estimated by using the large k_0 and large θ estimate given by Byron *et al* (1983). The three results differ significantly from one another for $\theta \gtrsim 90°$. In the region $\theta \simeq 90$–$110°$ different terms in (1) are of comparable magnitude and the dip in the $B2$ and MG results at $\theta \simeq 105°$ depicts the interference cancellation between them. Beyond $\theta \simeq 110°$, the $B2$ and MG results are almost identical. Here the first Born contribution has fallen down considerably and the higher order terms ($n > 2$) though important do not seem to contribute significantly.

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