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Phase shift analysis of hyperon-nucleon elastic scattering using optimized polynomial expansion techniques

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Abstract. A relatively stable method of phase shift analysis of hyperon-nucleon scattering proposed by us is applied to $\Sigma^+ p$ and Λp scattering. The analytic cut t-planes of analyticity of the helicity amplitudes are mapped into the interior of unifocal ellipses. The helicity amplitudes are then expressed as accelerated convergent expansions in the mapped variable. A definite economy is observed in the number of free parameters for fixed energy phase shift analysis of $\Sigma^+ p$ and Λp scattering at 40 and 100 MeV and 100 MeV respectively. Twenty six more phase shifts and coupling parameters corresponding to higher J values are also predicted.

Keywords. Hyperon nucleon elastic scattering; helicity amplitudes; polynomial expansion technique; phase shifts; coupling parameters.

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1. Introduction

Recently we proposed a possible stabilizing lever for the phase shift analysis of hyperon-nucleon (YN) scattering (Mohanty and Mohapatra 1985, referred to hereafter as paper I). However, little effort has been made to extract the phase shifts and coupling parameters of YN scattering primarily due to two reasons: (i) The number of free parameters that one is asked to handle is large (about 36) as one has to deal with atleast six helicity amplitudes; (ii) due to the short life time (Hauptman *et al* 1977) (10^{-10} sec) of hyperon beam the available error-affected data are very scanty. However, there have been earlier attempts to extract these parameters by using simple potential models (Bryan *et al* 1958; de Swart and Dullemond 1961; Nagels *et al* 1977, 1979) or by a multichannel ND^{-1} formalism in the framework of one-boson exchange model (Lettessier and Tounsi 1971). Some of the results earlier reported show that for these parameters, different sets of values, having equal reliability have been obtained. This, as discussed in paper I, is primarily due to the fact that the phase shift analysis of YN scattering is an ill-posed practical problem in the sense that with a few data points and large number of parameters it is always possible to squeeze in several sets of solutions into the error corridor (Ciulli 1973). An educated guess from the results of the potential models (Bryan *et al* 1958, deSwart and Dullemond 1961; Nagels *et al* 1977, 1979) is that such models perhaps will not be able to distinguish between these sets of solution. On the other hand, we have considered the fact that dynamical singularities of the scattering amplitude which reflect the nature of the forces responsible for the scattering are likely to contain information about the phase shifts and coupling parameters and thus we have used to our advantage the analytic structure of the helicity amplitudes for proposing a relatively stable procedure and estimate these parameters. In this paper we plan to apply the technique, proposed by us in paper I, to Σ^+p and Λp elastic scattering.

Section 2 contains a brief discussion of the results for Σ^+p scattering at energies 40 and 100 MeV while the results for Λp scattering at 100 MeV have been presented in § 3.

2. Σ^+p scattering

For Σ^+p scattering we have performed the phase shift analysis at two energies viz 40 and 100 MeV, following the mathematical and computational techniques described in paper I. No Coulomb corrections have been considered as they are negligible in this energy range (Lettessier and Tounsi 1971). The right hand and the left hand cuts for this scattering are given by

$$x_+ = 1 + 2M_p^2/k_{\Sigma^+}^2, \tag{1}$$

$$-x_- = -1 - (M_\Lambda + M_p)^2/k_{\Sigma^+}^2. \tag{2}$$

It was observed that at these two energies only two terms in each of the expansions were enough to give a good fit (figure 1) to the data, i.e. we had

$$\Phi_{iR} = a_{i0} + a_{i1}T_1(z), \tag{3}$$

$$\Phi_{il} = b_{i0} + b_{i1}T_1(z). \tag{4}$$

In fact we obtained χ^2/NDF equal to 0·11 and 0·14 in our fits at 40 and 100 MeV. Therefore at each of these energies we were using a total of 24 coefficients, which can

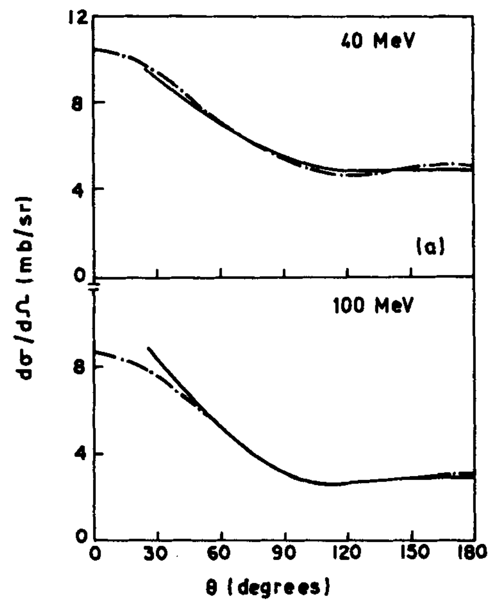

Figure 1. Differential cross-section curve at $E^{\Sigma}_{\text{lab}} = 40$ MeV and 100 MeV for Σ^+p–Σ^+p. (—·—) indicates our analysis while (——) indicates those of de Swart and Dullemond (1961).

be determined by only 12 phase shifts and coupling parameters. Thus effectively we are using only 12 free parameters. These 24 coefficients at each energy are given in table 1. The 12 phase shifts and coupling parameters (our input parameters) are given in table 2, along with the values of a few of the earlier workers for comparison (Bryan *et al* 1958; deSwart and Dullemond 1961; Lettessier and Tounsi 1971; Nagels *et al* 1977, 1979). We have taken care to convert the nuclear bar phase shifts and coupling parameters of Nagels *et al* (1977, 1979) to eigen phase shifts and coupling parameters (Pal 1982). Although the results of Lettessier and Tounsi (1971) and Nagels *et al* (1977, 1979) are not exactly for an incident Σ^+ lab energy of 40 and 100 MeV, yet they are very close and so we have included their results in table 2. It can be seen that there is some agreement between our values and the values obtained by the previous workers (Bryan *et al* 1958; deSwart and Dullemond 1961; Lettessier and Tounsi 1971; Nagels *et al* 1977, 1979). However, there are also cases where our magnitudes (i.e. $^3P_0^\Sigma$ and $^3\varepsilon_2^\Sigma$ at 40 MeV, $^1D_2^\Sigma$ and $^3\varepsilon_2^\Sigma$ at 100 MeV) as well as our signs (positive or negative) (i.e. $^3D_2^\Sigma$ and $^3\varepsilon_2^\Sigma$ at 40 MeV, $^3\varepsilon_2^\Sigma$ at 100 MeV) are quite different from theirs. One comment is in order here. The values of these parameters, which the various earlier workers obtained show significant variation even among themselves (Bryan *et al* 1958; deSwart and Dullemond 1961; Lettessier and Tounsi 1971; Nagels *et al* 1977, 1979) not only in magnitude but also sometimes in sign. This is to be expected (paper I) as such efforts necessarily

Table 1. Values of (a_{in}) and (b_{in}) obtained from an analysis of Σ^+p and Λp scattering.

	Coefficient (*n*)			
	a_{in}		b_{in}	
i	1	2	1	2
(a) Σ^+p scattering at 40 MeV				
1	1·9818	0·73349	−0·63045	0·33997
2	−0·13753	−1·1153	−0·62912	0·33406
3	−1·1342	−0·82918	0·30167	0·23012
4	−0·23595	−0·083147	0·2048	0·15649
5	0·38002	−0·34192	−0·081253	0·00114
6	0·38273	−0·34871	−0·079783	−0·018884
(b) Σ^+p scattering at 100 MeV				
1	1·3983	1·0679	−0·47812	0·36792
2	0·32845	−0·95801	−0·46962	0·36008
3	−0·93379	−0·62222	0·64175	0·43344
4	0·3508	0·34282	0·30543	0·23201
5	0·1282	−0·25593	−0·1527	−0·054761
6	0·11093	−0·25915	−0·12871	−0·033556

(c) Λp scattering at 100 MeV					
1	0·47548	0·27177	−0·00571	−0·56213	0·29289
2	−1·0317	0·2238	−0·42688	0·56211	0·2928
3	−0·21079	−0·19251	0·11837	0·14674	0·17468
4	−0·18116	0·11846	−0·18125	0·05416	0·07895
5	0·0739	−0·11781	0·0791	−0·56011	−0·03144
6	−0·0316	−0·6426	−0·0063	−0·05589	−0·03346

χ^2/NDF values for table 1a = 0·11, 1b = 0·14, and 1c = 0·28.

Table 2. Values (in degrees) for the 12 input phase shifts and coupling parameters.

Phase shifts and coupling parameters	Present analysis	Lettessier and Tounsi (1971)	Bryan *et al* (1958)	deSwart and Dullemond (1961)	Nagels *et al* (1977)
(a) 40 MeV $\Sigma^+ p$ scattering					
$^1S_0^\Sigma$	42·5	50·0	49·5	55·5	31·58
$^1P_1^\Sigma$	9·57	−15·0	10·5	11·0	26·18
$^3P_1^\Sigma$	−8·8	6·5	−6·9	−7·0	−5·09
$^1D_2^\Sigma$	0·001	−4·0	0·9	0·8	0·96
$^3D_2^\Sigma$	0·38	4·0	−0·8	−1·1	−1·21
$^3P_0^\Sigma$	39·0	9·0	9·4	10·7	8·8
$^3S_1^\Sigma$	−6·6	10·0	74·8	−17·3	−12·43
$^3\varepsilon_1^\Sigma$	18·2	7·0	−2·7	14·3	14·53
$^3D_1^\Sigma$	1·0	−2·0	−0·7	1·9	1·75
$^3P_2^\Sigma$	3·1	9·0	6·9	4·6	3·51
$^3\varepsilon_2^\Sigma$	34·73	11·5	−5·4	−13·7	−16·77
$^3F_2^\Sigma$	−1·9	0·2	−0·2	−0·2	−0·14
(b) 100 MeV of $\Sigma^+ p$ scattering					
$^1S_0^\Sigma$	36·7	44·0	32·2	36·4	14·6
$^1P_1^\Sigma$	17·2	−30·0	22·1	25·5	68·74
$^3P_1^\Sigma$	−21·1	15·0	−15·0	−13·7	−9·92
$^1D_2^\Sigma$	0·013	3·0	3·9	3·5	3·68
$^3D_2^\Sigma$	1·2	6·5	−4·0	−4·0	−2·95
$^3P_0^\Sigma$	27·0	17·5	17·9	10·5	8·14
$^3S_1^\Sigma$	−9·3	3·0	60·1	−28·3	−24·4
$^3\varepsilon_1^\Sigma$	17·5	18·0	−1·6	13·4	11·37
$^3D_1^\Sigma$	1·9	−9·0	−1·8	3·5	3·08
$^3P_2^\Sigma$	3·6	16·0	16·0	13·4	8·69
$^3\varepsilon_2^\Sigma$	35·24	19·0	−10·7	−13·7	−15·97
$^3F_2^\Sigma$	−2·4	−0·4	−1·4	−0·2	0·05

amount to extracting a large number of parameters from incomplete, error-affected data. However, we have brought the number of free parameters down merely to 12. Also, the fits obtained through this method are quite sensitive (paper I) to minute changes in the parameters. So we hopefully expect that our values for the 12 parameters are more reliable than those reported by earlier workers.

Since in the background of scanty available data our truncated series (3) and (4) are the best approximates of the actual helicity amplitudes of $\Sigma^+ p$ scattering at 40 and 100 MeV, they contain partial helicity amplitudes for all J values. So we have projected them out by using the orthogonality property of the reduced rotation matrix (paper I) and have obtained values for 26 more phase shifts and coupling parameters. Our predicted values are given in table 4. It is interesting to note that we have obtained

Table 3. Values (in degrees) for the 12 input phase shifts and coupling parameters at 100 MeV of Λp scattering.

Phase shifts and coupling parameters	Present analysis	Lettessier and Tounsi (1971)	deSwart and Dullemond (1961)	Nagels *et al* (1971)	Nagels *et al* (1979)
$^1S_0^\Lambda$	10·2	22·5	24·4	7·04	12·38
$^1P_1^\Lambda$	10·7	−7·0	13·0	9·03	−4·44
$^3P_1^\Lambda$	7·3	7·5	9·5	0·45	−2·19
$^1D_2^\Lambda$	1·24	2·0	0·6	2·39	1·95
$^3D_2^\Lambda$	1·37	1·0	1·7	2·44	2·88
$^3P_0^\Lambda$	12·0	18·0	−4·6	−1·39	−2·82
$^3S_1^\Lambda$	−7·5	32·0	−17·5	18·34	13·07
$^3\varepsilon_1^\Lambda$	−33·2	14·0	−10·7	18·63	−17·79
$^3D_1^\Lambda$	2·18	3·1	2·1	0·1	6·8
$^3P_2^\Lambda$	2·64	13·0	9·7	8·72	6·15
$^3\varepsilon_2^\Lambda$	3·7	3·5	3·1	−1·62	−2·71
$^3F_2^\Lambda$	−1·77	−0·3	0·0	0·20	0·22

values for some parameters ($^1\varepsilon_2$, $^1\varepsilon_3$, $^1\varepsilon_4$, $^1\varepsilon_5$, $^1\varepsilon_6$, $^1\varepsilon_7$) not reported by earlier workers (Bryan *et al* 1958; deSwart and Dullemond 1961) except Lettessier and Tounsi (1971) and Nagels *et al* (1977) (up to $J = 4$). The rest of our values compare favourably with those obtained by others (Bryan *et al* 1958; deSwart and Dullemond 1961; Lettessier and Tounsi 1971; Nagels *et al* 1977, 1979). Our values are hopefully more reliable because the helicity amplitudes constructed by us gave a fit with very low χ^2/NDF value.

3. Λp scattering

We have performed phase shift analysis of Λp scattering at 100 MeV using the same techniques as followed for $\Sigma^+ p$ scattering. The right hand and the left hand cuts for this scattering are given by

$$x_+ = 1 + 2M_p^2/k_\Lambda^2, \tag{5}$$

$$-x_- = -1 - 2(M_{\bar{\Lambda}} + M_{\bar{p}})^2/k_\Lambda^2. \tag{6}$$

In this case we needed three terms in the expansions for Φ_{iR} and two terms in the expansions of Φ_{iI} to obtain a good fit (χ^2/NDF = 0·28, figure 2) to the data, i.e. we had

$$\Phi_{iR} = a_{i0} + a_{i1}T_1(z) + a_{i2}T_2(z), \tag{7}$$

$$\Phi_{iI} = b_{i0} + b_{i1}T_1(z). \tag{8}$$

So at this energy we were using 30 coefficients. However, by using the relation between the R^J matrix and Φ^J (paper I), these 30 coefficients can also be determined by only 12

Table 4. Predicted values (in degrees) for phase shifts and coupling parameters.

Predicted values of phase shifts and coupling parameters	$(\Sigma^+ - p)$ 40 MeV	$(\Sigma^+ - p)$ 100 MeV	$(\Lambda - p)$ 100 MeV
(a) With 1 = J for $\Sigma^+ p$ and Λp scattering			
${}^1\varepsilon_2$	0·07	−0·37	0·007
${}^1\varepsilon_3$	3·6	0·84	0·23
${}^1F_3$	0·0	0·002	0·002
${}^1\varepsilon_4$	1·0	−3·0	0·24
${}^3F_3$	−0·19	−0·64	−0·14
${}^1G_4$	0·0	0·0	0·0
${}^1\varepsilon_5$	2·6	0·8	0·07
${}^3G_4$	0·11	0·38	0·08
${}^1H_5$	0·0	0·001	0·0
${}^1\varepsilon_6$	−3·9	−2·7	0·16
${}^3H_5$	−0·07	−0·25	−0·05
${}^3I_6$	−0·01	−0·002	0·001
${}^1\varepsilon_7$	−27·0	−8·4	−1·0
${}^3I_6$	0·07	0·18	0·04
(b) With 1 ≠ J for $\Sigma^+ p$ and Λp scattering			
${}^3D_3$	−1·2	−0·86	−0·62
${}^3\varepsilon_3$	4·3	12·2	7·1
${}^3G_3$	0·71	0·25	0·28
${}^3F_4$	0·74	0·84	0·56
${}^3\varepsilon_4$	5·6	4·7	3·3
${}^3H_4$	−0·39	−0·48	−0·36
${}^3G_5$	−0·48	−0·33	−0·24
${}^3\varepsilon_5$	6·6	15·0	9·2
${}^3I_5$	0·24	0·08	0·10
${}^3H_6$	−3·6	−4·3	−3·0
${}^3\varepsilon_6$	−2·8	−2·7	−2·6
${}^3J_6$	3·8	4·5	3·1

phase shifts and coupling parameters. Thus, again, we had effectively only 12 free parameters. These 30 coefficients are given in table 1c, and the 12 phase shifts and coupling parameters are given in table 3 along with the values of some earlier workers (Bryan *et al* 1958; deSwart and Dullemond 1961; Lettessier and Tounsi 1971; Nagels *et al* 1977, 1979). Here again, we have converted the nuclear bar phase shifts and coupling parameters of Nagels *et al* (1977, 1979) to eigen bar phase shifts and coupling parameters (Pal 1982). Their results are for 106·9 MeV lab energy which is close to the energy at which we have performed this analysis. So we include them in table 3 for comparison with our results. A general agreement is seen between our values and those of others although there are cases like ${}^3\varepsilon_1^\Lambda$ where our values differ in magnitude from earlier results. Our value of ${}^3F_2^\Lambda$ has a sign different from those of deSwart and Dullemond (1961) and Nagels *et al* (1977, 1979) but has the same sign as that of Lettessier and Tounsi (1971). Here again we have predicted values for 26 more phase shifts and coupling parameters (tables 4a and 4b).

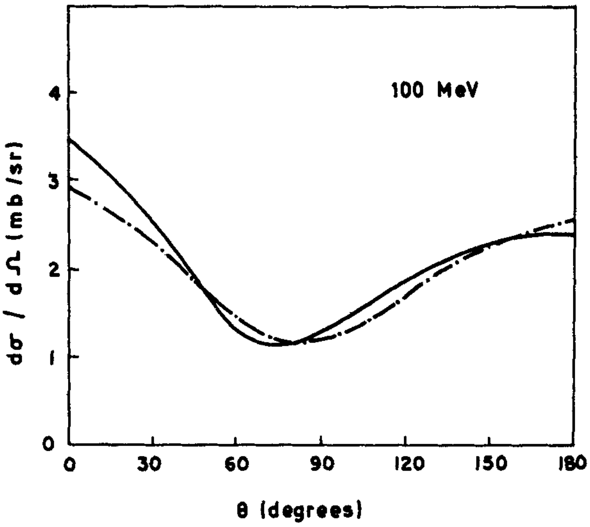

Figure 2. Differential cross-section curve at $E_{\rm lab}^{\Lambda} = 100$ MeV for Λp–Λp. Notations same as in figure 1.

4. Conclusion

In conclusion we note that by using a relatively stable method (Mohanty and Mohapatra 1985) we have, for the first time, performed a fixed energy phase shift analysis of $\Sigma^+ p$ and Λp scatterings by exploiting the analytic structure of the helicity amplitudes. Our low value of χ^2/NDF does show that our construction of helicity amplitudes is a reliable approximate of the actual helicity amplitudes. We have demonstrated considerable economy in the number of free parameters and hopefully this has decreased the ambiguity arising out of the use of a large number of input parameters. We have also been able to predict all other phase shifts and coupling parameters likely to be excited at those energies and it is to be hoped that the information storing and conveying capacity of the method make them quite reliable.

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