Pramāṇa – J. Phys., Vol. 27, No. 3, September 1986, pp. 383–391. © Printed in India.

A Study of the Higgs effect in the five-dimensional Kaluza-Klein theory

AMAR MAHESHWARI
International Centre for Theoretical Physics, Miramare, Trieste, Italy
Permanent address: Department of Physics, Regional College of Education, Mysore 570 006, India

MS received 19 February 1986

Abstract. The complete expression of the five-dimensional Einstein-Hilbert action as an expansion in fields in the Appelquist-Chodos parametrization of the Kaluza-Klein metric has been given in this paper. It is explicitly shown that a unitary gauge can be fixed in which in each of the charge sectors the vector and the scalar fields are absorbed as Goldstone modes leaving behind the Pauli-Fierz Lagrangian for massive charged spin-2 field.

Keywords. Higgs effect; Kaluza-Klein theory; spectrum analysis; unitary gauge.

PACS No. 04·50; 11·15; 12·10

1. Introduction

The five-dimensional Kaluza-Klein theory (Kaluza 1921; Klein 1926), although not a realistic theory, is typical of the higher dimensional models based on the Kaluza-Klein idea. It is sufficiently simple that it admits a complete analysis of its properties at least at the classical level. The spectrum analysis of the original five-dimensional (5D) theory has been given by Salam and Strathdee (1982) in their classic paper on the subject. In addition to the presence of the well-known zero mass modes, it was shown by Salam and Strathdee that the higher excitations describe an infinite tower of charged, massive, purely spin-2 particles. In an interesting paper, Dolan and Duff (1984) identified a Kac-Moody infinite parameter symmetry as the symmetry of the effective four-dimensional Lagrangian obtained by compactifying general relativity in five dimensions down to $M^4 \times S^1$. Dolan and Duff confirmed the spectrum analysis of Salam and Strathdee (1982) and rediscovered the SO(1,2) symmetry of the massive modes defined on the circle of the compactified dimension.

The Kaluza-Klein programme requires that after spontaneous compactification of the higher dimensions all fields including those contained in the higher dimensional metric should be expanded in harmonic expansion over the compactified space and an effective 4D theory to be obtained by integrating the harmonic functions. Although the programme is straightforward yet is difficult to carry out in practice because of the complexity of the procedure. The expression of the scalar curvature of the higher dimensional relativity explicitly appears in the literature only for the zero modes. Many interesting effects of Kaluza-Klein theories such as the mass generation in the tensor modes of each charge sector in the 5D theory by the absorption of vector and scalar

Goldstone bosons (Duff 1975) can be seen only by substituting harmonic expansions of the fields of the Kaluza-Klein ansatz in the 5D scalar curvature and integrating the Fourier coefficients defined with periodic boundary conditions on the circle of the compactified dimensions. The coupling of the U(1) gauge field belonging to the zero mass modes to the charged spin-2 excitations can be found only by carrying out this programme and may not turn out to be minimal.

In this paper we have given the expression of the 5D scalar curvature in terms of the tensor, vector and scalar fields as introduced by Appelquist and Chodos (1983) in their parametrization of the five-dimensional metric. This action is invariant under 5D general coordinate transformations. We then substitute the Fourier expansions of the tensor, vector and scalar fields in the action and integrate the dependence on the fifth coordinate. We show explicitly that there exists a unitary gauge in which in each of the charge sectors the vector and the scalar modes disappear by absorption into the tensor field and the bilinear part of the action reduces to the Pauli-Fierz (Pauli and Fierz 1939; Ogievetsky and Polubarinov 1965; Maheshwari 1972; Isham *et al* 1971) Lagrangian of a massive charged spin-2 field. The Higgs effects in sigma models coupled to gravity have also been independently investigated recently by Aulakh and Sahdev (1985).

In §2 we work out the curvature tensor for the 5D metric using the technique of horizontal lift basis (HLB) which has been used by Toms (1984) for writing the Lagrangian of the zero mass modes in Kaluza-Klein theories. In §3 we substitute the Fourier expansion of the fields in the 5D action and give expression of the bilinear fields in each charge sector to avoid the complication of coupling of fields belonging to different charge sectors, which are in the nonlinear interaction terms. We define the physical tensor field in terms of the tensor, vector and scalar fields of each charge sector and show explicitly that the vector and the scalar fields disappear as Goldstone modes from the action. In the Appendix we have given expressions of the 5D connection coefficients which have been used in calculating the expression of the scalar curvature density.

2. The Kaluza-Klein Lagrangian

To proceed, we parametrize the metric in the form (Appelquist and Chodos 1983; Unz 1985)

$$\hat{g}_{MN} = \phi^{-1/3}\begin{pmatrix} g_{\mu\nu} + \phi A_\mu A_\nu & \phi A_\mu \\ \phi A_\nu & \phi \end{pmatrix}, \tag{1}$$

and consider gravity in five dimensions described by the action

$$S = \frac{1}{L}\int \mathrm{d}^4x\,\mathrm{d}y\,(-\hat{g}_{MN})^{1/2}\,\hat{R}(\hat{g}_{MN}),$$

$$\equiv \frac{1}{L}\int \mathscr{L}_5\,\mathrm{d}^4x\,\mathrm{d}y. \tag{2}$$

We use coordinates $Z^M = (x^\mu, y)$ with $M = 1 \dots 5$ and $\mu = 1 \dots 4$. The 5D metric, tensors and connection coefficients will be distinguished by a caret on top of the corresponding variables in four dimensions constructed from the metric $g_{\mu\nu}$. We assume a ground state $M^4 \times S^1$, i.e. 4D Minkowski space times a circle of length L. The signature of the metric is $(-++++)$ and the 4D Minkowski metric tensor will be denoted by $\eta_{\mu\nu}$.

The metric components in (1) have been taken relative to the coordinate basis in which $\{dx^\mu, dy\}$ are the basis one-forms. The duals are $\{(\partial/\partial x^\mu), (\partial/\partial y)\}$ and form a basis for the tangent space. Following Toms (1984) we define a non-coordinate basis by the relations

$$\begin{aligned} dS^2 &= \hat{g}_{MN}\, dZ^M\, dZ^N \\ &= g_{\mu\nu}\phi^{-1/3}\, dx^\mu\, dx^\nu + \phi^{2/3}(dy + A_\mu\, dx^\mu)^2 \\ &= g_{\mu\nu}\theta^\mu\theta^\nu + \theta^5\theta^5, \end{aligned} \tag{3}$$

with

$$\theta^\mu = \phi^{-1/6}\, dx^\mu,$$

and

$$\theta^5 = \phi^{1/3}(dy + A_\mu\, dx^\mu). \tag{4}$$

In this basis the components of the metric tensor are given by

$$\hat{g}_{MN} = \begin{pmatrix} g_{\mu\nu} & 0 \\ 0 & 1 \end{pmatrix}. \tag{5}$$

The basis vectors e_μ and e_5, which are dual to θ^μ and θ^5, are

$$\begin{aligned} e_\mu &= \phi^{1/6}\frac{\partial}{\partial x^\mu} - \phi^{1/6}A_\mu\frac{\partial}{\partial y}, \\ e_5 &= \phi^{-1/3}\frac{\partial}{\partial y}. \end{aligned} \tag{6}$$

It is easy to check that

$$\langle \theta^\nu, e_\mu \rangle = \delta^\nu_\mu, \quad \langle \theta^5, e_\mu \rangle = 0, \quad \text{and} \quad \langle \theta^5, e_5 \rangle = 1. \tag{7}$$

The HLB is anholonomic and therefore the commutators of the basis vectors e_M are non-zero. The commutators of the basis vectors in the HLB fix the commutation-coefficients C^P_{MN}. By working out the commutators

$$\begin{aligned} [e_\mu, e_\nu] &= C^\lambda_{\mu\nu}e_\lambda + C^5_{\mu\nu}e_5, \\ [e_\mu, e_5] &= C^\lambda_{\mu 5}e_\lambda + C^5_{\mu 5}e_5, \end{aligned}$$

the C^P_{MN} are easily found to be given by the expressions

$$\begin{aligned} C^\lambda_{\mu\nu} &= \frac{1}{6}\phi^{-5/6}\left\{\left(\frac{\partial\phi}{\partial x^\mu}\delta^\lambda_\nu - \frac{\partial\phi}{\partial x^\nu}\delta^\lambda_\mu\right) - \frac{\partial\phi}{\partial y}(A_\mu\delta^\lambda_\nu - A_\nu\delta^\lambda_\mu)\right\}, \\ C^5_{\mu\nu} &= \phi^{2/3}\left\{-(\partial_\mu A_\nu - \partial_\nu A_\mu) + \left(A_\mu\frac{\partial A_\nu}{\partial y} - A_\nu\frac{\partial A_\mu}{\partial y}\right)\right\}, \\ C^\lambda_{\mu 5} &= -\frac{1}{6}\phi^{-4/3}\frac{\partial\phi}{\partial y}\delta^\lambda_\mu, \end{aligned}$$

$$C^5_{\mu 5} = \frac{1}{3}\phi^{-5/6}\left(A_\mu \frac{\partial \phi}{\partial y} - \frac{\partial \phi}{\partial x^\mu}\right) + \phi^{1/6}\frac{\partial A_\mu}{\partial y}, \tag{8}$$

$$C^P_{MN} = -C^P_{NM} \quad \text{and} \quad C_{MNP} = C^Q_{MN}\hat{g}_{QP}.$$

In an anholonomic basis the connection coefficients $\hat{\Gamma}_{MNP}$ are given by the following expression:

$$\hat{\Gamma}_{MNP} = \tfrac{1}{2}[e_P(\hat{g}_{MN}) + e_N(\hat{g}_{MP}) - e_M(\hat{g}_{NP}) + C_{MNP} + C_{MPN} + C_{PNM}], \tag{9}$$

and $\hat{\Gamma}^P_{MN} = \hat{g}^{PQ}\hat{\Gamma}_{QMN}$. The expression of the connection coefficients $\hat{\Gamma}^P_{MN}$ has been given in the Appendix. The curvature tensor in anholonomic basis is

$$\hat{R}^M_{PNQ} = e_Q(\hat{\Gamma}^M_{PN}) - e_N(\hat{\Gamma}^M_{PQ}) + \hat{\Gamma}^R_{PN}\hat{\Gamma}^M_{RQ} - \hat{\Gamma}^R_{PQ}\hat{\Gamma}^M_{RN} + \hat{\Gamma}^M_{PR}C^R_{NQ}. \tag{10}$$

Since we are interested in the scalar curvature, it can be calculated in any convenient basis. In the HLB because of the simplicity of the form of the metric tensor, equation (5), the scalar curvature can be obtained from the following two terms of the curvature tensor:

$$\hat{R} = g^{\nu\sigma}\hat{R}^\mu_{\nu\mu\sigma} + 2\hat{R}^\mu_{5\mu 5}. \tag{11}$$

Using the connection coefficients $\hat{\Gamma}^M_{PQ}$, which have been listed in the Appendix, and equations (8)–(11) the expression for the scalar $(-g_{MN})^{1/2}\hat{R}$ can be obtained by carrying out a straightforward calculation. Care has to be taken to discard terms which are either total four-divergence with respect to x^μ or a total derivative w.r.t. y so that an expression which consists of terms that are explicitly scalars both with respect to the 4D general coordinate transformations and the general coordinate transformations on the fifth coordinate can be obtained. In the following expressions the covariant derivatives indicated by a semicolon are with respect to the 4D metric $g_{\mu\nu}$ and the tensor indices are the 4D space-time indices. The Kaluza-Klein Lagrangian has been written as a sum of three parts; the first consists of terms bilinear in derivatives w.r.t. the space-time coordinates x^μ (this is the expression of the zero mode Lagrangian in which the metric components are assumed to be independent of the fifth coordinate y); the second consists of terms with one derivative w.r.t. x^μ and the other w.r.t. y; and the third consists of terms bilinear in derivatives w.r.t. to the fifth coordinate y. The second and the third type terms are a new result of this paper. The Kaluza-Klein Lagrangian is

$$\begin{aligned}
\mathscr{L}^{(5)} &= \sqrt{-g}\,[R + \tfrac{1}{4}\phi F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{6}\phi^{-2}\phi_{,\nu}\phi^{,\nu}] \\
&+ \sqrt{-g}\left[\frac{3}{2}A^\nu_{;\nu}g^{\lambda\tau}\frac{\partial g_{\lambda\tau}}{\partial y} + A_{\lambda;\tau}\frac{\partial g^{\lambda\tau}}{\partial y} - \frac{1}{2}A_{\nu;\sigma}g^{\nu\sigma}g^{\lambda\tau}\frac{\partial g_{\lambda\tau}}{\partial y}\right. \\
&\left. - \phi^{-1}\frac{\partial A_\lambda}{\partial y}\phi^{,\lambda} - \frac{1}{3}\phi^{-2}\frac{\partial \phi}{\partial y}A^\sigma\phi_{,\sigma} - \frac{1}{2}\phi F^{\sigma\lambda}\left(A_\sigma\frac{\partial A_\lambda}{\partial y} - A_\lambda\frac{\partial A_\sigma}{\partial y}\right)\right] \\
&+ \sqrt{-g}\left[\phi^{-1}\left\{\frac{1}{2}\frac{\partial \phi}{\partial y}\frac{\partial}{\partial y}(A_\lambda A^\lambda) + \frac{1}{2}\frac{\partial \phi}{\partial y}A^\lambda A^\tau\frac{\partial g_{\lambda\tau}}{\partial y}\right.\right. \\
&\left.\left. + \frac{1}{4}g^{\sigma\nu}g^{\lambda\tau}\frac{\partial g_{\sigma\lambda}}{\partial y}\frac{\partial g_{\nu\tau}}{\partial y} - \frac{1}{4}g^{\sigma\nu}g^{\lambda\tau}\frac{\partial g_{\sigma\nu}}{\partial y}\frac{\partial g_{\lambda\tau}}{\partial y}\right\}\right.
\end{aligned}$$

$$+\phi^{-2}\left\{\frac{1}{6}\left(\frac{\partial\phi}{\partial y}\right)^2 A^\sigma A_\sigma + \frac{1}{2}\frac{\partial\phi}{\partial y} g^{\sigma\nu}\frac{\partial g_{\sigma\nu}}{\partial y}\right\} - \frac{1}{3}\phi^{-3}\left(\frac{\partial\phi}{\partial y}\right)^2$$

$$+\frac{1}{2}\phi\left\{A^\mu A_\mu g^{\sigma\nu}\frac{\partial A_\sigma}{\partial y}\frac{\partial A_\nu}{\partial y} - A^\lambda A^\sigma \frac{\partial A_\lambda}{\partial y}\frac{\partial A_\sigma}{\partial y}\right\}$$

$$+\left\{\frac{1}{4}A^\sigma A_\sigma\left(g^{\lambda\beta}g^{\sigma\nu}\frac{\partial g_{\sigma\lambda}}{\partial y}\frac{\partial g_{\beta\nu}}{\partial y} - g^{\mu\nu}g^{\lambda\tau}\frac{\partial g_{\mu\nu}}{\partial y}\frac{\partial g_{\lambda\tau}}{\partial y}\right)\right.$$

$$+\frac{1}{2}\left(\frac{\partial}{\partial y}(A^\sigma A^\lambda)\frac{\partial g_{\sigma\lambda}}{\partial y} - \frac{\partial}{\partial y}(A^\sigma A_\sigma)g^{\lambda\tau}\frac{\partial g_{\lambda\tau}}{\partial y}\right)$$

$$\left.\left.+\frac{1}{2}A^\sigma A^\lambda g^{\tau\alpha}\frac{\partial g_{\alpha\sigma}}{\partial y}\frac{\partial g_{\tau\lambda}}{\partial y}\right\}\right], \tag{12}$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu.$$

The terms which are bilinear in derivatives w.r.t. y are the mass terms which can define a Higgs potential (Higgs 1964) for the spontaneous breakdown of the general covariance. Although this Lagrangian has a formidable appearance we show in the next section that after substituting the Fourier expansions of fields one can see another Kaluza-Klein miracle by going to a unitary gauge.

3. Spectrum analysis in unitary gauge

The next step to be carried out in the Kaluza-Klein programme is to Fourier expand the fields:

$$h_{\mu\nu}(x, y) = g_{\mu\nu} - \langle g_{\mu\nu}\rangle,\ A_\mu(x, y),$$

and

$$\Phi(x, y) = \phi - \langle\phi\rangle,$$

where the ground state manifold $M^4 \times S^1$ is determined by the vacuum expectation values

$$\langle g_{\mu\nu}\rangle = \eta_{\mu\nu}, \quad \langle\phi\rangle = 1. \tag{13}$$

In terms of the length L of the circle S^1 the Fourier expansions can be written as

$$h_{\mu\nu}(x, y) = \sum_{n=-\infty}^{\infty} h_{\mu\nu}^{(n)}(x)\exp(i2\pi ny/L),$$

$$A_\mu(x, y) = \sum_{n=-\infty}^{\infty} A_\mu^{(n)}(x)\exp(i2\pi ny/L),$$

$$\Phi(x, y) = \sum_{n=-\infty}^{\infty} \Phi^{(n)}(x)\exp(i2\pi ny/L). \tag{14}$$

with the reality conditions

$$h_{\mu\nu}^{(-n)}(x) = h_{\mu\nu}^{(n)*}(x), \quad A_\mu^{(-n)}(x) = A_\mu^{(n)*}(x), \quad \Phi^{(-n)}(x) = \Phi^{(n)*}(x). \tag{15}$$

Since

$$\int_0^L \exp\left(\frac{i2\pi}{L}(n+n')y\right)\mathrm{d}y = L\delta_{n+n',0}, \tag{16}$$

in the bilinear approximation to the 4D action different charge sectors are orthogonal to each other. Therefore we write the expression for the 4D Lagrangian $\mathscr{L}_4$ in terms of the fields $h_{\mu\nu}^{(n)}(x)$, $A_\mu^{(n)}(x)$ and $\Phi^{(n)}(x)$ of a non-zero charge sector, $n \neq 0$ and drop the superscript (n) from the 4D fields. By substituting the Fourier expansions, (14), in the Kaluza-Klein Lagrangian, (12), we find that the following terms contribute to charge n sector†

$$\begin{aligned}
\mathscr{L}_4^{(n)} &= \left(\int_0^L \mathscr{L}_5\,\mathrm{d}y\right)^{(n)} \\
&= [\{\tfrac{1}{2}h^{*\sigma\nu}_{,\lambda}\, h_{\sigma\nu}^{,\lambda} - h^{*\sigma\nu}_{,\nu} h_{\lambda\sigma}^{,\lambda} + \tfrac{1}{2}h^{*\sigma\nu}_{,\nu} h^{\lambda}_{\lambda,\sigma} \\
&\quad + \tfrac{1}{2}h^{\sigma\nu}_{,\nu} h^{*\lambda}_{\lambda,\sigma} - \tfrac{1}{2}h^{*\sigma}_{\sigma,\tau} h_{\lambda}^{\lambda,\tau}\} + \tfrac{1}{3}\Phi^{*,\mu}\Phi_{,\mu}] \\
&\quad + [A^{*\nu,\mu}A_{\nu,\mu} - A^{*\mu,\nu}A_{\nu,\mu} - ink\,(A^{\nu}_{,\nu}h^{*\lambda}_{\lambda} - A^{*\nu}{}_{,\nu}h^{\lambda}_{\lambda}) \\
&\quad + \frac{ink}{2}\,(h^{*\lambda\tau}(A_{\lambda,\tau} + A_{\tau,\lambda}) - h^{\lambda\tau}(A^{*}_{\lambda,\tau} + A^{*}_{\tau,\lambda})) \\
&\quad + ink\,(\Phi^{*}A^{\lambda}_{,\lambda} - \Phi A^{*\lambda}_{,\lambda}) + \frac{n^2k^2}{2}\,\{(h^{*\nu\tau}h_{\nu\tau} - h^{*\alpha}_{\alpha}h^{\beta}_{\beta}) \\
&\quad + (h^{*\alpha}_{\alpha}\Phi + h^{\alpha}_{\alpha}\Phi^{*}) - \tfrac{4}{3}\Phi^{*}\Phi\}],
\end{aligned} \tag{17}$$

where $k = 2\pi/L$.

We fix the unitary gauge by defining a tensor field $\psi_{\mu\nu}$, which absorbs the fields A_μ and Φ in $h_{\mu\nu}$, such that when the Lagrangian, (17), is expressed in terms of $\psi_{\mu\nu}$ the fields A_μ and Φ disappear completely like Goldstone fields leaving behind the Pauli-Fierz Lagrangian of charged massive spin-2 field $\psi_{\mu\nu}$. In the linear approximation the ansatz for $\psi_{\mu\nu}$ is

$$\psi_{\mu\nu} = h_{\mu\nu} - h^{\alpha}_{\alpha}\eta_{\mu\nu} + \eta_{\mu\nu}\left(\Phi - \frac{2i}{nk}A^{\alpha}_{,\alpha}\right) + \frac{i}{nk}(A_{\mu,\nu} + A_{\nu,\mu}), \tag{18}$$

and

$$h_{\mu\nu} = \psi_{\mu\nu} - \frac{1}{3}\eta_{\mu\nu}(\psi^{\alpha}_{\alpha} - \Phi) - \frac{i}{nk}(A_{\mu,\nu} + A_{\nu,\mu}). \tag{19}$$

It is a satisfying calculation to verify that on substituting the expression for the field $h_{\mu\nu}$ given in (19) on the right side of (17) all the terms containing A_μ disappear, and all the terms containing Φ including its kinetic and mass terms disappear except for one term in which Φ plays the role of an auxiliary field. The new expression of the Lagrangian is

$$\begin{aligned}
\mathscr{L}_4^{(n)} &= \tfrac{1}{2}[\psi^{*\sigma\nu}{}_{,\lambda}\psi_{\sigma\nu}{}^{,\lambda} - 2\psi^{*\sigma\nu}{}_{,\nu}\psi_{\lambda\sigma}{}^{,\lambda} \\
&\quad + \tfrac{1}{3}(\psi^{*\sigma\nu}{}_{,\nu}\psi^{\alpha}_{\alpha,\sigma} + \psi^{\sigma\nu}{}_{,\nu}\psi^{*\alpha}_{\alpha,\sigma}) \\
&\quad - \tfrac{1}{3}\psi^{*\alpha}_{\alpha,\lambda}\psi^{\beta,\lambda}_{\beta} + n^2k^2(\psi^{*\mu\nu}\psi_{\mu\nu} - \tfrac{1}{3}\psi^{*\alpha}_{\alpha}\psi^{\beta}_{\beta})] \\
&\quad + \tfrac{1}{3}[\psi^{*\sigma\nu}{}_{,\nu}\Phi_{,\sigma} + \psi^{\sigma\nu}{}_{,\nu}\Phi^{*}_{,\sigma}].
\end{aligned} \tag{20}$$

† In this expression the space-time tensor indices are Minkowskian and the Lagrangian is a scalar under Poincaré transformations.

It is the Pauli-Fierz Lagrangian for massive charged spin-2 field (Ogievetsky and Polubarinov 1965; Maheshwari 1972). To see whether the auxiliary field Φ interacts and to obtain the nature of the couplings of the field $\psi_{\mu\nu}$ we add the following source terms to $\mathscr{L}_4^{(n)}$:

$$\tfrac{1}{2}\psi^{*\mu\nu}(\theta_{\mu\nu}-\tfrac{1}{3}\theta^{\alpha}_{\alpha}\eta_{\mu\nu})+\Phi^{*}I+\text{c.c.} \tag{21}$$

The field equation for $\psi^{*\mu\nu}$ is easily seen to be

$$(\Box-m^2)\psi_{\mu\nu}=\theta_{\mu\nu}-\frac{1}{3m^2}(\eta_{\mu\nu}\Box-\partial_\mu\partial_\nu)\theta^{\alpha}_{\alpha}$$
$$+\tfrac{2}{3}(\eta_{\mu\nu}\Box-\partial_\mu\partial_\nu)\Phi-\frac{2}{3m^2}(\eta_{\mu\nu}\Box-\partial_\mu\partial_\nu)\Box\Phi. \tag{22}$$

This equation is compatible with the condition

$$\partial^\mu\psi_{\mu\nu}=0 \tag{23}$$

provided $\psi^{\mu\nu}$ is coupled to a conserved source (Ogievetsky and Polubarinov 1965; Isham *et al* 1971; Maheshwari 1972)

$$\partial_\mu\theta^{\mu\nu}=0. \tag{24}$$

If we define

$$P_{\mu\nu}=\tfrac{2}{3}(\eta_{\mu\nu}\Box-\partial_\mu\partial_\nu)\Phi, \tag{25}$$

it is identically conserved

$$\partial^\mu P_{\mu\nu}\equiv 0.$$

We can combine the $P_{\mu\nu}$ terms with $\theta_{\mu\nu}$ and define a new conserved source $\bar\theta_{\mu\nu}$,

$$\bar\theta_{\mu\nu}=\theta_{\mu\nu}+\tfrac{2}{3}(\eta_{\mu\nu}\Box-\partial_\mu\partial_\nu)\Phi, \tag{26}$$

the field equation (22) can then be expressed in the form

$$(\Box-m^2)\psi_{\mu\nu}=\bar\theta_{\mu\nu}-\frac{1}{3m^2}(\eta_{\mu\nu}\Box-\partial_\mu\partial_\nu)\bar\theta^{\alpha}_{\alpha}. \tag{27}$$

It has been observed before (Freund *et al* 1969) that the universal coupling of a pure massive spin 2 field is not to the energy momentum tensor but to a source $\Xi_{\mu\nu}$,

$$\Xi_{\mu\nu}\equiv\bar\theta_{\mu\nu}-\frac{1}{3m^2}(\eta_{\mu\nu}\Box-\partial_\mu\partial_\nu)\bar\theta^{\alpha}_{\alpha}. \tag{28}$$

The field equation for Φ^* gives the satisfying result that it is an auxiliary field and does not interact with the physical tensor field. It only contributes a term to the conserved source of $\psi_{\mu\nu}$. The equation for Φ^* is

$$\tfrac{1}{3}\psi^{\sigma\nu}_{,\sigma\nu}=I.$$

Since

$$\psi^{\sigma\nu}_{,\nu} = 0,$$

it requires that it cannot have a source I and is therefore only an auxiliary field as stated earlier. Although by working out $\partial \mathscr{L}_4 / \partial h_{\mu\nu}$ explicitly the contribution of the nonlinear terms to the field $\psi_{\mu\nu}$ in the unitary gauge can be obtained, the verification that under the revised ansatz also the A_μ and Φ fields would disappear from the Lagrangian is too tedious to be tried.

4. Conclusions

We have obtained the expression of the 5D Einstein-Hilbert action in the Kaluza-Klein ansatz of the metric tensor. On substituting the Fourier expansion of the fields in this action, it is explicitly verified that the zero charge sector consists of a graviton, a photon and Brans-Dicke scalar, and the non-zero charge sector consists of pure spin 2 fields only. The phenomenon of the absorption of charged vector and scalar fields in making a massive spin 2 field has been demonstrated. The calculation of the coupling of the vector field of the zero charge sector to the massive spin two field in the tower is under progress and will be reported elsewhere.

Acknowledgements

It is a pleasure to thank Professor John Strathdee for useful discussions and constructive comments on the manuscript. The author is also thankful to Professor Abdus Salam, the International Atomic Energy Agency and UNESCO for kind hospitality at the International Centre for Theoretical Physics, Trieste. Thanks are due to Drs J V Narlikar, N Mukunda and A K Sharma for their appreciation of my scientific needs and providing moral support. He is grateful to Dr P L Malhotra, National Council of Educational Research and Training, New Delhi for the grant of leave of absence that has made the visit to the ICTP possible.

Note added

After the completion of this work it has been brought to our notice that the expression of the Lagrangian of the 5D Kaluza-Klein theory using the Vielbein formalism has been obtained by Aulakh and Sahdev (1985).

Appendix

The expression of the 5D connection coefficients in terms of the 4D Christoffel symbols,

$$\Gamma^{\sigma}_{\mu\nu} = \frac{1}{2} g^{\sigma\lambda} \left(\frac{\partial g_{\lambda\mu}}{\partial x^{\nu}} + \frac{\partial g_{\lambda\nu}}{\partial x^{\mu}} - \frac{\partial g_{\mu\nu}}{\partial x^{\lambda}} \right)$$

and the fields $g_{\mu\nu}$, A_μ and Φ are given as follows:

$$\hat{\Gamma}^\sigma_{\nu\lambda} = \{\phi^{1/6}\Gamma^\sigma_{\nu\lambda} + \tfrac{1}{6}\phi^{-5/6}(\phi^{,\sigma}g_{\nu\lambda} - \phi_{,\nu}\delta^\sigma_\lambda)\}$$
$$+\left\{-\frac{1}{6}\phi^{-5/6}\frac{\partial\phi}{\partial y}(A^\sigma g_{\nu\lambda} - A_\nu\delta^\sigma_\lambda) - \frac{1}{2}\phi^{1/6}\left(g^{\sigma\mu}A_\lambda\frac{\partial g_{\mu\nu}}{\partial y}\right.\right.$$
$$\left.\left.+A_\nu g^{\sigma\mu}\frac{\partial g_{\mu\lambda}}{\partial y} - A^\sigma\frac{\partial g_{\nu\lambda}}{\partial y}\right)\right\},$$

$$\hat{\Gamma}^\sigma_{\nu 5} = \{-\tfrac{1}{2}\phi^{2/3}F^\sigma{}_\nu\} + \left\{\frac{1}{2}\phi^{-1/3}g^{\sigma\mu}\frac{\partial g_{\mu\nu}}{\partial y} + \frac{1}{2}\phi^{2/3}G^\sigma{}_\nu\right\},$$

$$\hat{\Gamma}^\sigma_{5\nu} = \{-\tfrac{1}{2}\phi^{2/3}F^\sigma{}_\nu\} + \left\{\frac{1}{2}\phi^{-1/3}g^{\sigma\mu}\frac{\partial g_{\mu\nu}}{\partial y} - \frac{1}{6}\phi^{-4/3}\frac{\partial\phi}{\partial y}\delta^\sigma_\nu + \frac{1}{2}\phi^{2/3}G^\sigma{}_\nu\right\},$$

$$\hat{\Gamma}^\sigma_{55} = \{-\tfrac{1}{3}\phi^{-5/6}\phi^{,\sigma}\} + \left\{\frac{1}{3}\phi^{-5/6}\frac{\partial\phi}{\partial y}A^\sigma + \phi^{1/6}g^{\sigma\mu}\frac{\partial A_\mu}{\partial y}\right\},$$

$$\hat{\Gamma}^5_{\mu\nu} = \{\tfrac{1}{2}\phi^{2/3}F_{\mu\nu}\} + \left\{-\frac{1}{2}\phi^{-1/3}\frac{\partial g_{\mu\nu}}{\partial y} + \frac{1}{6}\phi^{-4/3}\frac{\partial\phi}{\partial y}g_{\mu\nu} - \frac{1}{2}\phi^{2/3}G_{\mu\nu}\right\},$$

$$\hat{\Gamma}^5_{\mu 5} = \left\{\frac{1}{3}\phi^{-5/6}\frac{\partial\phi}{\partial x^\mu}\right\} + \left\{-\phi^{1/6}\frac{\partial A_\mu}{\partial y} - \frac{1}{3}\phi^{-5/6}\frac{\partial\phi}{\partial y}A_\mu\right\},$$

$$\hat{\Gamma}^5_{5\lambda} = 0,$$

$$\hat{\Gamma}^5_{55} = 0,$$

where $G_{\mu\nu} \equiv A_\mu\dfrac{\partial A_\nu}{\partial y} - A_\nu\dfrac{\partial A_\mu}{\partial y}$, and $F_{\mu\nu} \equiv \dfrac{\partial A_\nu}{\partial x^\mu} - \dfrac{\partial A_\mu}{\partial x^\nu}$.

References

Abdus Salam and Strathdee J 1982 *Ann. Phys.* **141** 316
Appelquist T and Chodos A 1983 *Phys. Rev.* **D28** 772
Aulakh C S and Sahdev D 1985 ICTP Trieste, preprint IC/85/4
Aulakh C S and Sahdev D 1985 ICTP Trieste, preprint IC/85/176 (to appear in *Phys. Lett.* B)
Dolan L and Duff M J 1984 *Phys. Rev. Lett.* **52** 14
Duff M J 1975 *Phys. Rev.* **D12** 3969
Freund P G O, Maheshwari A and Schonberg E 1969 *Astro. Phys.* **157** 857
Higgs P W 1964 *Phys. Rev. Lett.* **12** 132
Isham C J, Abdus Salam and Strathedee J 1971 *Phys. Rev.* **D3** 867
Kaluza Th. 1921 *Sitz. Preuss. Akad. Wiss. Berlin Math. Phys.* **K1** 966
Klein O 1926 *Z. Phys.* **37** 895
Maheshwari A 1972 *Nuovo Cimento* **A8** 319
Ogievetsky V I and Polubarinov I V 1965 *Ann. Phys.* **35** 167
Pauli W and Fierz M 1939 *Proc. R. Soc.* (*London*) **A173** 211
Toms D J 1984 *An introduction to Kaluza-Klein theories* (Singapore: World Scientific)
Unz R K 1985 SLAC preprint SLAC-PUB-3663